FLUID MECHANICS

Fluid mechanics is that branch of science which deals with the behaviour of the fluid (liquids or gases) at rest as well as in motion.

Thus this branch of science deals with the static, kinematics and dynamic aspects of fluid.

The study of fluids at rest is called fluid static.

The study of fluid in motion, where pressure force are not considered, is called fluid kinematics and if the pressure force are also considered for the fluid in motion, that branch of science is called fluid dynamics

* Properties of fluids

① Density or Mass density :-

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

The unit of mass density in SI unit is kg per cubic metre i.e. kg/m^3

The density of liquid may be considered as constant while that of gases changes with the variation of pressure and temp.

The value of density of water is $1\,gm/cm^3$ or $1000\,kg/m^3$

(ii) Specific Weight or Weight Density.

$$w = \frac{\text{weight of fluid}}{\text{Volume of fluid}}$$

$$w = \frac{(\text{Mass of fluid}) \times \text{deceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Vol. of fluid}}$$

$$= \rho \times g \quad \text{-----} \quad \because \frac{\text{Mass of fluid}}{\text{Vol. of fluid}} = \rho$$

$$\boxed{w = \rho g}$$

The value of specific weight or weight density for water is:-

$$9.81 \times 1000 \ N/m^3$$

(iii) Specific Gravity :-

Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid.

$$S(\text{for liquids}) = \frac{\text{Weight density of liquid}}{\text{Weight density of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density of gas}}{\text{Weight density of air}}$$

Note

{Specific gravity is also called relative density. It is dimensionless quantity.}

Thus, weight density of a liquid = $S \times$ weight density of water

$$= S \times 1000 \times 9.81 \ N/m^3$$

The density of liquid = $S \times$ Density of water

$$= S \times 1000 \ kg/m^3$$

If the specific gravity of a fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water.

For example, the specific gravity of mercury is 13.6, hence

density of mercury = $13.6 \times 1000 = 13600 \ kg/m^3$

10/1/2016

i) Introduction of fluid
ii) Types of fluid
iii) Types of flow
iv) Hydrostatic
v) Kinematic of fluid
vi) Dynamics of fluid
vii) Pipe flow
viii) Boundary layer formation
ix) Laminar & turbulent flow

Fluid

Fluid is defined as a substance which deforms continuously under the action of shear force or shear stress. No matter how small the shear stress may be. In other words the fluid is not able to resist & sustain shear stress when it is rest.

Fluid is define as a substance which is capable of flow this is because fluid is unable to resistance shear stress when it is at rest.

This however does not ~~not~~ mean that the fluid does not offer any resistance to the shear force infact there exist tangentially or shear stress b/w layer of the fluid.

Viscosity:-

It is defined as the property by virtue of which fluid offer the resistance to the movement of one layer of fluid over adjacent layer. It is denoted by ~~[illegible]~~ μ

$$\tau = \mu \frac{du}{dy}$$

where

μ = dynamic viscosity
du = small change in velocity
dy = ~~resistance~~ b/w two layer distance

Types of fluid

1) 1-D, 2-D, 3-D flow
2) Shear thinning fluid, shear thickening fluid, Newtonian fluid, Non-Newtonian fluid, compressible, incompressible fluid, ideal/ real fluid

Types of flow

1) Laminar flow
2) Turbulent flow
3) Steady & non-steady flow
4) Stream function flow, velocity function flow
5) 1D, 2D, flow
6) Boundary layer flow, Non-boundary layer flow etc.

Date 11/1/17

UNIT-I

* Properties of fluids *

① Viscosity :- viscosity comes into exist becau

① force of cohesive
② Molecular moment exchange b/w diff. layer of fluid & as the flow take place these effect appears as shearing stress b/w the layer of fluid

Acc. to newton's law —

$$\tau = \mu \frac{du}{dy}$$

~~dy~~ → dynamic viscosity
du/dy → ~~temp.~~ velocity gradient

Que:→ The distance b/w the fixed plate & moving plate is 0.025 mm. The upper plate is moving at a velocity of 60cm/sec. & the force applied on the top plate is 2N/m². Calculate the viscosity b/w the upper & lower plate.

$$\tau = \mu \frac{du}{dy}$$

$$2 = \mu \times \frac{60}{0.025}$$

$$\frac{2 \times 0.025}{60} = \mu \qquad \mu = 8.33 \ N s/m^2 \ N s/m^2$$

Classification of fluid → as D does not depend on time

Newtonian fluid — Non-newtonian fluid

obey Newton's law of viscosity — obey Power law

$$\tau = \mu \frac{du}{dy} \qquad \tau = \tau_y + k\left(\frac{du}{dy}\right)^n$$

eg:- water, kerosin, diesel, petrol, mercury, glycerin, ethyl alcohol, etc

n → flow behaviour index
k → consistency index
τ_y → shear yield stress

$\tau_y > 0$
$n < 1$
Time↑ ν↓

$\tau = 0$ → Viscosity = function of temp

$\nu = f(\text{temp}, \frac{du}{dy})$

Shear thinning — Shear thickening

* Shear thinning fluid:— ① Also known as Pseudo-plastic fluid in which $n \cancel{<} 1$

(ii) $\frac{du}{dy}$ ↑ ν↓ eg:- paint, blood, Milk,

* Shear thickening fluid :— Also known dilatant fluid

$n > 1$

~~dy~~ $\frac{du}{dy}$ ↓ ν↑ eg: Sugar solⁿ, quick sand

* Bingham plastic :-

When $n=1$, $k=\mu$, $\tau_y > 0$

$$\tau = \tau_y + \mu \frac{du}{dy}$$

eg:- tooth paste.

Ques.> The distance b/w the shaft & the sleeve is point 1.5 m if the shaft is rotates at a velocity of 50 m/sec when a force is applied 40 N to sleeve parallel to shaft. Determine the speed if a force of 200 N is applied. The fluid obey Newtons law of viscosity.

Solution>

$\boxed{\text{Ans: } 250}$

$du = 1.5$ m	$du = 1.5$
$u_1 = 50$ m/sec	$u_2 = ?$
$F_1 = 40$ N	$F_2 = 200$ N

$$\tau = \mu \frac{du}{dy}$$

$$\frac{F}{A} = \mu \frac{u}{y}$$

$$F = \frac{A \mu u}{y}$$

$F_1 \propto u_1$ $\quad F_2 \propto u_2$

$$\frac{F_1}{u_1} = \frac{F_2}{u_2}$$

Properties of fluid

* Mass density :- It is defined as the ratio of mass over volume. Denoted by ρ.

$$\rho = m/v$$

* Weight density :-

$$w = \frac{\text{weight}}{\text{volume}} = \frac{mg}{v}$$

$$w = \rho g.$$

* Specific volume :-

$$v = \frac{\text{Volume}}{\text{Mass}}$$

* Specific gravity :- (s) Ratio of mass density of test fluid to mass density of the standard fluid.

* Kinematic Viscosity :- (ν)

ratio of dynamic viscosity to mass density

$$\nu = \frac{\mu}{\rho} \quad \frac{m^2}{sec}$$

$$\frac{N s m^2}{m^2 \times kg}$$

$$\nu = \frac{\mu}{\rho} = \frac{Ns}{m^2} \times \frac{m^3}{kg}$$

$$kg \times \frac{m}{s} \times \frac{s \cdot m}{kg}$$

* Vapour Pressure :-

* Cavitation :-

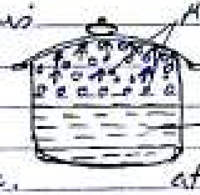
Classification of fluid

Newtonian Fluid	Non-Newtonian fluid
A real fluid, in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as N.F.	A real fluid, in which the shear stress is not proportional to the rate of shear strain, known as N.N.F.
or	or
A fluid which obey Newton's law of viscosity	A fluid which does not obey Newton's law of viscosity.
$\tau = \mu \frac{du}{dy}$	$\tau = \tau_y + K\left(\frac{du}{dy}\right)^n$
eg: Water, kerosin, mercury.	where, n - flow behavior index K → consistency index τ_y → shear yield stress

* Vapour Pressure :-

A change from the liquid state to the gaseous state is known as vaporization. The vaporization is depends upon the prevailing pressure and temp. condition. The vaporization occurs because of continuous escaping of the molecules through the free liquid surface.

Consider a liquid (say water) which is confined in a closed vessel. Let the temp of liquid is 20°c and pressure is atmospheric. The liquid will vaporise at 100°c. When, the vaporization takes place, the molecules escapes from the free surface of the liquid. These vapour molecules get accumulated in the space b/w the free liquid and top of the vessel. These accumulated vapour exert a pressure on the liquid surface. This pressure is known as Vapour Pressure of the liquid or this is the pressure at which the liquid is converted into vapour.

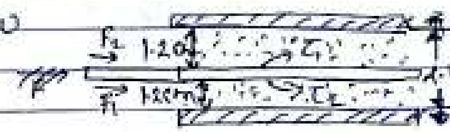


* Cavitation:-

Consider a flowing liquid in a system. If the pressure at any point in this flowing liquid becomes equal to or less than the vapour pressure, the vaporization of the liquid starts. The bubbles of these vapours are carried by the flowing liquid into the region of high pressure where they collapse, giving rise to high impact pressure. The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries gets eroded and cavities are formed on them. This phenomenon is known as cavitation.

Q. Two large surfaces is placed ~~to~~ 2.4 cm apart filled with glycerin having viscosity 8.4×10^{-1} Nsec/m². A thin plate having area 0.5 m² is placed in the middle of these two plane surface having velocity of 0.6 m/sec. What force is required to drag the plate.

Acc. to Newton's law



$$\tau = \mu \frac{du}{dy}$$

$$\tau = \frac{F}{A}$$

$$\tau_1 = \frac{F_1}{A}, \quad \tau_2 = \frac{F_2}{A}$$

$$\boxed{F = F_1 + F_2}$$

$F_1 = 21$ N

$F_2 = 21$ N

$\frac{F}{A} = \mu \frac{du}{dy}$

$F = A \mu \frac{du}{dy}$

$F = 0.5 \times 8.4 \times 10^{-1} \times \frac{1.2}{1.2}$

$F = 0.42$

$F = 0.42$ N

If the plate is fixed 0.8 cm apart from the lower end/surface.

$\tau = \mu \frac{du}{dy}$ ~~$= 8.4 \times 10^{-1} \times \frac{1.6}{0.8} = 1.68$~~

$\frac{F_1}{A} = \mu \frac{du}{dy}$

$F_1 = A \mu \frac{du}{dy}$

$= 0.5 \times 8.4 \times 10^{-1} \frac{0.6}{0.4}$

$= 31.5$ N

~~$F_2 = F - F_1$~~
~~$= 42 - 0.5$~~
~~$= 41.5$~~

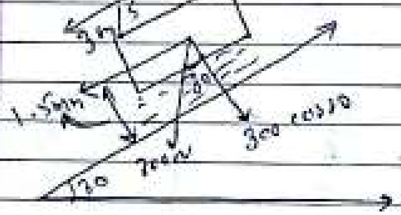
$F_2 = 0.5 \times 8.4 \times 10^{-1} \times \frac{0.6}{1.6}$

$= 0.1575$

$F = F_1 + F_2$

$= 47.25$ KN

Ques → Calculate the dynamic viscosity of an oil which is used for lubrication b/w square plate having area ~~of~~ 0.8 x 0.8 m² and inclined plane with an angle of 30° inclination 30°. The weight of the square plate is 300N and it slide down the inclined plane with a vel. of 0.3 m/sec. The thickness of ~~a~~ lubrication film is 1.5 mm.



$F_1 = 300 \cos 30° = 259.8$

$F_2 = 300 \sin 30° = 150$

$\tau = \mu \frac{du}{dy}$

$\frac{F_2}{A} = \mu \frac{du}{dy}$

$\frac{F_2}{A} \times \frac{dy}{du} = \mu$

$\frac{150 \times 1.5}{0.8 \times 0.8 \times 0.3} = \mu = \frac{F_2 dy}{A du} = \frac{150 \times 1.5 mm}{0.8 \times 0.8 m^2 \times 0.3} =$

$\mu = 1.171 N/m$

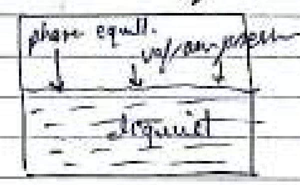
$\mu = \frac{150}{0.8 \times 0.8 m^2} \times \frac{0.3 m}{sec}$

1.171 N/m

Vapour pressure & Cavitation

All liquid have a tendency to evaporize. Vapour pressure of liquid defined as the pressure exerted by vapour of the liquid which are in phase equilibrium with a liquid itself. Vapour pressure ↑ with ↑ of temp. If the vapour pressure of the liquid becomes equal to external absolute pressure exerted upon the liquid then the phenomena of boiling taken place.

$\text{liquid} \underset{\text{condensation}}{\overset{\text{vaporization}}{\rightleftharpoons}} \text{vapour}$



Cavitation:—

Now considering a flowing liquid in a system if the pressure at any point in this flowing liquid becomes equal to the V.P. then vaporization of the liquid start. The bubbles of these vapour are carried by the flowing liquid into the region of high pressure where they collapse, giving rise to high ~~in~~ impact pressure. The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries get eroded and cavities are formed on them which is known as cavitation.

Ques:-> If the velocity profile of a fluid over a flat plate is parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/sec. Calculate the vel. gradient at a distance of 0, 10, 20 cm from the plate. the $\mu = 0.85\ N\,sec/m^2$. Assume the parabolic eq^n is $u = ay^2 + by + c$

$\frac{du}{dy}$

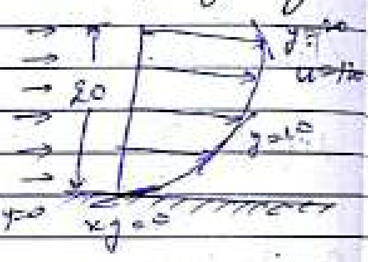
$u = ay^2 + by + c$

Ot $\frac{du}{dy} = 2ay + b$

At $y = 0$

$u = 0$

$\boxed{0 = c}$



At $y = 20$, $u = 120$ | $120 = a(20)^2 + b(20) + 0$
$120 = 400a + 20b$

At $y = 10$

$\frac{du}{dy} = 0$

$u = a(10)^2 + \frac{db}{dy}$

$u = a\,100 + 0$

$\left.\frac{du}{dy}\right|_{y=20} = 0$

$2ay + b = 0$

$400a + 20b = 120$

$a = -0.3$

$b = 12$

$\left(\frac{du}{dy}\right)_{y=0,10,20}$

Given, $u = ay^2 + by + c$

at $y = 20$, $u = 120$

$\boxed{120 = 400a + 20b}$

At $y = 20$, $\frac{du}{dy} = 0$ {due to maxi. condition}

$\frac{du}{dy} = 2ay + b$

$0 = 2a(20) + b$

$0 = 40a + b$

$\boxed{b = -40a}$

So, $120 = 400a + 20(-40a)$

$120 = 400a - 800a$

$120 = -400a$

$a = -\frac{120}{400} = -0.3$

$\boxed{a = -0.3}$

$\therefore$ $b = -40(-0.3)$

$\boxed{b = 12}$

At

$\left(\frac{du}{dy}\right)_{y=0} = 2ay + b = 2(-0.3)(0) + 12$

$= 12$

$\left(\frac{du}{dy}\right)_{y=10} = 2(-0.3)(10) + 12$

$= 6$

$\left(\frac{du}{dy}\right)_{y=20} = 2(-0.3)(20) + 12 = 0$

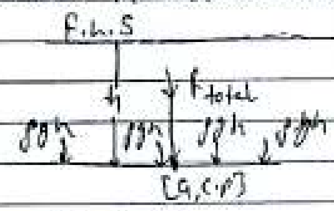
13/1/2017

- **Hydrostatics forces on the surface.**

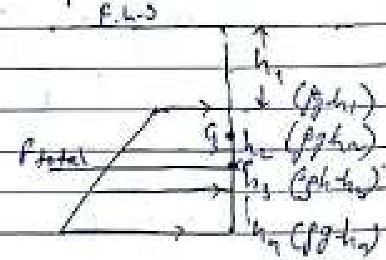
Whenever, any static mass of fluid comes into contact with surface either plane or curved it exerts forces upon the surface. The magnitude of these forces is known as total pressure force denoted by F_{total}.

The point of application of total pressure force on the surface is known as centre of pressure.

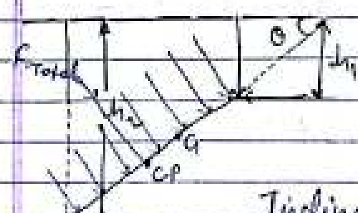
* Pressure distribution for inclined surface, vertical & horizontal surface :-



Horizontal



Vertical

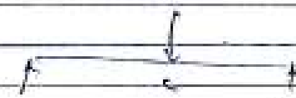


Inclined

Free liquid surface

$$U = \frac{W^2 L^3}{6EI}$$

$$y_B = \frac{WL^3}{3EI}$$



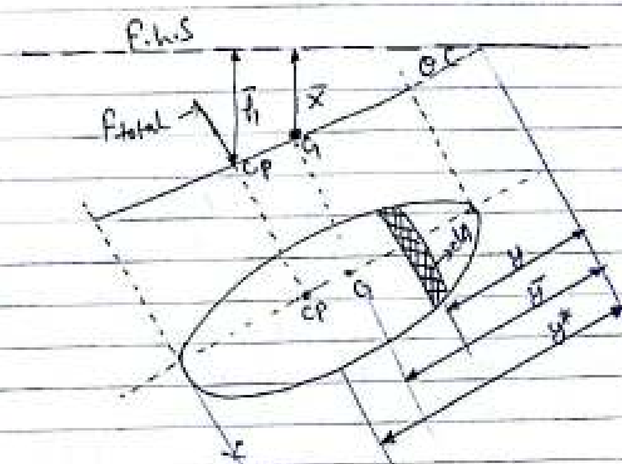
$$y_c = \frac{WL^3}{48EI}$$

$$U = \frac{W^2 L^3}{96EI}$$

FLS (Free liquid surface)

- **Force Exerted on an inclined Surface** -

Consider a body as shown in body -



Consider a small elementary area dA, $\bar{x}$ = depth of G below F.L.S

$\bar{h}$ = depth of C.P below F.L.S

C.P → centre of pressure point

$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$, $F_{total} = \rho g \bar{x} A$

I_G → moment of inertia about G.

$I_G (\square) = \frac{bd^3}{12}$

$I_G (\square) = \frac{b^4}{12}$

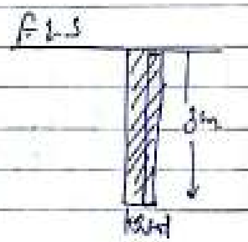
$I_G (\bigcirc) = \frac{\pi d^4}{64}$

Special case

(1) When $\theta = 0$
$\bar{h} = \bar{x}$

(2) when, plate is vertical, $\theta = 90°$
$\bar{h} > \bar{x}$

Ques:> A rectangular plane surface is 2m wide and 3m deep. It lies in vertical plane in water. Determine F_{total} and C.P. on the plane surface when its upper edge is horizontal and coincide with water
(2) 2.5 m F.W.S



Solution>

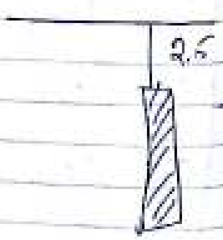
$F_{total} = \rho g \bar{x} A$
$= 1000 \times 9.8 \times 1.5 \times 6$
$= 88200$

$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$, $I_G = \frac{bd^3}{12} = \frac{2(3)^3}{12}$

$= \frac{4.5 \times 1 \times 88200\ 1.5}{6 \times 88200\ 1.5}$

$\boxed{\bar{h} = 2m}$

$= 2$

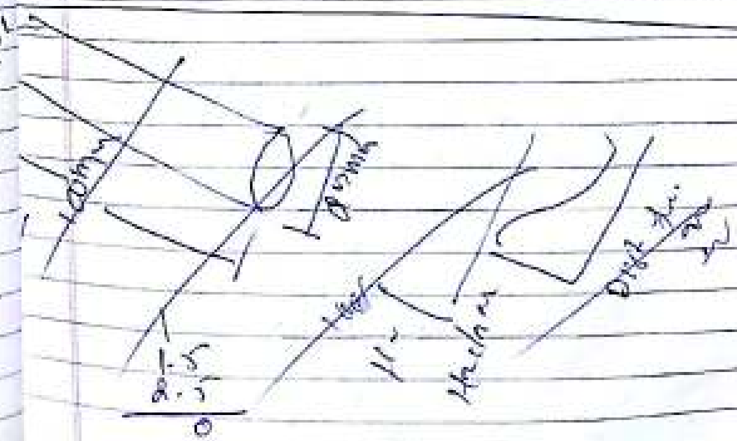


$F_{total} = \rho g \bar{x} A$
$= 1000 \times 9.8 \times 4 \times 6$
$= 39200 \quad 235200$
$= 2.4 \times 10^4$ kN

$\boxed{2.4 \times 10^4}$

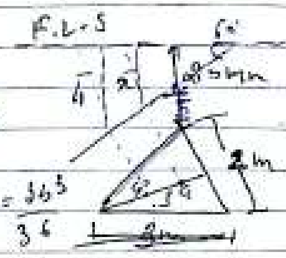
$\bar{h} = \frac{4.5 \times 1}{6 \times 4} + 4$

$\boxed{\bar{h} = 4.1875}$



17/1/2017

Ques→ A triangular plate 2m base, 3m height is immersed into the water at an angle of 60°. As shown in figure. Calculate the total force acting on the plate & location of C.P.



$F_t = \rho g A \bar{x}$

$= 1000 \times 9.81 \times \frac{1}{2} \times 2 \times 3 \times 3.36$

$= 10.08 \times 10^4$ N

$I_G = \frac{bh^3}{36}$

$\bar{h} = \frac{I_G \sin^2\theta}{A \times \bar{x}} + \bar{x}$

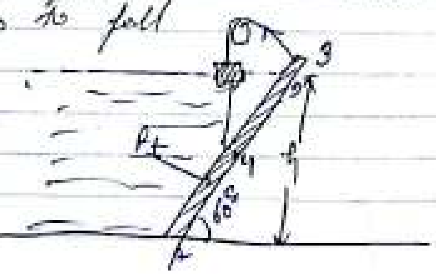
$I_G = \frac{bh^3}{36} = \frac{2(3)^3}{36} = 1.5$

$\bar{h} = \frac{1.5 \times \sin^2(60)}{3 \times 3.36} + 3.36$

$\frac{x}{3} = \sin 60$

$\frac{\sqrt{3}}{2} \times 3$

Ques→ A rectangular gate 6m × 2m is inclined at 60° & is hinged at the base. It contains on one side to prevent the gate for falling. A 500 kg is attached at the upper end by mean of pully allowing the gate to be man low & neglecting friction of the hinged and the pully. Find depth of the water at which gate begins to fall



Taking moment about A

$F_t \times AC - 500g \times 5 = 0$

$F_t \times AC = 500g \times 5$

$\boxed{A = AD \times 2}$

$\frac{h}{AD} = \sin 60$

$AD = \frac{2h}{\sqrt{3}}$

$A = \frac{2h}{\sqrt{3}} \times 2 = \frac{4h}{\sqrt{3}}$

By similarity —

$\bar{x} = h/2$

$\boxed{AC = AD - CD}$

$\bar{h} = \frac{I_G \sin^2\theta}{A \times \bar{x}} + \bar{x}$

$I_G = \frac{bd^3}{12}$

$= \frac{2 \times (AD)^3}{12}$

$= \frac{2 \times \left(\frac{2h}{\sqrt{3}}\right)}{12}$

$= \frac{4h}{\sqrt{3}} \times 12$

$= \frac{48h}{\sqrt{3}}$

$\bar{h} = \frac{\frac{48h}{\sqrt{3}} \times \sin^2(60)}{\frac{4h}{\sqrt{3}} \times \frac{48h}{\sqrt{3}} \, h/2} + h/2$

$= \frac{48 \times 0.75}{4 \times h/2} + h/2 = \frac{36}{\frac{4h}{2}} + h/2$

$= \frac{72}{4h} + \frac{h}{2}$

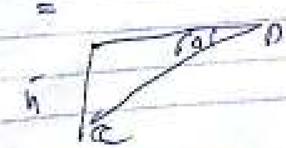
$= \frac{144 + 4h^2}{8h}$

$=$

$\boxed{\bar{h} = \frac{2h}{3}}$

$\frac{\bar{h}}{CD} = \sin 60$

$CD = \frac{4h}{3\sqrt{3}}$



$$AC = \frac{2h}{\sqrt{3}} - \frac{4h}{3\sqrt{3}} = \frac{6\sqrt{3}h - 4\sqrt{3}h}{9} = \frac{2\sqrt{3}}{9}h$$

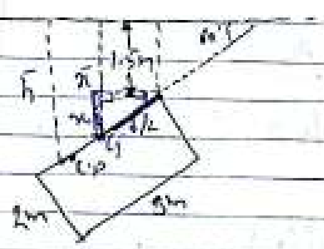
$$1000 \times 10 \times \frac{4h}{\sqrt{3}} \times \frac{h}{2} \times \frac{2h}{3\sqrt{3}} = 5000 \times 10 \times 5$$

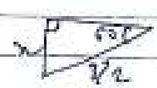
$\boxed{h = 3.83\,m}$ Ans

$$\frac{8h^3}{12} \quad 8h^3 = 600 =$$

Date 20/11/2016

Ques → A rectangular plate having dimension 2m × 3m is immersed in a water makes an angle 60°. Calculate F_{total} & C.O.P.





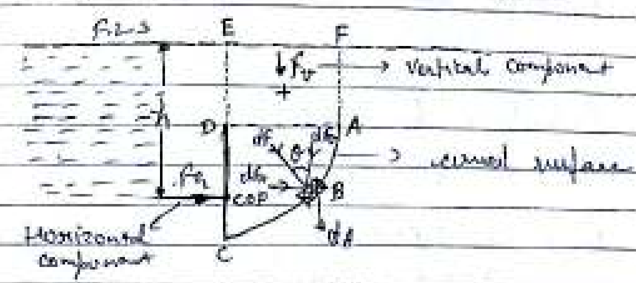
$$\frac{x}{3/2} = \sin 60^\circ$$
$$x = 1.299$$
$$\bar{x} = 1.299 + 2.799$$

$$F_t = \rho g A \bar{x} = 1000 \times 10 \times 6 \times \bar{x}$$
$$= 1000 \times 10 \times 6 \times 2.799$$
$$= 16.7940\,KN$$

$$I_G = \frac{bd^3}{12} = \frac{3 \times (2)^3}{12} = 2$$

$$\bar{h} = \frac{2 \times 0.75}{6 \times 2.799} + 2.799$$
$$= 2.89\,h$$
$$\boxed{h = 2.88\,m}$$

• Hydrostatic forces on a curve surface •



dA → small elementary area
F_h → total pressure force in x-direction
F_v → total pressure force in y-direction
F_R → resultant force → $F_R = \sqrt{F_h^2 + F_v^2}$

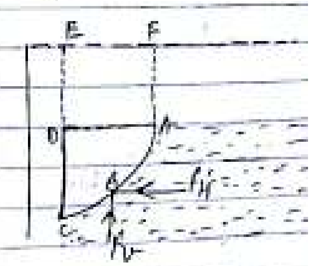
The projection of ABC in a vertical plane CD the trace of which by (CD)

F_v = the total weight of the liquid in the vol. (ABCDEF)

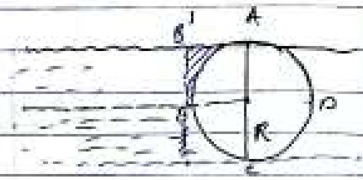
In other words F_v represents the weight of the liquid by the curved surface ABC till the free liquid surface.

* Case-II :-

When the curved surface is subjected to the fluid pressure acting upon the underside.



Ques→ Calculate the total vertical component of hydrostatic force when a cylindrical gate is subjected to a following condition.



Hydrostatic force on a curved surface of $F(AB)\downarrow$

$F_{AB}\downarrow = \cancel{\text{Vol.}[BB'A]}$
= weight of liquid
$= mg$
$= \rho V g = \rho g\,[BB'A]$

$\boxed{m = \rho V}$

$F_{AB}\downarrow = \rho g\,[R^2 - \frac{1}{4}\pi R^2] \times l$ —— ①

Force exerted on curved surface BC

$F_{BC}\uparrow$ = weight of liquid
$= mg$
$= \rho V g = \rho g\,[B'ABC]$

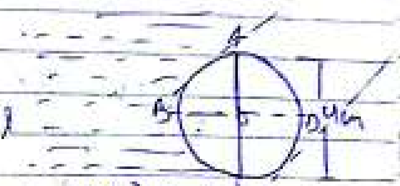
$F_{BC}(\uparrow) = \rho g\,[R^2 + \frac{1}{4}\pi R^2] \times l$ —— ②

$(F_V)_{total} = F_{BC}\uparrow - F_{AB}\downarrow$
$= \rho g \frac{\pi}{4} R^2 \times 2$

$\boxed{(F_V)_{total} = \rho g \frac{\pi}{2} R^2 \times l}$

Find the magnitude and the direction of force due to water ~~are~~ acting on a roller gate of cylinder form which have $d = 4m$. When the gate place on a dam in such a way that the water going to glass. Take length of the of the gate = 8m.

Case - I



$F_{AB}\downarrow = \rho g\,[R^2 - \frac{1}{4}\pi R^2]\,l$
$= 1000 \times 10\,[(2)^2 - \frac{1}{4} \times \pi \times (2)^2] \times 8 \quad l = 8m$
$= 68672.58$

$F_{(BC)}\uparrow = 1000 \times 10\,[2^2 + \frac{1}{4} \times \pi \times (2)^2] \times 8$
$= 571.32\ KN$

$\boxed{F_V = 502.65\ KN}$ $\boxed{\text{Ans: u.3}}$

$F_V = \rho g \frac{\pi}{2} R^2 \times l = 1000 \times 10 \times \frac{\pi}{2} \times (2)^2 \times 8$
$= 502.65\ KN$

$F_H = \rho g A \bar{x} = \rho g$

$F_H(AD) = \rho g \bar{x} A = 1000 \times 10 \times 1 \times 16 = 160\ KN$

$F_H(DC) = 1000 \times 10 \times 3 \times 16 = 480\ KN$

$F_H = F_H(AD) + F_H(DC) = 640\ KN$

$F_R = \sqrt{409600 + 252657.6} = \sqrt{662257.6}$

$\tan\theta = \tan^{-1}\left[\frac{502.4}{813.63}\right]$

$= 31.69$

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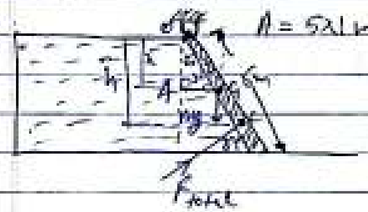
Ques:→ A hinged gate of length 5m is inclined at 30° with the horizontal & with water man on its left, as shown in fig. the density of water 1000 kg/m³. The mini. mass of gate in kg per unit width require to keep it close is



$A = 5 \times 1\ m^2$

Taking moment about O

$mg \times AG = F_t \times CO$

$mg \times \bar{x} = \rho g A \bar{h} \times CO$

$\boxed{A = 5}$

$\bar{h} = \frac{I_G \sin^2\theta}{A \times \bar{x}} + \bar{x}$

$\Rightarrow I_G = \frac{bd^3}{12} = \frac{5^3}{12} = 10.42$

$\bar{h} = \frac{10.42 \sin 30}{5 \times 1.44} + 1.44$

$\boxed{\bar{h} = 2.16}$

$\boxed{A = 9621}$ mass kg

$\frac{x}{2.5} = \tan 30°$

$x = 2.5 a$

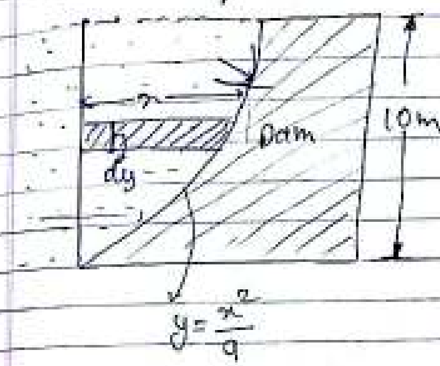
$\boxed{\bar{x} = 1.44}$

$m \times 10 \times 1.44 = 1000 \times 10 \times 5 \times 1.44 \times 4.5$

$\frac{P}{H} = \frac{2.16}{x} = \cos 30$

$\frac{2.16}{\cos 30} = x$

Calculate F_V & F_H if a dam is subjected to following condition. Find the magnitude & the direction of the resultant water pressure acting on a curved face of dam which shape are to the relation $y = x^2/9$. The height of the water retain by the dam 10m consider the width of dam is unity.



$y = \frac{x^2}{9}$

$\boxed{F_H = \rho A g \bar{x}} = 1000 \times 10 \times 1 \times \cancel{9.81} \times 5 = \cancel{490.5\ kN}$

$= 500\ kN$

$F_V = \cancel{\rho A g \bar{x}}\ \rho g V$

$dA = \int_0^{10} x\, dy$

$y = \frac{x^2}{9}$

$x^2 = 9y$

$x = 3\sqrt{y}$

$dA = \int_0^{10} 3\sqrt{y}\, dy$

$= 3\int_0^{10} \sqrt{y}\, dy = 3\int_0^{10} (y)^{1/2} dy$

$= 3\left[\frac{y^{3/2}}{3/2}\right]_0^{10}$

$= 3\left[\frac{(10)^{3/2}}{3/2}\right]$

$\boxed{dA = 63.25\ m^2}$

$F_V = \rho g V$

$= \rho g\ A \times l$

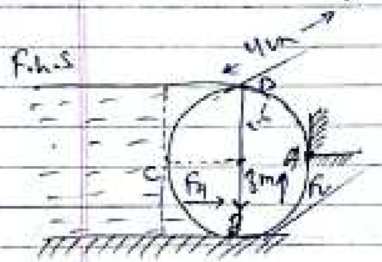
$= 1000 \times 10 \times 10 \times 63.24$

$= 632.45\ kN$ (2)

$F_R = \sqrt{(F_V)^2 + (F_H)^2}$

$= 806.22\ kN$

Q→ A cylinder 3m dia, & 4m long, retains water on one side as shown in the figure. The cylinder is supported as shown in the fig. Calculate the horizontal reaction at (A) & vertical reaction at (B). If the weight of cylinder is 196.2 KN. Ingnore friction.



$F_{AB} \downarrow =$

$\boxed{(R)_B = 59516 \downarrow}$

$F_V = \rho g \frac{\pi}{2} R^2 \times l$

$= 1000 \times 9.81 \times \frac{\pi}{2} \times (3/2)^2 \times 4$

$= \cancel{92.457\ KN}$

$= \cancel{138.685\ KN}\ 141.371\ KN$

$R_B = F_V - R_g$

$= 141.371 - 192.2$

$= 50.829\ KN$

$\downarrow(F_{DC}) = \rho g \left[R^2 - \frac{1}{4}\pi R^2\right] \times l$

$= 1000 \times 10 \left[(1.5)^2 - \frac{1}{4} \times \pi \times (1.5)^2\right] \times 4$

$= 19.314\ KN$

$\uparrow F_{CB} = 1000 \times 10 \left[(1.5)^2 + \frac{1}{4} \times \pi \times (1.5)^2\right] \times 4$

$= 160.685\ KN$

$F_H = \rho g A \bar{x}$

$= 1000 \times 10 \times (4 \times 3) \times 1.5$

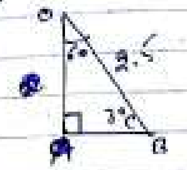
$= 180.0\ KN$

$F_H = R_A = 180\ KN$

Taking moment about O.

F.b.d.

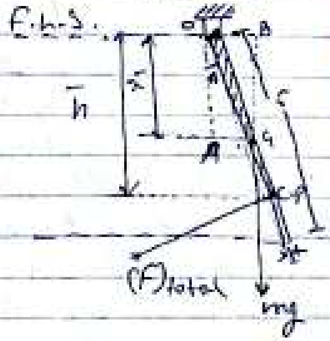
$(mg)(OA) + F_{total} \times \bar{h} = 0$



$\boxed{AB = OB}$

$\frac{AB}{2.5} = \cos 30°$

$\boxed{AB = 2.165}$



$h = \frac{I_G \sin^2\theta}{A \times \bar{x}} + \bar{x} \Rightarrow I_G = \frac{bd^3}{12} = \frac{1(5)^3}{12} = 10.45$

$= \frac{10.45 \times \sin^2 60°}{5 \times 2.5} + 2.5$

$\boxed{\bar{h} = 3.396}$

$(F)_{total} = \rho g A \bar{x}$

$= 1000 \times 10 \times 5 \times 2.5$

$= 12.5\ KN$

Now, $m \times 10 \times (2.165) - (3.396\ m) \times 12.5 = 0$

$m = \frac{(3.396)(12.5\ KN)}{10 \times (2.165)} = 1.96$

Date 25/1/2017

Buoyancy & Flotation

Whenever any body is completely or partially immersed in a fluid, it is subjected to an upward force which tends to lift it up in, other words the tendency of a body immersed in a fluid to be lifted upon by force in opposition to the force of gravity.

It is the upwards force acting upon the body under the condition of buoyancy. Because buoyancy forces acts opposite to the gravity, in by the body under goes apparent loss in the body whenever it is immersed in fluid.

~~Principle of flotation~~:—

* **Archimedes principle**:—

It state that buoyancy force (FB) acting on a body is equal to weight of the fluid displaced by immersed position of a body

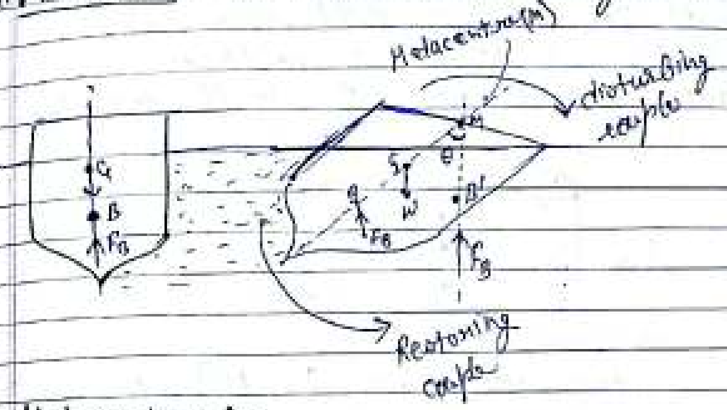
F_B = weight of fluid displaced / body volume.
= mg

* **Principle of flotation**:—

It state that for a body floating in a liquid, the weight of the body must be balanced by the F_B. This principle is known as principle of flotation.

$mg = F_B = \omega V$ ∴ $\rho g v = \omega v$ $\rho g = \omega$

* **Metacentre & metacentric height**:—



* **Metacentre**:—

It is defined as a pt of intersection of the line of action of F_B passing through the new centre F_B and the axis of the body passing through B & G.

It is denoted by (M).

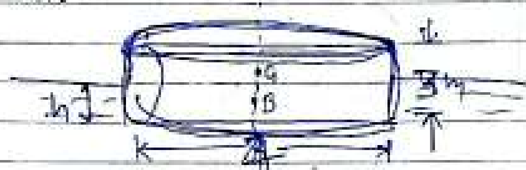
* For stable condition M above G
* and GM should be +ve, where GM is known as metacentric height denoted by (GM)

$$\overline{GM} = \frac{I}{V} - BG \quad [V \to \text{displaced volume}]$$

Unstable condition →
for unstable cond^n.
G above M, $\overline{GM}$ should be negative.

$\boxed{\text{Weight density of water} = 1000 \times 9.81\ N/m^2}$

Ques> A solid cylinder having dia 4m & length 3m is immersed in a fluid. Find the cond'n of the cylinder. Having specific gravity of the cylinder is 0.6



$\rho_{cy} = 0.6 \times 1000$
$= \cancel{6000}\ 600\ kg/m^3$

$GM = \frac{I}{V} - BG$

$I = \frac{\pi d^4}{64} = \pi$

$I = 4\pi$

Weight of the body = mg

For floating cond'

$\cancel{mg = \rho g v}$

$mg = wv$

$\cancel{mg = wv}$

$mg = \rho g v$

$\cancel{mg = \rho g v}$

$m = \rho v$

$\cancel{m = \rho v}$

$\rho v = \rho v$

$\cancel{\rho v = \rho v}$

$600 \times \frac{\pi}{4} d^2 \times 3 = 1000 \times \frac{\pi}{4} d^2 \times h$

$h = 1.8$

$\boxed{h = 1.8}$

$BG = 1.5 - 0.9$
$= 0.6$

$GM = \frac{I}{V} - BG$

$GM = -0.5$ Ans

27/1/2017

Centre of buoyancy $= \frac{h}{2}$

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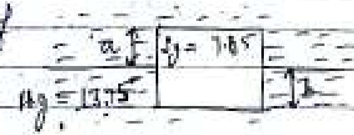
Q- A block of steel specific gravity 0.75 7.85 floats at a mercury water interface as shown fig. What the ratio a & b for this cond'n.

Ans sp. g. ρ = to 13.57.

Acc. to the principle of flotation, down F_B.



$\downarrow (mg)_{body} = F_B \uparrow$
$= (F_{B_1})_{water} + (F_{B_2})_{water} \uparrow$

$(7.85 \times 1000 \times A(a+b)g = 1000 \times g \times A.a + 13.57 \times 1000 \times g \times A$

$7.85(a+b) = a + 13.57b$

$7.85a + 7.85b = a + 13.57b$

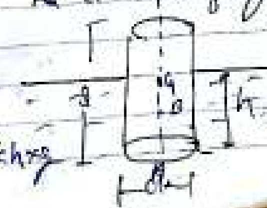
$7.85a - a = 13.57 \times b - 7.85b$

$6.85a = 5.72b$

$\frac{a}{b} = \frac{5.72}{6.85}$

$\boxed{\frac{a}{b} = 0.835}$

Ques> A wooden cylinder sp. gravity 0.6 & circular in crossection is required to float in oil having sp. gravity 0.90. Find the ratio of l/d of cy. to float to its longitudinal axis vertical to oil, where l = length of height of cy. & d = dia of cy.



Floating condition,

$mg = wv$

$0.6 \times 1000 \times \frac{\pi}{4} d^2 \times l \times g = 0.9 \times 1000 \times \frac{\pi}{4} d^2 \times h \times g$

$0.6 \times l = 0.9 \times h$

$\therefore \boxed{h = 2/3\, l}$

If eq. ~ floating it means GM is greater than zero.

$GM = \frac{I}{V} - BG$

$I = \frac{\pi d^4}{64}$ $GM = \frac{\frac{\pi d^4}{64}}{\pi r^2 h} - 0.167 l$

$BG = \frac{d}{2} - \frac{h}{2}$

$= \frac{l - (2/3 l)}{2} = \frac{0.33 l}{2}$

$\boxed{GB = 0.167\, l}$

$\boxed{\frac{d}{l} > 3/4}$ Ans

$\boxed{GM > 0}$

$GM = \frac{\pi d^4}{64} \times \cancel{\ldots} = 0.167 l$

$GM = \frac{\frac{\pi d^4}{64}}{\frac{\pi}{4} d^2 \times h} - 0.16 l$ $= \frac{\pi d^4}{64} \times$

$= \frac{\cancel{\pi} d^{\cancel{4}\,2}}{64 \times 16} \times \frac{4}{\cancel{\pi d^2} \times h} - 0.16 l$

$= \frac{d^2}{16 \times h} - 0.16$

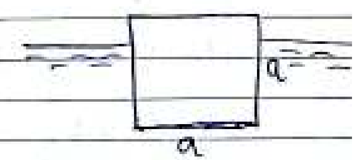
$= \frac{d^2}{16 \times (2/3 l)} - 0.16 l$

$= \frac{d^2}{10.66 l} - 0.16 l$

Q: A cube of side (a) floats with its vertical one axis in a liquid of sg. fluid is l_s. If the sp. g (cube) $= C_s$ ratio $l_s : C_s$, metacentric height $= 0$

Sol^n (Sp. gravity)$_{liquid}$ $= l_s$

(Sp. gravity)$_{cube}$ $= C_s$



Condition of floating.

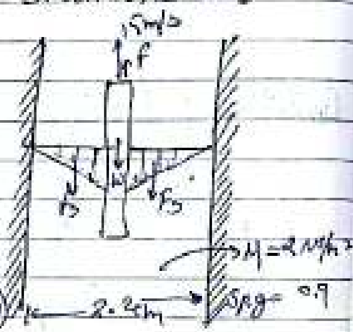
$mg = wt$

$\rho V g = wt$

$l_s \times 1000 \times a^3 \times g = \rho g \times V$

Date
1/2/2017

Que-> Consider a flat plate having dimension 1.2m × 1.2m × 0.2 cm is placed in the middle of the two flat surfaces which are filled with a liquid having viscosity 2 N-s/m², sp. gravity 0.9. If the weight is 40N & a plate is moving vel 0.15 m/sec in upward direction calculate net force required on a flat plate to maintain its speed.



$F_{Net} = F_B - [2 F_S + W]$

$F_B = w \forall$

$= \rho g [1.2 \times 1.2 \times 10^{-2}]$

$= 0.9 \times 1000 \times 9.8 (0.0144)$

$= 25.4\ N \uparrow$

$\tau = \frac{F}{A}$

$\therefore F_S = \tau \times A$

$= \left[\mu \frac{du}{dy}\right] \times A$

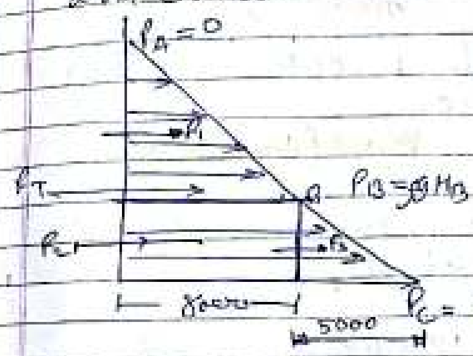
$= 2 \times \frac{0.15}{1 \times 10^{-2}} \times 1.2 \times 1.2$

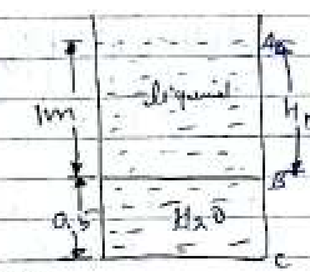
$= 43.2\ N$

$F_{Net} = 25.4 - 2(43.2) - 40$

$= -101\ N$

Q> A tank contains water upto a height of 0.5m above the base. Another liquid of sp. g is 0.8 is filled on the top of the water 1m height. Calculate total pressure on the one side of tank. Position C.P. for one side of the tank which 2m width.





$P_B = 0.8 \times 1000 \times 10 \times 1 = 8000\ N/m^2$

$P_C = 8000 + 1000 \times 10 \times 0.5 = 13000\ N/m^2$

$F_{total} = F_1 + F_2 + F_3$ $= \left(\frac{1}{2} \times 8000 \times 1\right) + (8000 \times 0.5)2 + \left(\frac{1}{2} \times 5000 \times 0.5\right)$

$= \cancel{7250\ N}$

$= 18500\ N$

$m \times \frac{N}{m^2} \times m$

$\Sigma M_A = 0$

$F_1 \times \frac{2}{3}(H_B) + F_2\left[H_B + \frac{0.5}{2}\right] + F_3\left[H_B + \frac{2}{3} H_C\right] + F_T \times \bar{x} = 0$

$8000 \times \frac{2}{3}(1) + 8000\left(1 + \frac{0.5}{2}\right) + 2500\left[1 + \frac{2}{3}(0.5)\right] + 18500\bar{x}$

$18666.67 + 18500\,\bar{x} = 0$

$\bar{x} = 1.009$

Fluid Kinematic

It is a branch of science which deals with the geometry of motion of fluid without any references of forces causing the motion i.e. deals with the description of motion of fluid in ~~terms~~ terms of ~~type~~ time & space.

Mathematically vel. & acc^n is a fun^n. of ~~type~~ time & space.

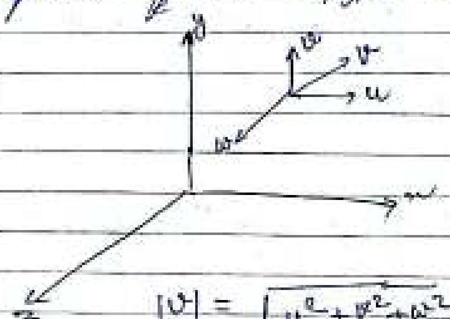
$V_a = f\,[\text{time}, \text{space}]$

The velocity of the fluid is defined by following relationship:—

$$V = u\hat{i} + v\hat{j} + w\hat{k}$$

where,—

u, v, w are the velocity component in x, y, z direction



$$|V| = \sqrt{u^2 + v^2 + w^2}$$

:- Types of flow :-

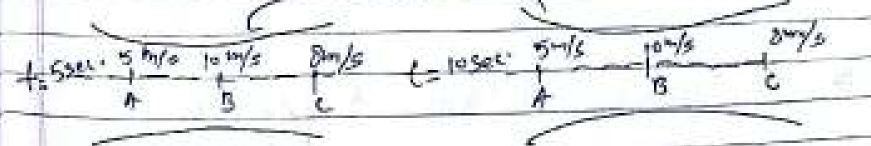
1. Steady and un-steady flow :—
2. Uniform and non- Uniform
3. One- dimensional, 2D, & 3D flow
4. Laminar & turbulent flow
5. Compressible and incompressible flow
6. Internal & external flow
7. Rotational & irrotational flow.

*** Steady flow :—** Steady flow in which the properties of flow such as velocity, acc^n, density at any pt ~~does~~ do not vary w.r.t. time at any ~~time~~ pt.

There should be no change in direction & magnitude of the vector quantity.

For un-steady flow the flow properties change w.r.t time at any pt.

Steady + Non-uniform



For steady flow

$$\frac{\partial v}{\partial t} = \frac{\partial a}{\partial t} = \frac{\partial P}{\partial t} = 0$$

for unsteady flow

$$\frac{\partial v}{\partial t} \neq 0 \quad \frac{\partial P}{\partial t} \neq 0$$

$$\frac{\partial a}{\partial t} \neq 0$$

* Uniform & Non-uniform *

A flow is uniform if the velocity of the flow does not change from pt to pt in a space, for uniform flow there should not change in the magnitude as well as direction w.r.t space at any stand of time.

For non-uniform flow velo. changes from pt. to pt in space.

A $5\,m/s$ B $5\,m/s$ C $5\,m/s$ A $5\,m/s$ B $5\,m/s$ C $5\,m/s$

steady + uniform.

* 1-D, 2-D & 3-D flow *

for 1-D flow → $V = u\hat{i}$

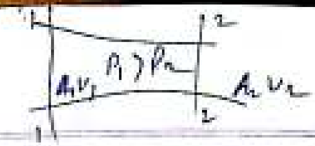
2-D flow → $V = u\hat{i} + v\hat{j}$

3-D flow → $V = u\hat{i} + v\hat{j} + w\hat{k}$

Continuity Equation $\boxed{A.V = C}$

The eqn which is based on conservation of mass is known as continuity equation.

In this eqn. the quantity of fluid per second is constant



$\dot{m}_1 + \dot{m}_2 = \rho A V$

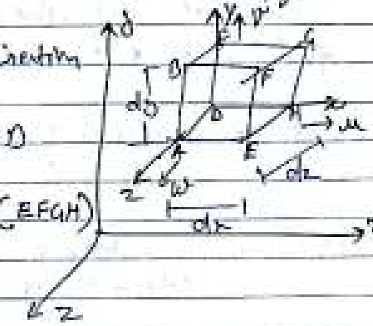
Acc. to continuity eqn. $A_1V_1 = A_2V_2$,
where

A_1, A_2 → cross section area at pt 1-1, 2-2
V_1, V_2 → velo. at 1-1, 2-2

* Continuity equation for 3-D flow :-

Consider a small fluid elementary fluid vol. having dimension dx, dy, dz and vel. and u, v, w as shown in figure.

Mass flow rate in x-direction



$$\dot{m}_x = \rho\, dz\, dy\, u \quad (ABCD)$$

$$\dot{m}_{x+dx} = \dot{m}_x + \frac{\partial \dot{m}_x}{\partial x} \cdot dx \quad (EFGH)$$

$$[\dot{m}_x]_{net} = \dot{m}_x - \dot{m}_{x+dx}$$
$$= \cancel{\dot{m}_x} - \cancel{\dot{m}_x} - \frac{\partial \dot{m}_x}{\partial x}\, dx$$
$$= -\frac{\partial \dot{m}_x}{\partial x} \cdot dx$$
$$= -\frac{\partial}{\partial x} \rho\, dx . dy dz . u$$

$$[\dot{m}_x]_{net} = \frac{-\partial}{\partial x}(\rho u) . dx . dy . dz \quad \text{--- (1)}$$

Similarly, in y-direction

$$[\dot{m}_y]_{net} = \frac{-\partial}{\partial y}(\rho v) . dx . dy\, dz \quad \text{--- (2)}$$

In z- direction w

$$\left(\dot{m}_z\right)_{net} = -\frac{\partial}{\partial x}\left(\rho w\right) dx.dy.dz \text{ --- } (3)$$

* Apply conservation of mass principle. The net mass flow rate in x, y, z direction is equal to increase of mass in small element are dx, dy, & dz

The rate of increase of mass in the fluid element is equal to —

$$\text{Rate of increase of mass [CV]} = \frac{\partial \rho}{\partial t}(dx\,dy\,dz) \text{ --- } (4)$$

⇒ Due to law of conservation of mass
1+2+3 = 4

$$\left[-\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial x} - \frac{\partial(\rho w)}{\partial x}\right] \cancel{dx.dy.dz} = -\frac{\partial \rho}{\partial t}.\cancel{dx.dy.dz}$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial x} + \frac{\partial(\rho w)}{\partial x} + \frac{\partial \rho}{\partial t} = 0$$

Above eqn is known as 3-D continuity equation

Special Case—

(1) for steady flow — $\frac{\partial \rho}{\partial t} = 0$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial x} + \frac{\partial(\rho w)}{\partial x} = 0$$

(2) for steady & incompressible flow —

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} = 0 \quad \text{--} [\rho \rightarrow const.]$$

3/2/2017

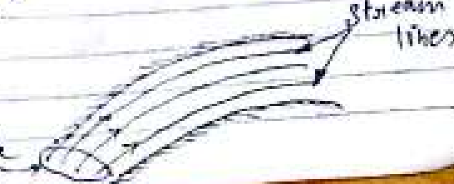
Laminar and Turbulent Flow

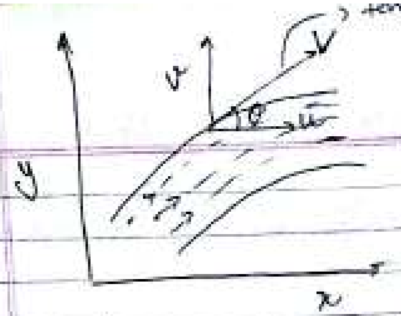
Laminar flow is a flow in which fluid particles are flowing in thin sheets or layers or laminae sliding over one another thus there is no bulk mixing b/w the adjacent fluid layer. It occurs at low vel.

Turbulent flow is a random disorder flow which occurs at higher flow vel. compare to laminar flow. There is a bulk mixing b/w adjacent fluid layer.

DD

Stream Lines :- It is an imaginary line draw into the flow field in such a manner ~~such~~ that tangent to stream line at any time gives the direction of velocity of flow at that pt. Since, stream lines is tangent to the velocity vector at every pt therefore there can be no component of vel. at right angle to the stream line.





2-D

$\boxed{\vec{V} = u\hat{i} + v\hat{j} + 0\hat{k}}$

consider a stream lines as shown dig figure in 2-D.

From geometry $\quad v = V\sin\theta \; - ①$

$\quad u = V\cos\theta \; - ②$

① ÷ ②

$\tan\theta = \frac{v}{u}$

$\frac{v}{u} = \frac{dy/dt}{dx/dt}$

$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$

stream line eqn in 3D

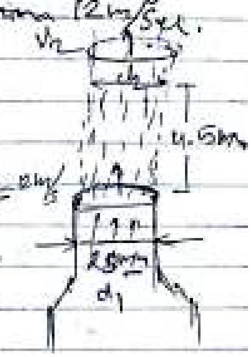
u, v, w component of vel in x, y, z direction

⊙ Stream bundle :-

It is an imaginary tubes form by a gp of stream line passing through a small closed curved which may or may not be circular.

Ques> A jet of water from a 25mm diameter nozzle is directed vertically upward. Assuming that jet remain circular & neglecting any loses of energy

what will be dia at pt 4.5 m above the nozzle if the vel. with which the jet leaves the nozzle is ~~12mm~~ 12 m/sec.



$V^2 = u^2 - 2gh$

$V_2^2 = V_1^2 - 2gh$

$V_2 = \sqrt{V_1^2 - 2gh}$

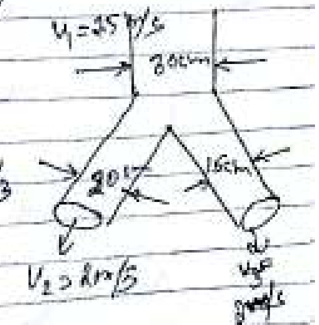
$= \sqrt{(12)^2 - 2\times10\times4.5} = 7.34\ m/s$

$A_1V_1 = A_2V_2$

$\frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} d_2^2 \times V_2$

$\frac{\pi}{4}(25\times10^{-3})^2 \times 12\frac{m}{s} = \frac{\pi}{4}\times(d_2)^2 \times 12\frac{m}{s}$

Q> A 30cm dia of pipe containing water, branches into two pipes of dia 20cm & 15cm respectively. If the vel. in 30cm dia of pipe is 2.5 m/s, find the discharge in this pipe also determine the vel. in 15cm pipe if the of 20 dm is 2 m/s velocity.



$A_1V_1 = A_2V_2 + A_3V_3$

$\frac{\pi}{4}(30)^2 \times 2.5 = \frac{\pi}{4}(20)\times\frac{2m}{s} + \frac{\pi}{4}(15)^2 \times V_3$

~~1.7671 = 0.031 + 0.1767~~

$\frac{\pi}{4}(30\times10^{-3})^2 \times 2.5 = \frac{\pi}{4}(20\times10^{-3})^2\times2 + \frac{\pi}{4}(15\times10^{-3})^2 \times V_3$

$1.76\times10^{-3} = 6.283\times10^{-4} + 1.767\times10^{-4} V_3$

$V_3 = 6.44$

$Q = 0.1767$

Acceleration of fluid particles

Consider a flow in 3-D having vel. u, v, w and acc^n are a_x, a_y, a_z are the acc^n in x, y, z direction.

$a = a_x i + a_y j + a_z k$ — 3D

$V = ui + vj + wk$ — 4D

$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$ (resultant acc^n)

Mathematically –

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = \underbrace{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}_{\text{convective acc}^n} + \underbrace{\frac{\partial w}{\partial t}}_{\text{local acc}^n}$$

⇒ Special case :–

If the flow is steady flow

$$\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial w}{\partial t} = 0$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

vel. direction

51

Acc. can also be defined in terms of tangential & normal component. Tangential component is developed when the magnitude of velocity changes w.r.t time. Tangential component is always tangent to stream line. Normal component is developed when the direction vel. change w.r.t time. This happen when fluid particles travel along the curved path.

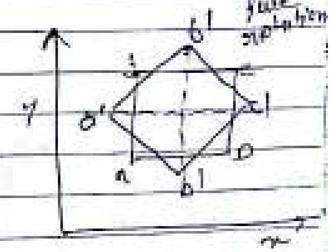
Rotational flow and Irrotational flow

* Rotational flow: It is defined as the movement of fluid element in such way that both the axis horizontal & vertical rotates in same direction

$\omega = \omega_x i + \omega_y j + \omega_z k$

$a = a_x i + a_y j + a_z k$

$v = ui + vj + wk$



$|\omega| = \sqrt{(\omega_x)^2 + (\omega_y)^2 + (\omega_z)^2}$

$u = \frac{dx}{dt}$, $a_x =$

$v = \frac{dy}{dt}$, $a_y =$

$w = \frac{dz}{dt}$, $a_z =$

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \quad \tau = \mu\frac{du}{dy}$$

Rotation
Rotation
Torsion
Shear-Stress

or

$$\omega = \frac{1}{2}\begin{bmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{bmatrix}$$

⇒ For ideal flow $\mu = 0$ flow will be irrotational & if $\frac{du}{dy} = 0$, flow will be irrotational.

For ideal

$$\omega_x = \omega_y = \omega_z = 0$$

$$\boxed{\mu = 0 \quad \frac{du}{dy} = 0}$$

:- Velocity Potential Function :-

It is defined as a scalar func of space and time such that its -ve derivatives w.r.t. any direction gives the component of vel. along that direction.

It is denoted by ϕ.

$$\frac{-\partial\phi}{\partial x} = u, \quad \frac{-\partial\phi}{\partial y} = v, \quad \frac{-\partial\phi}{\partial z} = w \quad \text{--- (A)}$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$= \frac{1}{2}\left[\frac{-\partial^2\phi}{\partial x\partial y} + \frac{\partial^2\phi}{\partial y\partial x}\right]$$

angular velocity $\omega_z = 0$ [irrotational flow]

⇒ For steady + incompressible flow :-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

From (A)

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad \text{[laplace equation]}$$

$\therefore \omega_z = 0$ [irrotational] {because the above eqn is laplace equation}

Equipotential Lines :- ϕ = constant

It is a line along which the value of ϕ is constant.

$$\phi = c$$
$$d\phi = 0$$

$$d\phi = \frac{\partial\phi}{\partial x}\cdot dx + \frac{\partial\phi}{\partial y}\cdot dy$$

$$0 = -u\,dx - v\,dy$$

$$\boxed{\frac{dy}{dx} = \frac{-u}{v}} \rightarrow \text{equipotential line equation}$$

$$m_2 = \frac{-u}{v}$$

Stream function ψ

It is defined as a scalar function of space & time such that its partial derivative w.r.t. to any direction gives the component of velocity at right angle to this direction.

$$\frac{\partial \psi}{\partial x} = v \quad , \quad \frac{\partial \psi}{\partial y} = -u$$

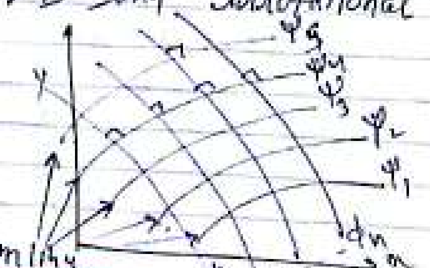
Stream line :-

$$\psi = C$$
$$d\psi = 0$$
$$d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$$
$$0 = v\,dx - u\,dy$$

$$\boxed{\frac{dy}{dx} = \frac{v}{u}} \rightarrow \text{stream line equation.}$$

Flow Net

It is an imaginary grid form by drawing series of stream line & equipotential lines. It is drawn for steady, 2-D and irrotational flow.



$$\boxed{m_1 \times m_2 = -1}$$

Q. Find the eqn of stream line if the velocity field is given by the following eqn which is passing through pt (1,1). $\vec{V} = x\hat{i} - y\hat{j}$. [1,1]

$$\frac{dy}{dx} = \frac{v}{u}$$
$$\vec{V} = u_x\hat{i} + v_y\hat{j}$$
$$u_x = x$$
$$v_y = -y$$

$$\frac{dy}{dx} = \frac{-y}{x} = y.$$
$$\int \frac{1}{y} dy = \int \frac{1}{x} dx$$
$$\int y^{-1} dy = \int x^{-1} dx$$
$$\ln y + \ln x = \ln c$$
$$yx = c$$
$$\boxed{c = 1}$$
$$x \cdot y - 1 = 0$$

Q. Calculate the vel. & acc^n. of a fluid partical which passing through a pt (2,1,3) at $t = 1$ s. Velocity field is equal to

$$\vec{V} = 4x^3\hat{i} - 10x^2y\hat{j} + 2t\hat{k}$$
$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k} \qquad t = 1$$

$$u = 4x^3, \quad v = -10x^2y$$
$$w = 2t$$

$$|V| = \sqrt{u^2 + v^2 + w^2} = \sqrt{(4x^3)^2 + (-10x^2y)^2 + (2t)^2}$$
$$= \sqrt{[4(2)^3]^2 + [10(2)^2 \times 1]^2 + [2 \times 1]^2}$$

$a_x = 1.531$

$$\boxed{|V| = 51.21 \text{ m/s}}$$

$$\frac{du}{dt} = 4x \cdot 3x^2 / -2\dot{x}x$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{du}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$= 4x^3 \cdot x^2 + (10x^3 y) 18x^2 + 2$$

$$= \cancel{32x^5 - 10x^4 + 2} \quad 41x^3 + 2$$

$$= \cancel{32(2)^5 - 10(2)^4 + 2}$$

$a_x = 1536$, $a_y = 320$, $a_z = 2$

$$\boxed{a = 1568}$$

Ques> A flow field is given by the following eq^n

$$\vec{V} = x^2 y\,\hat{i} + y^2 z\,\hat{j} - (2xyz + yz^2)\hat{k}$$

Prove that this is the case of steady + incompressible flow. Also calculate the velocity & acc^n ~~at~~ (1,2,3)

for steady + incompressible.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$u = x^2 y \Rightarrow \cancel{(\ldots)} = 12$

$v = y^2 z \Rightarrow \cancel{(\ldots)} = 0$

$w = 2xyz + yz^2$

$$\cancel{2xy} + 2yz + 2xy - 2yz = 0$$

$$a_x = u\frac{du}{\partial x} + v\frac{du}{\partial y} + w\frac{du}{\partial z}$$

$$a_x = (x^2 y)\, 2xy + y^2 z\,(2yz) + 2xy - 2yz\ldots$$

$$\cancel{a_x = (1^2 \times 3)\, 2 \times 1 \times 3 + \ldots\ 2(2)(3)}$$

$a_x = 18$

$a_y = \ldots$ 1,2,3 5 7

$a_z = \ldots$

$$\boxed{a_x = 20}$$

$$a_x = u\frac{du}{\partial x} + v\frac{du}{\partial y} + w\frac{du}{\partial z}$$

$$a_x = (x^2 y)(2xy)$$

Ques> The vel. potential function is given by $\phi = 5x^2 - 5y^2$, calculate vel. components at the pt (4,5).

$$u = 5x^2, \quad v = 5y^2$$

$$\frac{du}{\partial x} = 10x, \quad \frac{du}{\partial y} = 10y \qquad u = -\frac{\partial \phi}{\partial x}, \quad v = -\frac{\partial \phi}{\partial y}$$

-40

50

$u = 40, \quad v = -50$

Q) Prove that flow will be irrotational flow if $u = y^3/3 + 2x - x^2 y$ and $v = xy^2 - 2y - x^3/3$. Also prove continuity eq^n.

$$\boxed{\omega_z = 0}$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0$$

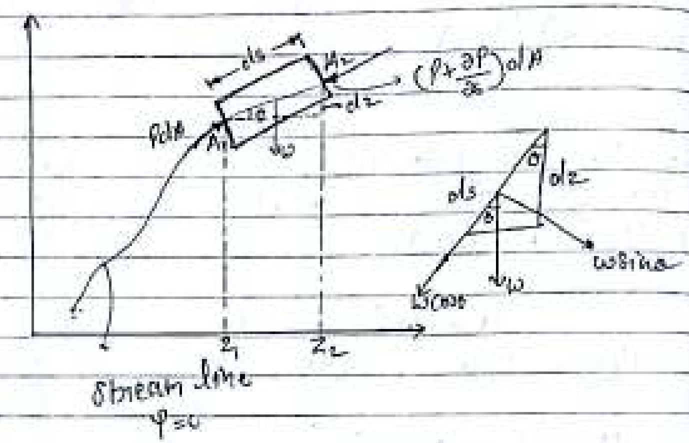
$$= \frac{1}{2}\left[(y^2 - 2 - 0) - \left(\tfrac{1}{3} \cdot 3y^2 + 0 - x^2\right)\right] = 0$$

$$= \frac{1}{2}\left[\cancel{y^2} - \cancel{x^2} - \cancel{y^2} - \cancel{x^2}\right] =$$

$= 0$

14/2/2017

Fluid Dynamics (Bernoulli's Eqn)



Consider a stream line in which ψ = const. as shown in fig.
Consider a small elementary area dA having length ds and weight 'w'.

Consider all forces along stream line.
Resolve all forces along stream line

$PdA - [P + \frac{\partial P}{\partial s} \cdot ds] \cdot dA - w\cos\theta = ma.$ $\quad w = mg$

$\quad w = \rho V g$

$\quad w = \rho dA ds g$

$PdA - PdA - \frac{\partial P}{\partial s} \cdot ds \cdot dA - \rho dA \cdot ds\, g \cos\theta = ma$

$-\frac{\partial P}{\partial s} \cdot ds \cdot dA - \rho g dA \cdot ds \cos\theta - \rho dA \cdot ds (a) = 0$

$\because a = \frac{dv}{dt} \times \frac{ds}{ds}$

$ds\, dA \left(-\frac{\partial P}{\partial s} - \rho g \cos\theta - \rho a\right) = 0$

$= v\frac{dv}{ds}$

$-\frac{\partial P}{\partial s} - \rho g \cos\theta = \rho v \frac{dv}{ds}$

$\cos\theta = \frac{dz}{ds}$

$\frac{\partial P}{\partial s} + \rho g \cdot \frac{dz}{ds} + \rho v \frac{dv}{ds} = 0$

Now divide above eqn by $\cdot \rho$

$\frac{\partial P}{\rho} + g dz + v dv = 0$

By integrating we get

$\int \frac{\partial P}{\rho} + \int g dz + \int v dv = 0$

$\frac{P}{\rho} + gz + \frac{v^2}{2} = 0$

$\frac{P}{\rho} + \frac{v^2}{2} + gz = 0$

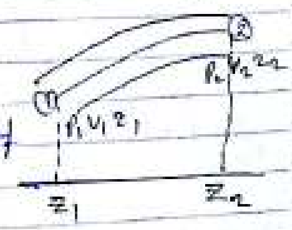
Now, divide it by g.

$$\boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + z = 0}$$

Assumption :-
1. Flow occurs only in stream line.
2. Flow should be incompressible (ρ)
3. Flow should be irrotational.
4. Ideal flow.

* Application of Bernoulli's equation for a real fluid :-

Apply Bernoulli's eqn b/w ① & ②

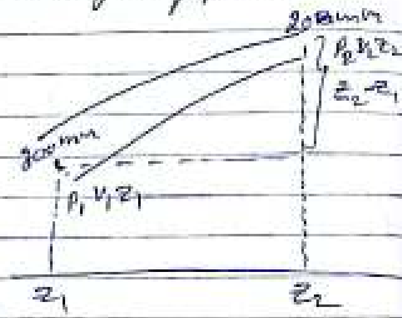


$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$

where, h_f is head loss due to pipe friction.

This equation is valid for real fluid

Ques-> If water is flowing through a pipe having diameter of 300mm & 200mm respectively. At the bottom and upper. The intensity of pressure at the bottom end is 24.525 N/cm² & pressure at upper end 9.81 N/cm². Determine the difference in datum head if flow rate of the pipe is 0.04 m³/s.



$P_1 = 24.525$ N/cm²
$P_2 = 9.81$ N/cm²

$Q = A_1 V_1$

$0.04 = \frac{\pi}{4}(300)^2 \times V_1$

$V_1 = \frac{(0.04)(4)}{\pi(300)^2} = 0.565$

$V_2 = \frac{(0.04)(4)}{\pi(200)^2}$

$\boxed{V_1 = 0.565 \text{ ms}^{-1}}$ $\boxed{V_2 = 1.27 \text{ ms}^{-1}}$

Apply Bernoulli's equation -

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_1}{\rho g} + \frac{P_2}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = z_2 - z_1$$

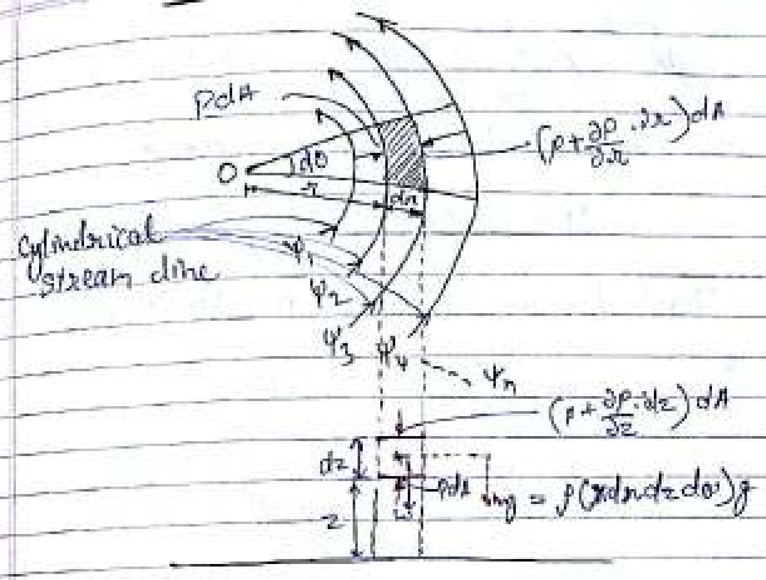
$$\frac{1}{\rho g}(P_1 - P_2) + \frac{1}{2g}(V_1^2 - V_2^2) = \Delta z$$

$$\frac{1}{10 \times 9.81}(24.525 - 9.81) + \frac{1}{2 \times 9.81}(0.565^2 - 1.27^2)$$

$\boxed{\Delta z = 14.9 \text{m}}$

VORTEX FLOW

Cylindrical radial Cylindrical



Consider a cyl. stream lines $\psi_1, \psi_2, \psi_3 \dots \psi_n$ as shown in fig. Consider a small elementary area b/w two stream lines

=> Resolving all the forces along horizontal plane (in radial direction)

$$\left[p + \frac{\partial p}{\partial r} \cdot dr\right] dA - p \cdot dA = \frac{mv^2}{r}$$

$$\frac{\partial p}{\partial r} \cdot dr \cdot dA = \frac{mv^2}{r}$$

$$\frac{\partial p}{\partial r} dr \times dr \cdot dz = \rho \, r d\theta \, dz \, dr \cdot \frac{v^2}{r}$$

$$\frac{\partial p}{\partial r} = \frac{\rho v^2}{r}$$

$$\partial p = \frac{\rho v^2}{r} dr \quad \text{---} (1)$$

resolving all the forces in z-direction

$$p\,dA - \left(p + \frac{\partial p}{\partial z}\cdot dz\right) dA = \rho\, r d\theta \cdot dr \cdot dz\, g$$

$$-\frac{\partial p}{\partial z}\, dz\, r d\theta \cdot dr = \rho\, r d\theta \cdot dr\, dz\, g$$

$$\partial p = -\rho g\, dz \quad \text{--- (2)}$$

Total pressure change in vortex flow

$$\boxed{\partial p = \frac{\rho v^2}{r}\, dr - \rho g\, dz} \quad \text{--- (3)}$$

↳ This eqn is known as pressure distribution eqn in vortex flow motion.

Free Vortex Flow

It is a flow in which fluid rotates without any external expenditure of energy. Here the mass of the fluid rotates either because of motion given to it earlier or because of external mechanical system.

eg:- wash basin flow, cyclones, whirlpool in the river etc.

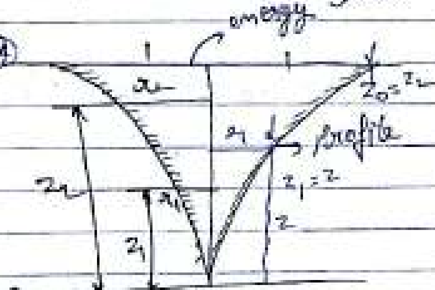
Vortex motion can be classified in two

① Spherical Spiral or

(ii) Cylindrical Vortex

cylindrical vortex is one in which the pattern of motion can be describe by co-centric stream line or vortex is defined as a rotating mass of fluid and defined as a its motion is known as vortex motion.

* Pressure distribution in free vortex :-



$$\partial p = \frac{\rho v^2}{r}\, dr - \rho g\, dz \quad \text{--- (A)}$$

$$\int_{p_1}^{p_2} \partial p = \int_{r_1}^{r_2} \frac{\rho c^2}{r^3}\, dr - \int_{z_1}^{z_2} \rho g\, dz$$

$$p_2 - p_1 = \rho c^2 \left[\frac{1}{r_1^2} - \frac{1}{r_2^2}\right] - \rho g (z_2 - z_1)$$

angular momentum $= mvr$

$T = 0$

$\frac{dL}{dt} = 0$

$L = C$

$mvr = C$

$vr = C$

$V = C/r$ --- (1)

$$p_2 + \rho v_2^2 + \rho g z_2 = \rho v_1^2 + p_1 + \rho g z_1$$

Divide by ρg

$$\boxed{\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_1}$$

Consider a two points at a distance r where $z_1 = z$.

at r tends to ∞

$$z_2 = z_0$$

$$\boxed{p_1 = p_2}$$

$$p_2 - p_1 = \frac{\rho}{2}\left[V_1^2 - V_2^2\right] - \rho g (z_2 - z_1)$$

$$0 = \frac{\rho}{2}\left[\frac{c^2}{r_1^2} - \frac{c^2}{r_2^2}\right] - \rho g (z_1 - z_1)$$

at $r_2 \to \infty$, $z_2 = z_0$

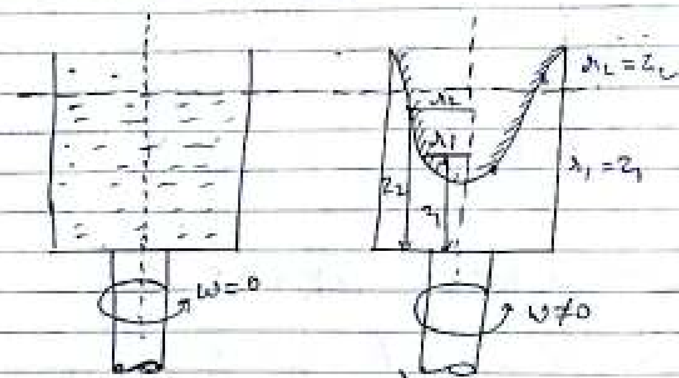
at $r_1 = r$, $z_1 = z$

$$0 = \frac{\rho c^2}{2 r_1^2} - \rho g (z_0 - z)$$

$$0 = \frac{c^2}{g z r^2} - z_0 + z$$

$$\boxed{z = z_0 - \frac{c^2}{2 g r^2}}$$ parabolic eq^n

* Forced Vortex Flow *



We know that in case of forced vortex flow ω = constant

$v = r\omega$

$\omega = \frac{v}{r}$

$$\partial p = \frac{\rho v^2}{r} dr - \rho g dz$$

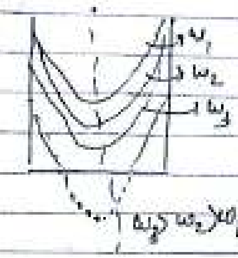
$$\int_1^2 \partial p = \int_1^2 \frac{\rho r^2 \omega^2}{r} \, dr - \int_1^2 \rho g dz$$

$$\int_{p_1}^{p_2} \partial p = \int_{r_1}^{r_2} \rho \omega^2 r dr - \int_{z_1}^{z_2} \rho g dz$$

$$p_2 - p_1 = \frac{\rho \omega^2}{2}\left[r_2^2 - r_1^2\right] - \rho g (z_2 - z_1)$$

This eq^n is known as forced vortex flow eq^n

$\boxed{p_2 = p_1}$



$$0 = \frac{\rho \omega^2}{2}\left[r_2^2 - r_1^2\right] - \rho g \left[z_2 - z_1\right]$$

$$\frac{\rho \omega^2}{2}\left[r_2^2 - r_1^2\right] = \rho g \Delta z$$

$$\Delta z = \frac{\omega^2}{2g}\left[r_2^2 - \cancel{r_1^2}\right]$$

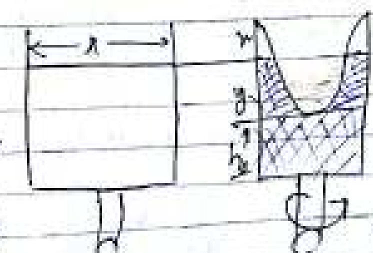
$$\boxed{\Delta z = \frac{\omega^2}{2g} r^2}$$

$\frac{1}{2} \pi r^2 z$ → Area of parabola

For a force vortex flow prove that rise of liquid at the end is equal to fall at the centre.

$\boxed{\text{Prove} \rightarrow x = y}$

→ Apply conservation of volume



$$V_i = \pi r^2 (h + y) \quad \text{--- (1)}$$

$$V_f = \pi r^2 (h + x + y) - \frac{1}{2} \pi r^2 (x + y) \quad \text{--- (2)}$$

① = ②

$\pi r^2(h+y) = \pi r^2(h+x+y) - \frac{1}{2}\pi r^2(x+y)$

$\cancel{\pi r^2}(h+y) = \cancel{\pi r^2}\left[h+x+y-\frac{1}{2}(x+y)\right]$

$h+y = h+x+y-\frac{x}{2}-\frac{y}{2}$

$\therefore \cancel{h}+y = \cancel{h}+\frac{x}{2}+\frac{y}{2}$

$\therefore \frac{y}{2} = \frac{x}{2}$

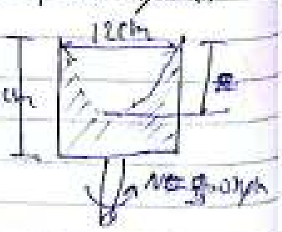
$\boxed{y = x}$

$$\boxed{\Delta P = \frac{\rho\omega^2}{2}\left[r_2^2 - r_1^2\right] \quad \boxed{\Delta P \propto \omega^2 \quad \Delta P \propto r^2}}$$

16/2/17

Ques 7 → A cyl. vessel 12cm in diameter & 30cm deep is filled with water upto the top level. Find the quantity of the liquid in the vessel in when it is rotated about its vertical axis with a speed of 300 rpm & 600 rpm.

Forced vortex



$V = V_c - V_p$

$V_c = \pi r^2 h$

$= \pi \times \left(\frac{0.12}{2}\right)^2 \times 0.30$

$= \cancel{6.785 \times 10^{-3}}$ 3.39×10^{-3} . $[\;\;1100]$

$V_{parabola} = \frac{1}{2}\pi r^2 z = \frac{1}{2}\times\pi\times\left(\frac{0.12}{2}\right)^2 \times 0.184$

$= 1.01\times10^{-3}\,m^3$

$z = \frac{\omega^2 r^2}{2g}$

$\omega = \frac{2\pi N}{60} = \frac{2\pi\times300}{60} = 31.4$ rad/sec.

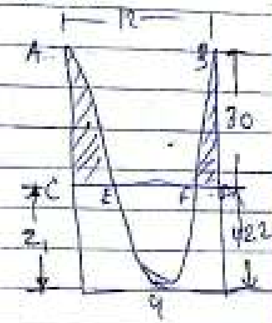
$\therefore z = 0.18$ mm

$V_{final} = 3.39\times10^{-3} - 1.01\times10^{-3} = 2.38\times10^{-3}$

At $N = 600$ rpm

$\omega = \frac{2\pi N}{60} = 62.8$ rad/sec.

$z = \frac{\omega^2 r^2}{2g} = 72.2$ cm



$V_f = V_{ABCD} - \left[V_{AGB} - V_{EFG}\right]$

$V_{ABCD} = \pi r^2 h = \pi\times\left(\frac{0.12}{2}\right)^2\times .30 = \cancel{0.34}$

$= \boxed{3.393\times10^{-3}\,m^3}$

$V_{AGB} = \frac{1}{2}\pi r^2 z = \frac{1}{2}\times\pi\times\left(\frac{0.12}{2}\right)^2\times z$

$z = \frac{\omega^2 r^2}{2g} = \frac{(62.83)^2(0.12/2)^2}{2g} = \cancel{0.44}$ 0.724 m

$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times600}{60} = 62.83$ rad/sec

$\boxed{V_{AGB} = 4.094\,m^3}$

$V_{EFG} = \frac{1}{2}\pi r_1^2\times(z_1) = \boxed{1.41\times10^{-3}}$

$V_f = 4.09$

$z_1 = \frac{\omega^2 r_1^2}{2g} = 0.424$ m

$\frac{(62.83)^2\times r^2}{2\times9.81} = 0.424$

$r = 0.046$ m

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Impulse Momentum Equation

(Force exerted on the pipe bends)

Consider a pipe as shown in the fig.

If consider two pt at section (1,1) & (2,2) showing velocity at section (1,1), V_1, A_1, P_1 & section (2,2) $V_2 A_2 P_2$

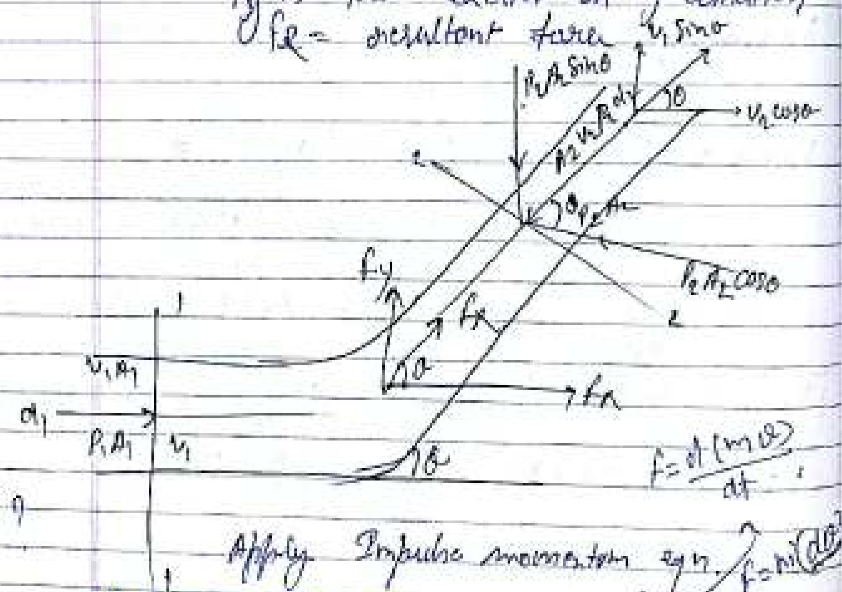
If the pipe in bend at an angle θ with on horizontal axis

$$F_R = \sqrt{F_x^2 + F_y^2}$$

where, F_x = force exerted on x-direction

$F_y \rightarrow$ force exerted on y-direction

F_R = resultant force



$$F = \frac{d(mv)}{dt}$$

Apply Impulse momentum eqn. $F = m(dv)$

Force exerted = $\dot{m}$ [change in velocity]

$\Rightarrow$ Resolving all the forces in x-direction.

$$P_1 A_1 - P_2 A_2 \cos\theta - F_x = \dot{m}\,[\text{change in velocity}]$$

$$\dot{m} = \rho A V = \rho Q$$

$$P_1 A_1 - P_2 A_2 \cos\theta - F_x = \rho Q\,[V_2 \cos\theta - V_1] \quad \text{--- (1)}$$

where, $-ve$ F_x shown force exerted on the fluid.

for bend we take $+ve$

$\Rightarrow$ Resolving all the force in y-direction.

$$0 - P_2 A_2 \sin\theta - F_y = \rho Q\,[V_2 \sin\theta - 0] \quad \text{--- (2)}$$

Ques $\Rightarrow$ A 45° reducing bend is connected to a pipe line, the diameter at inlet & outlet are 600, 300 respectively. Find the force exerted on the bend if pressure at inlet i.e. $P_1 = 8.829$ N/cm² rate of flow of water $Q = 0.6$ m³/s, find the resultant force at the bend.

$Q = 0.6$ m³/s

$P_1 = 8.829 \times 10^4$ N/m²

$d_1 = 600$ mm

$d_2 = 300$ mm

$Q = A_1 V_1 = A_2 V_2$

$0.6 = \frac{\pi}{4} d_1^2 V_1$

$V_1 = \frac{(0.6)4}{\pi d^2} = 2.122 \times 10^{-1}$

$V_2 = \frac{0.6 \times 4}{\pi \times 300^2} = 8.488 \times 10^{-6}$

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$

$\frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{2.12 \times 10^{-6}}{2 \times 9.81} = \frac{P_2}{1000 \times 9.81} + \frac{8.48 \times 10^{-6}}{2 \times 9.81}$

$0.0991 + 1.070 \times 10^{-7} = \frac{P_2}{1000 \times 9.81} + 4.32 \times 10^{-?}$

$P_2 = 98.21$

$Q = 0.6\, m^3/s$

$P_1 = 8.829 \times 10^4\, N/m^2$

$d_1 = 600\, mm = 600 \times 10^{-3}\, m$

$d_2 = 300\, mm = 300 \times 10^{-3}\, m$

$Q = A_1 V_1 = A_2 V_2$

$0.6\, m^3/s = \frac{\pi}{4} d_1^2 V_1$

$\frac{0.6 \times 4}{\pi \times d_1^2} = V_1$

$\boxed{V_1 = 2.12\, m^3}$

$0.6 = \frac{\pi}{4} d_2^2 V_2$

$\frac{0.6 \times 4}{\pi \times d_2^2} = V_2$

$\boxed{V_2 = 8.49\, m/s}$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{(2.12)^2}{2 \times 9.81} = \frac{P_2}{1000 \times 9.81} + \frac{(8.49)^2}{2 \times 9.81}$$

$$\boxed{P_2 = 84.497 \times 10^3}$$

Now, find F_x & F_y

$$P_1 A_1 - P_2 A_2 \cos\theta - F_x = \rho Q [V_2 \cos\theta - V_1]$$

$$(8.829 \times 10^4)(0.283) - (84.5 \times 10^3)(0.07)\cos(45^\circ) - F_x = 1000(0.6)[8.49 \cos 45^\circ - 2.12]$$

$22055.12 - F_x = 2601.51$

$\boxed{F_x = 19483.11}$

$P_2 A_2 \sin\theta - F_y = \rho Q (V_2 \sin\theta - 0)$

$\boxed{F_y = 32086.63}$

$F_R = \sqrt{F_x^2 + F_y^2} = 37538.81$

4.27

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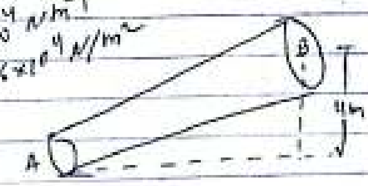
70 Date 17/2/2017

$\boxed{\ \int = 0.37 \times 100\ }$

71

① Properties of fluid	⑤ Fluid kinematics ✓
② Manometer	⑥ Laminar ✓
③ Hydrostatic ✓	⑦ Pipe flow
④ Meta-centre	⑧ Fluid dynamics ✓

Ques) A pipe line carrying oil of sp. gr. 0.87 changes in diameter from 200mm at a position (A) to 500mm dia at a position (B) which is 4m at a higher level. If P_A & P_B are $9.81\, N/cm^2$ & $5.886\, N/cm^2$ respectively. The $Q = 0.2\, m^3/s$. Determine the loss of head & direction of flow.



$P_A = 9.81\, N/cm^2 = 9.81 \times 10^4\, N/m^2$

$P_B = 5.886\, N/cm^2 = 5.886 \times 10^4\, N/m^2$

$(D)_A = 200\, mm$

$(D)_B = 500\, mm$

$Q = 0.2\, m^3/s$

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_f$$

$Q = A_1 V_1$

$0.2 = \frac{\pi}{4} d_1^2 \times V_1$

$\frac{0.2 \times 4}{\pi \times d_1^2} = V_1$

$\boxed{V_1 = 6.36\, ms^{-1}}$

$\boxed{\text{energy at A}}$ $\boxed{h_f = 2.6\, m}$

Energy at A =

$$\frac{9.81 \times 10^4}{0.87 \times 1000 \times 9.81} + \frac{(6.36)^2}{2 \times 9.81} + 0 = 10.077$$

$\frac{0.87}{1000}$

$= 5.7 \times 10^{-4}$

$V_2 = \frac{0.2 \times 4}{\pi \times (d_2)^2}$

$\boxed{V_2 = 0.509\, ms^{-1}}$ 1.01

Energy at B

$10.01 + h_f$

$E_A = 12m$

$E_B = 10m$

$E_A = E_B + h_f$

$=$

Pressure head = $\frac{P}{\rho g}$

(A)

Given, A conical tube of length 2m is fixed vertically with its smaller end upward. The vel. of flow at smaller end is $5\,ms^{-1}$ while at the lower end it is $2\,ms^{-1}$. Pressure head at the smaller end is 2.5m of the liquid column. If the head loss b/w the lower end & upper end is $h_f = \frac{0.35(v_1-v_2)^2}{2g}$

$v_1 \rightarrow$ vel. of the smaller end

$v_2 \rightarrow$ " " bigger end.

Find the pressure head at the lower end.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

$$2.5 + \frac{5^2}{2\times9.81} + 0 = \frac{P_2}{\rho g} + \frac{2^2}{2\times9.81} + 2 + h_f$$

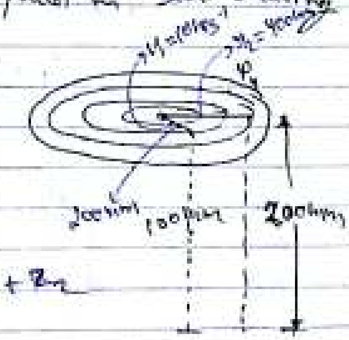
$2.703 =$

$$h_f = \frac{0.35(5-2)^2}{2\times9.81}$$

$= 0.16$

Q.) ~~A open circular cyl. vessel with~~

In a free cyl. vortex flow at a point in a fluid at radius of 200mm and at a height of 100 mm, the vel. & pressure are $10\,ms^{-1}$ & ~~117.72~~ 117.72 kN/m^2. Find the pressure at the radius of 400 mm & at a height 200mm. If the fluid is air having density is 1.24 kg/m^3



Torque is const.

$\therefore \; v_1 r_1 = v_2 r_2$

$\boxed{v_2 = 5\,m/s}$

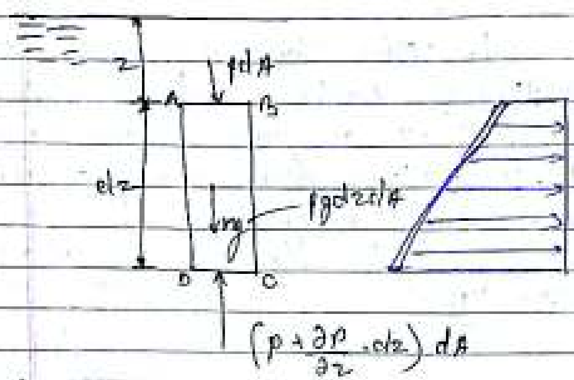
$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{117.72}{1.24\times9.81} + \frac{10^2}{2\times9.81} + 100 = \frac{P_2}{2\times9.81} + \frac{5^2}{2\times9.81} + 200$$

$\boxed{P_2 = 117.67\,N/m^2}$

Date 17/8/2017

Hydrostatic Law



⇒ Consider a fluid element ABCD having area dA & height dz as shown in the figure.

Resolving all the forces in y-direction. But [force in x-direction = 0]

$$pdA - (p+\frac{\partial p}{\partial z}.dz)dA + \rho g\,dA\,dz = 0$$

$$-\frac{\partial p}{\partial z}.dA.dz + \rho g\,dA.dz = 0$$

$$\frac{\partial p}{\partial z} = \rho g$$

$$\partial p = \rho g\,dz$$

Differentiating –

$$\boxed{p = \rho g z}$$

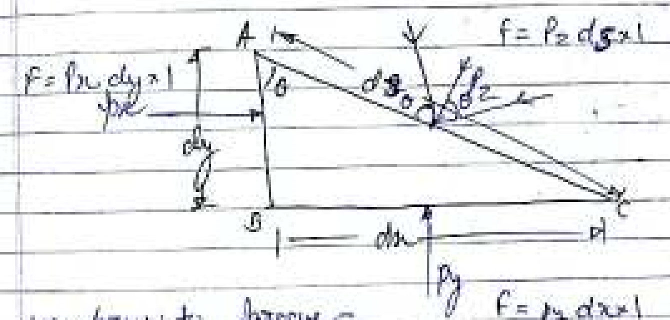
Hydrostatic law states that pressure in a static fluid increase in a vertical downward direction. It means z & p both increase.

It is applicable for both compressible and incompressible fluid

Pascal Law

It states that pressure at a point in a fluid at rest has the same magnitude in all direction. In other word, if a certain pressure is applied at any pt in static fluid, it gets equally transmitted in all the direction.

⇒ Consider a fluid element ABC



we have to prove –

$$\boxed{P_x = P_y = P_z}$$

Resolve all the forces in x-direction.

$$P_x\,dy \times 1 = P_z\,ds.1.\cos\theta$$

$$P_x.dy = P_z.ds.\cos\theta$$

$$P_x.dy = P_z.ds.\frac{dy}{ds}$$

$$P_x = P_z \quad \text{---} \quad (1)$$

Ref. Resolve all the forces in y-direction

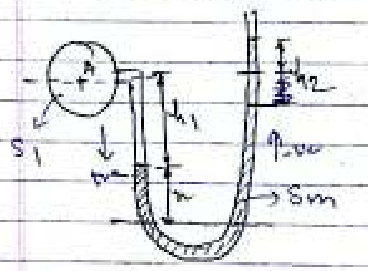
$$p_y \cdot dx \cdot 1 = p_z\, ds \cdot \sin\theta$$
$$p_y\, dx = p_z \cdot ds \cdot \frac{dx}{ds}$$

$$\boxed{p_y = p_z} \quad \text{--- (II)}$$

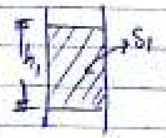
From equation (I) & (II)

$$\boxed{p_x = p_y = p_z}$$

U-tube Manometers



(I)



$$\approx \boxed{h_w = h_1 S_1}$$ height of water column

Write gauge equation in terms of water column

$$h_w + h_1 S_1 + x S_m - (x + h_1 + h_2)\, S_m = 0$$

Pitot Tube

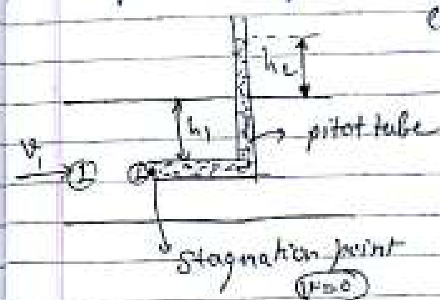
→ For velocity measurement.

It is a instrument used for measurement of vel. of flow in open channel as well as in pipes.

Pitot tube work under the principle of stagnation pressure. Stagnation pressure is such achieve when flowing fluid is retarded to zero vel. in a frictionless process.

Stagnation pressure at which the velocity zero.

case-1) open pipe



⇒ Apply Bernoulli's equation at point (1) & (2)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\boxed{V_2 = 0}$$

$$\boxed{Z_1 = Z_2}$$

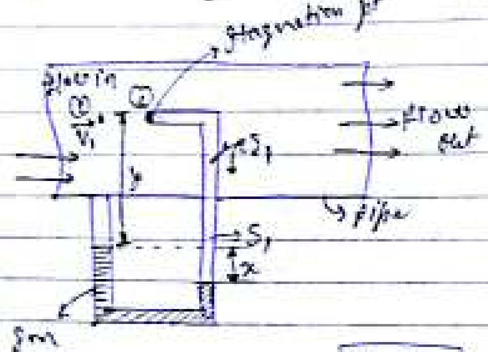
we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g}$$

$$h_1 + \frac{V_1^2}{2g} = (h_2 + h_1)$$

$$\boxed{V_1 = \sqrt{2 g h_2}}$$

Case II Now, for velocity measurement closed pipe



$\boxed{z_1 = z_2}$

Apply Bernoulli's theorem b/w (1) & (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \qquad \boxed{V_2 = 0}$$

$$\frac{P_2}{\rho g} - \frac{P_1}{\rho g} = \frac{V_1^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_1^2}{2g}$$

$$h = \frac{V_1^2}{2g} \quad \text{---- (A)}$$

$$\boxed{V_1 = \sqrt{2gh}}$$

⇒ Writing gauge eqn in terms of water column.

$$h_{w_1} + yS_1 + xS_m - (x+y)S_1 = h_2\omega$$

$$h\omega_1 + xS_m - xS_1 = h_2\omega$$

$$\cancel{h_{w_1} + xS_m - x}$$

$$h_2\omega - h_1\omega = x(S_m - S_1) \quad \text{---- (1)}$$

$$(h_2 - h_1)S_1 = x(S_m - S_1)$$

$h_1\omega = h_1 S_1$
$h_2\omega = h_2 S_2$

$$hS_1 = x(S_m - S_1)$$

$$h = \frac{x(S_m - S_1)}{S_1}$$

$$\boxed{h = x\left(\frac{S_m}{S_1} - 1\right)}$$

Reference: Lecture R.N. Laminar

Date 22/2/2017

Laminar Flow

Laminar flow in which the fluid flowing this sheet or layers or laminae, smoothly sliding over one another.

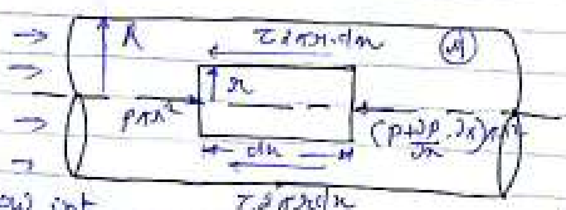
In a laminar flow viscous shear stress b/w layer of the fluid reserved) result in decrease of energy of the flowing fluid.

Due to this pressure of energy decrease which results in a negative pressure gradient along the direction flow.

⊛ Steady uniform flow through a circular pipe

Consider a pipe having diameter 'd' as shown in the figure.

Consider a small elementary strip having radius 'r'.



$\tau = \frac{F}{A} = \mu \frac{\partial u}{\partial y}$

Resolving all the forces in x-direction is,

$\boxed{F_x = 0}$

$p\pi r^2 - \left(p + \frac{\partial p}{\partial x} \cdot dx\right)\pi r^2 - \tau 2\pi r dx = 0$

$-\frac{\partial p}{\partial x} dx \cdot \pi r^2 = \tau \cdot 2\pi r dx$

$$\boxed{\left(-\frac{\partial p}{\partial x}\right)\left(\frac{r}{2}\right) = \tau} \quad \text{----} (1)$$

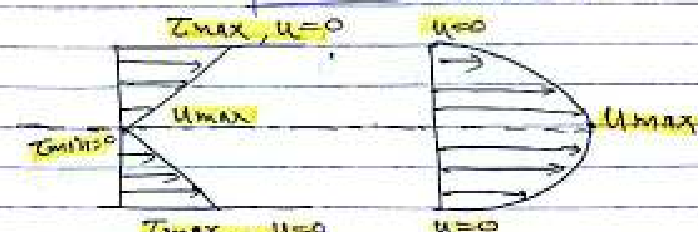
$\boxed{\tau \propto r}$

τ_{min} at $r = 0$

$\boxed{\tau_{min} = 0}$

τ_{max} at $r = R$

$$\boxed{\tau_{max} = \left(\frac{-\partial p}{\partial x}\right)\left(\frac{R}{2}\right)}$$



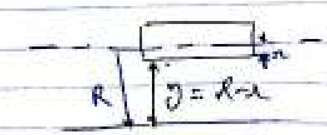
Shear stress Distribution. velocity profile

⇒ We know that acc. to Newtons law of viscosity

$\tau = \mu \frac{du}{dy}$ ---- (2)

From (2) & (1)

$\tau = -\mu \frac{du}{dr} = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}$



$\boxed{dy = -dr}$

$\int du = \int \frac{\partial p}{\partial x} \cdot \frac{r}{2\mu} dr$

$$\int du = \frac{\partial p}{\partial x}\frac{1}{2\mu}\int r\,dr$$

$$u = \frac{\partial p}{\partial x} \times \frac{1}{2\mu} \times \frac{r^2}{2}$$

$$\boxed{u = \frac{\partial p}{\partial x} \cdot \frac{1}{4\mu} r^2 + C_1} \text{ ---- (A)}$$

at $r = R$
$u = 0$

$$C_1 = \frac{-\partial p}{\partial x} \cdot \frac{1}{4\mu} R^2$$

From eqn (A), $u = \frac{\partial p}{\partial x} \cdot \frac{1}{4\mu} r^2 - \frac{\partial p}{\partial x} \cdot \frac{1}{4\mu} R^2$

$$\boxed{u = \frac{-\partial p}{\partial x} \cdot \frac{1}{4\mu} (R^2 - r^2)} \text{ ---- (B)}$$

$$\boxed{u \propto r^2}$$

U_{min} at $r = R$

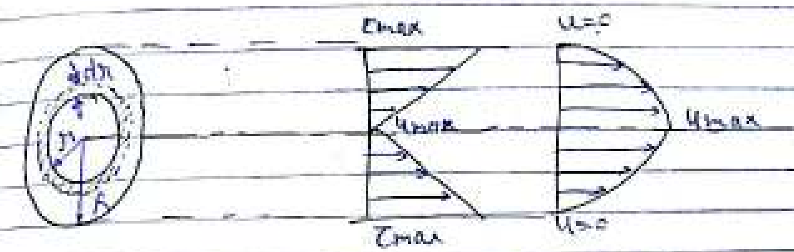
$$\boxed{u = 0} \quad \boxed{u_{min} = 0}$$

u_{max} at $r = 0$

$$\boxed{u_{max} = \left(\frac{-\partial p}{\partial x}\right) \frac{1}{4\mu} R^2} \text{ ---- (C)}$$

From equation (B) the velocity profile is parabola

Discharge through a Circular pipe



We know the discharge through a elementary area:- $dA = 2\pi r\,dr$

$$dQ = A u$$
$$= 2\pi r\,dr \left[\frac{-\partial p}{\partial x} \frac{(R^2 - r^2)}{4\mu}\right]$$

Total discharge

$$\int_0^Q dQ = \int_0^R 2\pi r\,dr \left[\frac{-\partial p}{\partial x} \cdot \frac{1}{4\mu} (R^2 - r^2)\right]$$

$$Q = 2\pi\,dr \int_0^R \left(\frac{-\partial p}{\partial x}\right) \frac{1}{4\mu} (rR^2 - r^3)$$

$$Q = 2\pi \left(\frac{-\partial p}{\partial x}\right) \frac{1}{4\mu} \int_0^R (rR^2 - r^3)\,dr$$

$$Q = 2\pi \left(\frac{-\partial p}{\partial x}\right) \frac{1}{4\mu} \left[\frac{r^2 R^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$Q = \frac{-\partial p}{\partial x} \times \frac{\pi}{2\mu} \left[\frac{R^4}{4}\right]$$

$$\boxed{Q = \left(\frac{-\partial p}{\partial x}\right) \frac{\pi R^4}{8\mu}} \text{ ---- (D)}$$

$Q = A\bar{u}$, average velocity

$Q = A\bar{u}$

from (D)

$$\left(\frac{-\partial p}{\partial x}\right)\frac{\pi R^4}{8\mu} = \pi R^2 \bar{u}$$

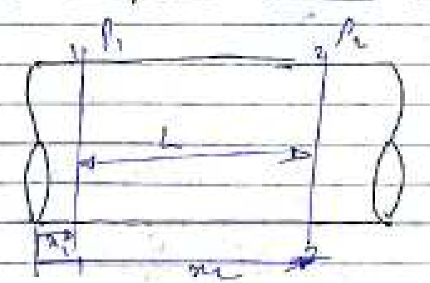
$$\bar{u}_{avg} = \frac{-\partial p}{\partial x} \times \frac{R^4}{8\mu \times R^2}$$

$$\boxed{\bar{u}_{avg} = \frac{-\partial p}{\partial x} \times \frac{R^2}{8\mu}} \quad \text{--- (E)}$$

$$\boxed{u_{max} = \frac{-\partial p}{\partial x} \times \frac{R^2}{4\mu}} \quad \text{---}$$

$$\boxed{\frac{\bar{u}_{avg}}{\bar{u}_{max}} = \frac{1}{2}}$$

* Pressure Drop in laminar Flow :—



Apply Bernoulli theorem b/w 1-1 & 2-2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2} + h_f$$

$\boxed{Z_1 = Z_2}$ $\boxed{\text{uniform cross section are}}$ $(V_1 = V_2)$

$\boxed{A_1 V_1 = A_2 V_2}$ $\boxed{V_1 = V_2}$

85

$$\therefore \; h_f = \frac{P_1 - P_2}{\rho g}$$

Is known as head loss b/w 1-1 & 2-2

We know that, $\bar{u}_{avg} = \frac{-\partial p}{\partial x} \cdot \frac{R^2}{8\mu}$

$$\left(\frac{-\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2} = \frac{32\mu\bar{u}}{D^2}$$

$$\frac{P_1 - P_2}{x_2 - x_1} = \frac{32\mu\bar{u}}{D^2}$$

Assume, $x_1 - x_2 = l$

$$P_1 - P_2 = \frac{32\mu\bar{u} \times L}{D^2}$$

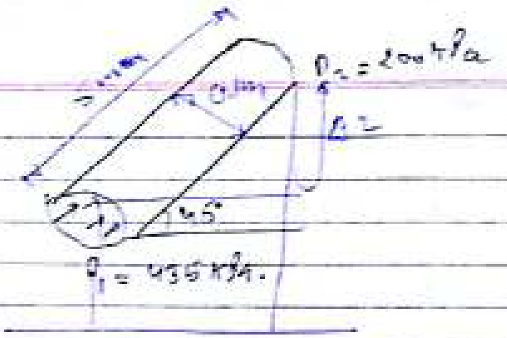
But, we know that

$$\boxed{h_f = \frac{32\mu\bar{u}L}{\rho g D^2}}$$ ← Hagen Pressure drop equation

$\boxed{h_f \propto \bar{u}}$ laminar (h_f) | turbulent (h_f)

Ques => A laminar flow is flowing through a circular pipe having the diff. b/w two (d = 5m) end $P_1 = 435$ kPa, $P_2 = 200$ kPa. Find the discharge through the pipe if it inclined 45° with the horizontal if the dia of the pipe is 100 mm having viscosity 0.8 N s/m². Calc Q = ? through the pipe.

Solution:



$Q = \frac{-\partial p}{\partial x} \cdot \frac{\pi D^4}{128\mu}$ not applicable

Applying Bernoulli's theorem b/w 1-1, 2-2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

$h_f = \frac{P_1 - P_2}{\rho g}$

$\boxed{Q = 0.7 m^3/s}$

$$P_1 - P_2 = \frac{32 \mu \bar{u} l}{D^2}$$

$$435 - 200 = 32 \times 0.5 \times \bar{u} \times 5$$

$$\boxed{\begin{aligned} Q &= A\bar{u} \\ &= \frac{\pi}{4}(.1)^2 \bar{u} \end{aligned}}$$

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_f$$

uniform crossection $V_1 = V_2$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + (z_2 - z_1) + \frac{32 \mu \bar{u} l}{\rho g D^2}$$

$$\frac{4.17}{\cancel{0.043}} = \frac{1.962}{\cancel{0.02}} + 5\sin 45 + \frac{32 \times 0.5 \times \bar{u} \times 5}{1000 \times 9.81 \times 2} \qquad \bar{u} = 15.1 m/s$$

27

$$\frac{2.203}{\cancel{0.023}} = 3.54 + \frac{\cancel{0.5} \times 12\,\bar{u}}{19620}$$

$$-\frac{1.332}{\cancel{0.023}} = \frac{69454.3 + 123\bar{u}}{19620}$$

451.26 =

If the fluid is flowing from 2-2 to 1-1 calculate pressure at point (2) if the average velocity of flow 15 m/s

$P_2 = ?$

$P_2 = 612$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + (z_2 - z_1) + \frac{32 \mu \bar{u} l}{\rho g d^2}$$

$$\frac{435}{1000 \times 9.81} = \frac{200}{1000 \times 9.81} + 5\sin(45°) + \frac{32 \times 0.5 \times \bar{u} \times 5}{1000 \times 9.81 \times 0.1^2}$$

$$0.044 = 0.0203 + 3.54 + 1.31\,\bar{u}$$

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_f$$

Due to uniform crossection $V_1 = V_2$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + z_2 - z_1 + h_f$$

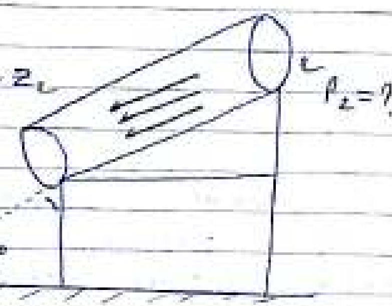
$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + 5\sin 45 + \frac{32 \mu \bar{u} l}{\rho g D^2}$$

$$\frac{435\times10^3}{1000\times9.81} = \frac{200\times10^3}{1000\times9.81} + 3.54 + \frac{32\times0.8\times5\times\bar{u}}{1000\times9.81\times(0.1)^2}$$

$\boxed{\bar{u} = 15.58}$

$$Q = A\bar{u}$$
$$= \frac{\pi}{4}d^2\times\bar{u}$$
$$= \frac{\pi}{4}(0.1)^2\times15.58$$
$$= 0.12\ m^3/s$$

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1+h_f = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$



$$\frac{P_1}{\rho g}+(z_1-z_2)+h_f = \frac{P_2}{\rho g}$$

$$\rho g\left(\frac{P_1}{\rho g}+(z_1-z_2)+h_f\right) = P_2$$

$$h_f = \frac{32\mu\bar{u}L}{\rho g\times d^2}$$

$\tau_{max} = \frac{-\partial p}{\partial x}\cdot\frac{R}{2}$ at $r=R$

$\tau_{min} = 0$ at $r=0$

$$= \frac{32\times0.8\times15.58\times5}{1000\times9.81\times(0.1)^2}$$
$$= 20.71$$

(1) $\tau = \frac{-\partial p}{\partial x}\cdot\frac{r}{2}$ (4) $Q = \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right)R^4$

(2) $u = \frac{-\partial p}{\partial x}\cdot\frac{1}{4\mu}(R^2-r^2)$ (5) $\frac{\bar{u}}{u_{max}} = \frac{1}{2}$

(3) $\bar{u} = \frac{-\partial p}{\partial x}\cdot\frac{R^2}{8\mu}$ (6) $\frac{P_1-P_2}{w} = h_f$

(7) $\frac{P_1-P_2}{h_f} = \frac{32\mu\bar{u}L}{\rho g d^2}$

Laminar flow ($h_f \propto V$)

Turbulent flow ($h_f \propto V^2$) $\boxed{(h_f)_{Turb.} > (h_f)_{Laminar}}$

Q.) A laminar flow which is flowing through a circular pipe having dia 50mm & length 300m. If the sp. density of oil is 0.9 & $\mu = 0.1\ N s/cm^2$. If the flow through a pipe is $0.0035\ m^3/s$. Calculate the shear stress at the wall & also calculate the pressure diff. in a length of 300m.

$d = 50mm$
$L = 300m = 300\times10^3\ mm$
$\mu = 0.1\ N s\,cm^{-2}$
$Q = 0.0035\ m^3 s^{-1}$

$\tau = \frac{-\partial p}{\partial x}\cdot\frac{r}{2}$ $Q = \left(\frac{-\partial p}{\partial x}\right)\frac{\pi R^4}{8\mu}$

$\frac{0.0035\times8\mu}{\pi R^4} = \frac{-\partial p}{\partial x}$

$\tau = 2.24\times10^{-7}\times\frac{0.5}{2}$ $\frac{-\partial p}{\partial x} = 0.28\times10^{-7}$

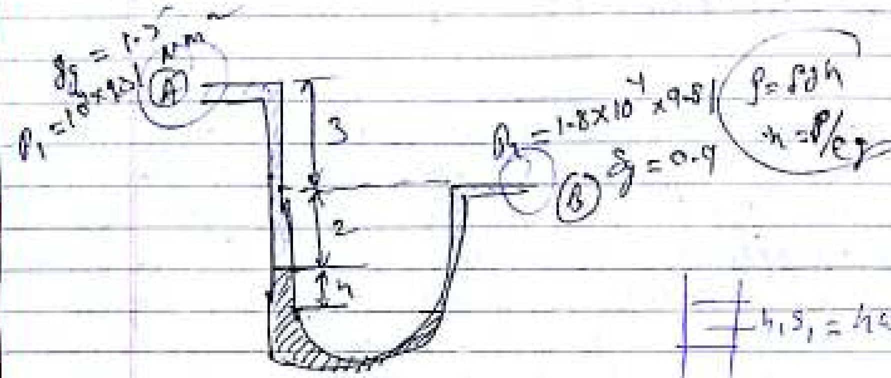
$\tau = 2.85\times10^{-8}$

$\boxed{\tau = 22.82\ Pa}$ $Q = \left(\frac{\partial p}{\partial x}\right)\frac{\pi R^4}{8\mu}$

$0.0035\times$

$\boxed{\Delta P = 6844\ N/m^2 \quad 68.4\ N/cm^2}$

Q-> Calculate the diff. b/w mercury level shown in figure!



$p = \rho g h$

$h = P/\rho g$

$h_1 S_1 = h_{2O\,stm}$

$h_{A\,H_2O} + (3+2)1.5 + h \times 13.6 - (2+h)0.9 = h_{B\,H_2O}$

$10.005 + 7.5 + 13.6h - 1.8 - 0.9 \times h = 18$

$h_A = \frac{P}{\rho g}$

$P_A = h_A \rho g$

$10^4 \times 9.81 = h_A\, 1.5 \times 1000 \times 9.81$

$h_A = 6.67m$

$P_B = h_B \rho_B g$

$10^4 \times 9.81 \times 1.8 = h_B \times 0.9 \times 1000 \times 9.81$

$\boxed{h_B = 20m}$

$\therefore h_{A\,H_2O} = h_A S_A = 6.67 \times 1.5$

$h_{A\,H_2O} = 10.005$

$h_{B\,H_2O} = h_B S_B$

$= 20 \times 0.9$

$= 18$

$\boxed{h = 18\ cm}$

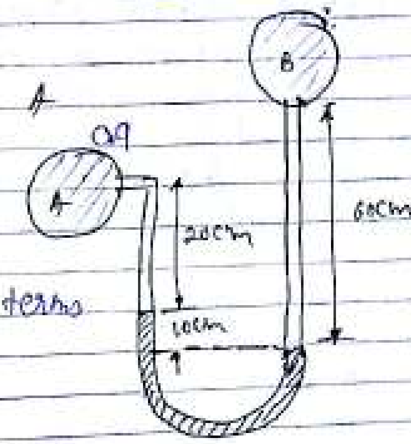
Calculate pressure at A

given

$P_B = 9.81\ N/cm^2$

Write gauge eq^n in terms of water column

$S_A = 1$



$h_{A\,H_2O} + 0.20 \times 0.9 + 0.10 \times 13.6 - 0.60 \times 1 = h_{B\,H_2O}$

$P_{B\,H_2O} = h_{B\,H_2O} \times \rho_{H_2O} \times g$

$9.81 = h_{B\,H_2O} \times 1000 \times 9.81$

$(h_B)_{H_2O} = 10$

$h_{A\,H_2O} = 9.06\ m$

$P_A = (h_A)_{H_2O} = 4.51$

$9.06 = h_1 \times 0.9$

$h_1 = 10.06$

$P_A = \rho g h$

$= 0.9 \times 1000 \times 9.81 \times 10.06$

$= 8.8$

Ques→ A fluid of viscosity 0.7 Ns/m² sp. gr. 1.3 is flowing through of circular pipe having dia. = 100 mm. The Max Shear stress at the pipe wall = 196.2 N/m². Calculate pressure gradient & average velocity.

Solution→ $\mu = 0.7\ Ns/m^2$

Sp. gr. = 1.3

$d = 100mm = 100 \times 10^{-3} m$

$\tau_{max} = 196.2\ N/m^2$

$\tau_{max} = \frac{-\partial p}{\partial x} \cdot \frac{R}{2}$ 3.5

$\frac{196.2 \times 2}{} = \frac{-\partial p}{\partial x}$

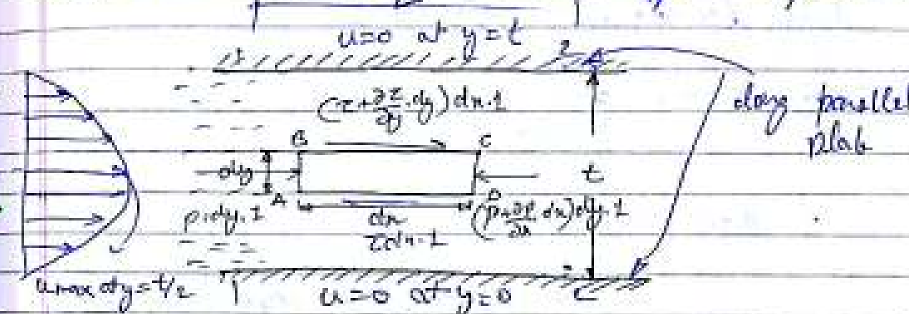
100×10^{-3}

$\frac{-\partial p}{\partial x} = ~~78~~\ 7848$

Date 2/3/2017

* Laminar flow between two infinite long parallel plate :-

Consider two infinite long parallel plate as shown in the fig.
Consider the small elementary area dx.dy and the distance b/w two parallel plate is t.



Resolving all the forces in x-direction.
$\Sigma F_x = 0$

$$p\,dy.1 - \left(p + \frac{\partial p}{\partial x} \cdot dx\right) dy.1 - \tau\, dx.1 + \left(\tau + \frac{\partial \tau}{\partial y} \cdot dy\right) dx.1 = 0$$

$$\frac{\partial p}{\partial x} \cdot dx \cdot dy \cdot 1 = \frac{\partial \tau}{\partial y} \cdot dy\, dx \cdot 1$$

$$\boxed{\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}} \quad --- (1)$$

Acc. to Newton law of viscosity —

$$\tau = \mu \frac{du}{dy}$$

$$\boxed{\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}} \quad --- (2)$$

Scanned by CamScanner

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial x}$$

Now, integrating we get –

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial P}{\partial x}\cdot y + C_1$$

Again

$$u = \frac{1}{\mu}\frac{\partial P}{\partial x}\cdot y^2/2 + C_1 y + C_2 \quad \text{---} \; (3)$$

Apply boundary condition –

(i) $u=0, \; y=0$

$$\boxed{C_2 = 0}$$

(ii) $u=0, \; y=t$

$$0 = \frac{1}{\mu}\frac{\partial P}{\partial x}\cdot\frac{t^2}{2} + C_1 t$$

$$u = \frac{-1}{2\mu}\frac{\partial P}{\partial x}\left[ty - y^2\right] \quad \text{———} \; (4)$$

$u \propto y^2$ (parabolic profile)

U_{max} at $y = t/2$

$$u = -\frac{1}{2\mu}\frac{\partial P}{\partial x}\left\{t\left(t/2\right) - \frac{t^2}{4}\right\}$$

$$u = -\frac{1}{8\mu}\cdot\frac{\partial P}{\partial x}\cdot t^2 \; \text{-----} \; (5)$$

Small discharge

$$dQ = u \cdot dA$$

$$\int_0 dQ = \int_0^t \frac{1}{-2\mu}\frac{\partial P}{\partial x}\left[ty - y^2\right] dy \cdot 1$$

$$Q = \frac{1}{-2\mu}\frac{\partial P}{\partial x}\int_0^t \left(ty - y^2\right) dy$$

$$Q = \frac{1}{-2\mu}\frac{\partial P}{\partial x}\left\{\frac{ty^2}{2} - \frac{y^3}{3}\right\}_0^t$$

$$= \frac{-1}{2\mu}\frac{\partial P}{\partial x}\left\{\frac{t^3}{2} - \frac{t^3}{3}\right\}$$

$$Q = \frac{-1}{12\mu}\cdot\frac{\partial P}{\partial x}t^3 \quad \text{———} \; (6)$$

$$\bar{u} = \frac{Q}{A} = \frac{-\frac{1}{12\mu}\cdot\frac{\partial P}{\partial x}}{t \cdot 1} = \frac{-t^2}{12\mu}\cdot\frac{\partial P}{\partial x}$$

$$\boxed{\bar{u} = \frac{-t^2}{12\mu}\cdot\frac{\partial P}{\partial x}} \quad \text{——} \; (7)$$

$$\boxed{\frac{U_{max}}{\bar{u}} = -\frac{3}{2}} \qquad \boxed{\bar{u} = \frac{2}{3}U_{max}}$$

Apply bernoulli's theorem in b/w (1-1) & (2-2)

$$\frac{P_1}{\rho g} + \cancel{\frac{v_1^2}{2g}} = \frac{P_2}{\rho g} + \cancel{\frac{v_2^2}{2g}} + h_f$$

$$h_f = \frac{P_1 - P_2}{\rho g} \quad \text{----} \; (8)$$

$h_f \rightarrow$ head loss b/w (1-1) & (2-2)

$$\bar{u} = -\frac{1}{12\mu}\cdot\frac{\partial p}{\partial x} t^2$$

$$\frac{\partial p}{\partial x} = \frac{-12\mu\bar{u}}{t^2}$$

$$\int_1^2 \partial p = \int_1^2 \frac{-12\mu\bar{u}}{t^2}\, dx$$

$$P_2 - P_1 = \frac{-12\mu\bar{u}}{t^2}[x_2 - x_1]$$

$$P_1 - P_2 = \frac{12\mu\bar{u}L}{t^2}$$

Putting in eqn (8)

$$\boxed{h_f = \frac{12\mu\bar{u}L}{\rho g t^2}} \text{---}$$

Laminar flow through circular pipe

1. $u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$
2. $U_{max} = -\frac{1}{4\mu}\cdot\frac{\partial p}{\partial x}R^2$
3. $\frac{U_{max}}{\bar{u}} = 2$
4. $\bar{u} = \frac{-1}{8\mu}\cdot\frac{\partial p}{\partial x}R^2$
5. $q = \frac{-\partial p}{\partial x}\cdot\frac{\pi R^4}{8\mu}$
6. $h_f = \frac{32\mu\bar{u}L}{\rho g d^2}$

Laminar flow through two parallel plate

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]$$

$$U_{max} = \frac{-1}{8\mu}\cdot\frac{\partial p}{\partial x}\cdot t^2$$

$$\frac{U_{max}}{\bar{u}} = 3/2$$

$$\bar{u} = \frac{-1}{12\mu}\cdot\frac{\partial p}{\partial x}\cdot t^2$$

$$q = -\frac{1}{12\mu}\cdot\frac{\partial p}{\partial x}t^3$$

$$h_f = \frac{12\mu\bar{u}L}{\rho g t^2}$$

$[\mu_1 < \mu_2 \;,\; u_1 < u_2]$

Q. In a pipe flow having dynamic viscosity 12×10^{-4} N·s/m² is flowing with an average velocity 20 m/s. If the r = 1mm calculate pressure gradient.

$$\bar{u} = \frac{-1}{8\mu}\cdot\frac{\partial p}{\partial x}\cdot R^2$$

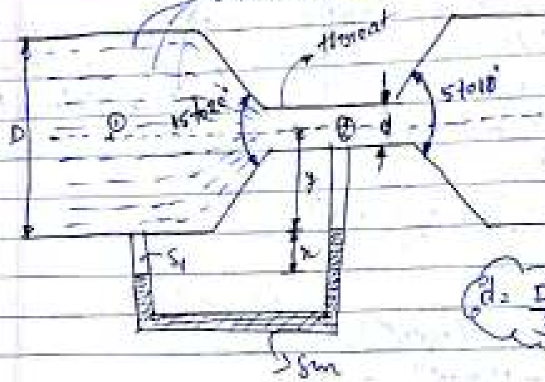
$$20 = \frac{-1}{8(12\times10^{-4})}\cdot\frac{\partial p}{\partial x}\times(1)^2$$

$$20\times8\times12\times10^{-4} = \frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial x} = 19.2 \text{ kPa } MN/m^3$$

Date 10/2/2018

* VENTURIMETER * → For discharge measurement



$d = \frac{D}{3}$

If x = distance of mercury head

Consider a pt ① & ② apply Bernoulli theorem b/w ① & ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

Pressure head $\left\{ \frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} \right.$

$$\frac{P_1 - P_2}{w} = \frac{V_2^2 - V_1^2}{2g}$$

Say,

$$\boxed{h = \frac{V_2^2 - V_1^2}{2g}}$$

where,

h → venturi head

Write gauge eqn in terms of water column

$h_{1_{H_2O}} + (x+y)S_1 + xS_m - yS_1 = h_{2_{H_2O}}$

$h_{1w} + xS_1 - xS_m = h_{2w}$

$h_{1w} - h_{2w} = x(S_m - S)$

$$\boxed{\begin{aligned} h_{1w} &= h_1 S_1 \\ h_{2w} &= h_2 S_1 \end{aligned}}$$

$S_1[h_1 - h_2] = x(S_m - S)$

$h_1 - h_2 = x\left(\frac{S_m}{S} - 1\right)$

$$\boxed{h = x\left[\frac{S_m}{S} - 1\right]}$$

$h = \frac{V_2^2 - V_1^2}{2g}$

$Q = A_1 V_1 = A_2 V_2$

$\frac{Q}{A_1} = V_1, \quad V_2 = Q/A_2$

$h2g = Q^2/A_2^2 - Q^2/A_1^2$

$$\boxed{Q = \frac{\sqrt{2gh} \cdot A_1 A_2}{\sqrt{A_1^2 - A_2^2}}}$$ Theoretically.

* Cd = coefficient of discharge & it is the ratio of Q actual divided by Q theoretical

$$Cd = \frac{Q_{actual}}{Q_{theoretical}}$$

$$Q_{Actual} = Cd \times \frac{\sqrt{2gh}\; A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

Q⇒ A horizontal venturimeter is fitted in pipe line having sp. gr. 0.8 . A diff. of mercury column is 2.5cm (x) of density of mercury 13.6 . Calculate actual Q if Cd = 0.98

Sp. gr. = 0.8
x = 2.5 cm = 2.5mm
Cd = 0.98

D = 100mm
d = 50 mm

$$h = x\left[\frac{S_m}{S} - 1\right]$$

$$= 2.5\left[\frac{13.6}{0.8} - 1\right]$$

$$\boxed{h = 40mm}$$

$$Q_{Actual} = \frac{0.98 \times \sqrt{2 \times 9.8 \times 40} \times \frac{\pi}{4}\times(100)^2 \times \frac{\pi}{4}(50)^2}{\sqrt{\frac{\pi}{4}(100)^2 - \frac{\pi}{4}(50)^2}}$$

$= 289338.3101$

Ques⇒ A laminar flow is flowing in circular pipe having dia 20mm if the max. velocity is 1.5 m/s . calculate the average vel. & radius at which it occurs.

d = 20mm
$u_{max} = 1.5$ m/s

$$u_{max} = \cancel{-\frac{1}{4\mu}}$$

$$\frac{u_{max}}{\bar{u}} = 2$$

$$\frac{1.5}{\bar{u}} = 2$$

$$\boxed{\bar{u} = 0.75 \text{ m/s}}$$

$$u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2)$$

$$u_{max} = -\frac{1}{4\mu}\cdot\frac{\partial p}{\partial x}R^2$$

$$\frac{\bar{u}}{u_{max}} = \left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$\frac{0.75}{1.5} = \left[1 - \left(\frac{r}{100}\right)^2\right]$$

$r = 70$ mm

$$\frac{0.75}{1.5} = 1 - \frac{r^2}{(100)^2}$$

$$\frac{r^2}{100^2} = 1 - \frac{0.75}{1.5}$$

$$\boxed{r = 70.7\text{ mm}}$$

Q⇒ Calculate the pressure gradient along flow, average vel. and the discharge of for an oil of viscosity 0.20 Ns/m² flowing b/w two stationary parallel plate 1m wide & 10mm apart. The vel. mid way b/w the plate 2m/s.

$\mu = 0.20$ Ns/m²
$u_{max} = 2$ m/s

$$u_{max} = -\frac{1}{8\mu}\cdot\frac{\partial p}{\partial x}t^2$$

$$2\text{ m/s} = \frac{1}{8(0.2)}\cdot\frac{\partial p}{\partial x}\cdot(10mm)$$

$$-\frac{\partial p}{\partial x} = 3.2 \times 10^{-4}$$

$= 3200 \cdot (3200)$ N/m

$$\frac{u_{max}}{\bar{u}} = 3/2$$

$$\frac{2}{\bar{u}} = 3/2$$

$\bar{u} = 1.33$ m/s

$$Q = -\frac{1}{12\mu}\frac{\partial P}{\partial x}t^3$$

$$= -\frac{1}{12(0.5)} \times 3200 \times (10 \times 10^{-3})^3$$

$$= \frac{1}{12 \times 0.02} \times 3200 \times (0.01)^3$$

$$= -1.33 \times 10^{-3}$$

* Flow through pipes *

Pipe is a closed force zone which is used for transporting fluid under pressure. The fluid flowing through a pipe line is always subjected to viscous resistance & frictional resistance. The resistance to the flow of fluid is general known as frictional resistance. Because of this resistance the ~~are~~ energy of flowing fluid & due to which pressure of the fluid decreases known as pressure gradient.

In a pipe flow there are two types of losses known as ~~min~~ minor losses & major losses.

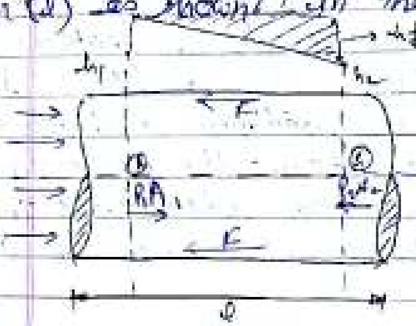
* Minor losses:-

1) Due to sudden enlargement
2) Due to sudden constraction
3) Due to sudden exit
4) Due to pipe fitting
5) Due to bend of the pipe.

* Major losses:-

* Darcy weisbatch equation. *

Consider a pipe having dia (d) & length (L) as shown in the figure.



$h_1 = \frac{p_1}{\omega}$ but $h_2 = p_2/\omega$

$h_1 > h_2$

$h_1 \rightarrow$ pressure height at pt ①

$h_2 \rightarrow$ " " " ②

but $h_1 > h_2$, from figure.

$$h_1 - h_2 = h_f \quad \text{------} ①$$

$h_f \rightarrow$ head loss due to pipe friction.

Apply bernoulli's theorem b/w ① & ② pt

$$\frac{p_1}{\omega} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\omega} + \frac{V_2^2}{2g} + z_2 + h_f$$

$$\boxed{h_f = \frac{p_1 - p_2}{\omega}} \quad \text{------} ①$$

Consider only frictional force — frictional force

Subtraction, $\Sigma F_x = 0$

$$p_1 A_1 = p_2 A_2 + F$$

$$F = (p_1 - p_2) A \quad \text{---} \quad A_1 = A_2 = A$$

$$F = h_f \cdot \omega \cdot A$$

$$\boxed{h_f = \frac{F}{\omega \cdot A}} \quad \text{------} ②$$

As per darcy, $\boxed{\text{frictional force} = f' \times \text{wetted area} \times V^n}$

where, $n \Rightarrow 2$ (for turbulent flow)

$$\boxed{F = f' \times P \times L \times V^2} \quad \text{------} ③$$

From eq^n ③ & ②

$$h_f = \frac{f' \times P \times L \times V^2}{\omega \cdot A}$$

where,

$m = A/P$ → perimeter

$m \rightarrow$ hydraulic mean depth

$$m = \frac{\pi/4 \, d^2}{\pi d} = d/4$$

$$h_f = \frac{f'}{\omega} \times \frac{4}{D} \cdot L V^2$$

As for darcy –

$$\boxed{\frac{4f'}{w} = \frac{f}{2g}}$$

$$\therefore \boxed{h_f = \frac{f \times L V^2}{2gd}}$$ → Darcy - weisbach equation

$f \rightarrow$ friction factor
$f' \rightarrow$ friction coefficient

$$\boxed{f = 4f'}$$

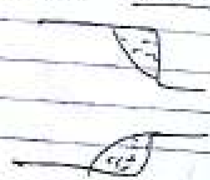
$$\boxed{\begin{aligned} h_f &\propto L \\ h_f &\propto v^2 \\ h_f &\propto 1/d \end{aligned}}$$

* Energy Grade line *

*① Minor losses due to sudden enlargement :–

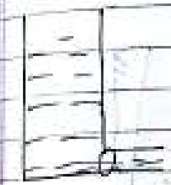
$$\boxed{h_f = \frac{(v_1 - v_2)^2}{2g}}$$

② Minor losses due to sudden contraction :–



$$\boxed{h_f = \frac{0.5V^2}{2g}}$$

③ Minor losses due to sudden exit :–



$$\boxed{h_f = \frac{0.5V^2}{2g}}$$

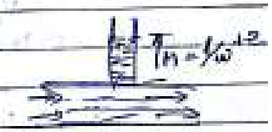
* Energy grade line * or Grade line

⇒ Energy lines are of two types

① h.g.l known as hydraulic grade line

It is a graphical representation of variation of piezometric head at diff pt along the length of the containing a fluid.

piezometric head = $P/w + z$



where, $\frac{P}{w} \rightarrow$ pressure head
$z \rightarrow$ potential head

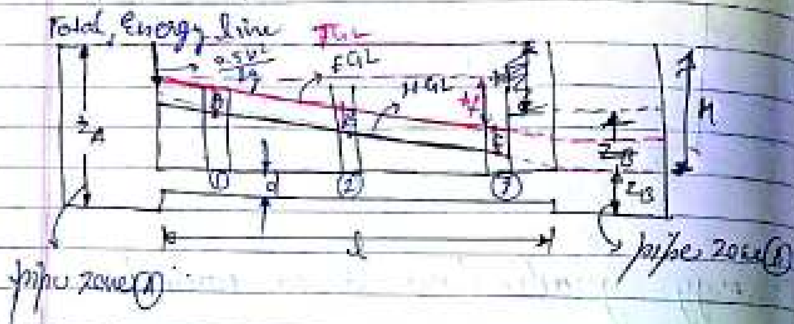
② Energy grade line :–

E.G.L is known as energy grade line

It is graphical representation of total head at any pt along the length of the pipe containing a fluid.

$$E.G.L = \underbrace{\frac{P}{w}}_{\text{pressure head}} + \overbrace{\frac{V^2}{2g}}^{\text{velocity head}} + z$$

* Graphical representation of HGL & EGL *



Consider a pipe line having length (l) and dia (d) as shown.

Consider three piezometer located at the point ①, ② & ③ as shown.

$$Q = AV$$

$$V_1 = V_2 = V_3 = V \quad \text{, because } A_1 = A_2 = A_3$$

& discharge will be same.

$$HGL = \frac{P}{\omega} + Z$$

$$EGL = \frac{P}{\omega} + \frac{V^2}{2g} + Z$$

$Z_A \rightarrow$ water level at A
$Z_B \rightarrow$ " " " B

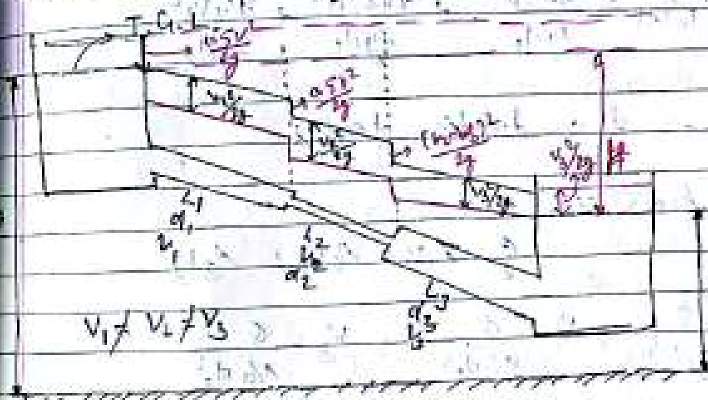
$$\boxed{Z_A - Z_B = H}$$

$$\boxed{Z_A - Z_B = \underbrace{\frac{0.5V^2}{2g}}_{\text{minor loss}} + \underbrace{\frac{4fLV^2}{2gd}}_{\text{major loss}} + \frac{V^2}{2g}} \rightarrow \text{minor loss}$$

* Pipe Network *

Series Combination :-

Consider a series network having three pipe line as shown in figure.



$V_1 \neq V_2 \neq V_3$

$$Z_A - Z_B = H$$
$$= \Sigma \text{ total loss}$$
$$= \frac{0.5V^2}{2g} + \underbrace{\frac{4fL_1V_1^2}{2gd_1}}_{M} + \frac{0.5V^2}{2g} + \underbrace{\frac{4fL_2V_2^2}{2gd_2}}_{M} + \frac{(V_2-V_3)^2}{2g} +$$
$$\underbrace{\frac{4fL_3V_3^2}{2gd_3}}_{M} + \frac{V_3^2}{2g}$$

Equivalent pipe :-

It is a pipe of uniform diameter having the loss of head & discharge equal to that of the compound consisting of different pipes of different diameter.

Note:→ neglecting minor losses

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Consider three pipes having dia d_1, d_2 & d_3 and length L_1, L_2 & L_3 & velocity V_1, V_2 & V_3.

Total head loss (H_{Total})

$$H_{Total} = \frac{4fL_1V_1^2}{2gd_1} + \frac{4fL_2V_2^2}{2gd_2} + \frac{4fL_3V_3^2}{2gd_3}$$

where, $f \rightarrow$ friction factor

We know that

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

$$V_1 = \frac{Q}{\frac{\pi}{4}d_1^2}, \quad V_2 = \frac{Q}{\pi/4\, d_2^2}, \quad V_3 = \frac{Q}{\pi/4\, d_3^2}$$

$$H_{Total} = \frac{fL_1Q^2}{2g\, d_1^5 \times (\pi/4)^2} + \frac{fL_2Q^2}{2g\, d_2^5 (\pi/4)^2} + \frac{fL_3Q^2}{2g\, d_3^5 (\pi/4)^2} \quad \text{---} (1)$$

for equivalent pipe:-

My $h_{f_{equ.}} = \frac{f\, L_{equ.} \times V_{equ.}^2}{2g \times d_{equ.}}$

But, we know that:-

$$Q = A_{eqn.} \times V_{eqn.}$$

$$V_{equ.} = \frac{Q}{\frac{\pi}{4} d_{eqn}^2}$$

$$h_{f_{eqn.}} = \frac{f \cdot L_{eqn.} \cdot Q^2}{2g \cdot d_{eq}^5 (\pi/4)^2} \quad \text{---} (2)$$

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from (1) & (2)

$H_{Total} = h_{f_{equ.}}$, we get

$$\boxed{\frac{L_{eq.}}{D_{eq}^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}}$$

Dupit equation

In general form -

$$\boxed{\frac{L_{eq.}}{D_{eq}^5} = \sum_{n=1}^{n} \frac{L_n}{d_n^5}}$$

Date 17/3/2017

coeff. (4f)

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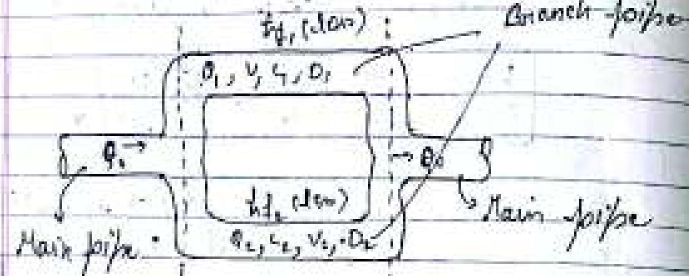
Parallel pipe network



For parallel network discharge in the main pipe divided it self in the branches but the head loss in the parallel network remain same (according to darcy)

$$Q_0 = Q_1 + Q_3 \quad \text{---} \ (1)$$

$$h_{f_1} = h_{f_2} = h_f \quad \text{----} \ (2)$$

$$\frac{4fL_1V_1^2}{2gd_1} = \frac{4fL_2V_2^2}{2gd_2} \quad \text{----} \ (3)$$

where

L

⇒ Equivalent pipe :-

In case of equivalent pipe

$$Q_{eq.} = Q_1 + Q_2$$

$$A_{eq}\cdot V_{eq} = A_1V_1 + A_2V_2$$

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$$Q_1 = A_1V_1$$

$$\boxed{V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{\pi/4\, d_1^2}}$$

Similarly

$$\boxed{V_2 = \frac{Q_2}{\pi/4\, d_2^2}}$$

$$A_{eq}.\ V_{eq} = A_1V_1 + A_2V_2$$

$$\cancel{\frac{\pi}{4}} d_{eq}^2 \cdot V_{eq} = \cancel{\frac{\pi}{4}} d_1^2V_1 + \cancel{\frac{\pi}{4}} d_2^2V_2$$

$$\boxed{d_{eq}^2 V_{eq} = d_1^2V_1 + d_2^2V_2} \quad \text{----} \ (4)$$

Acc. to darcy eqn.

$$h_{f_1} = \frac{4fL_1V_1^2}{2g\, d_1} \Rightarrow V_1 = \sqrt{\frac{2gh_{f_1}}{f}}\ \sqrt{\frac{d_1}{L_1}}$$

Similarly

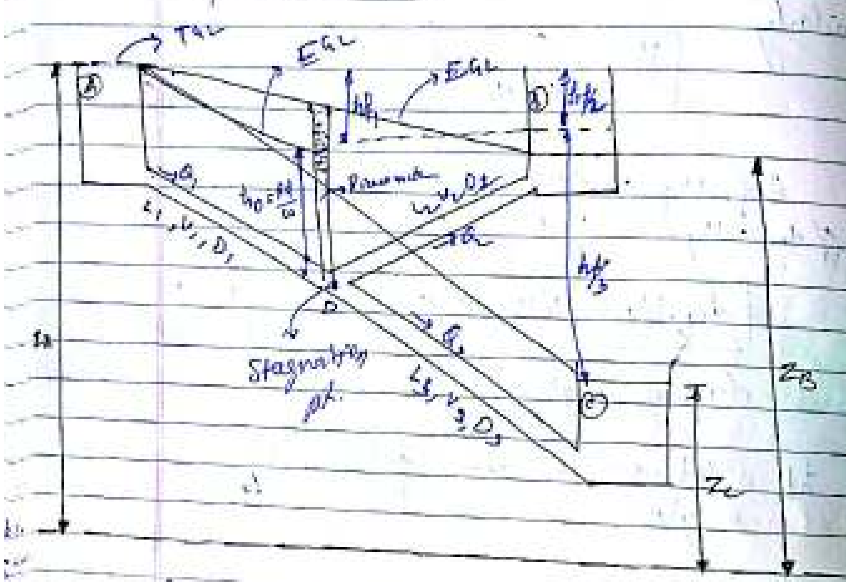
$$V_2 = \sqrt{\frac{2gh_{f_2}}{f}}\ \sqrt{\frac{d_2}{L_2}}$$

From eqn (4)

$$d_{eq}^2\sqrt{\frac{2gh_{f_{eq}}}{f}}\ \sqrt{\frac{D_{eq}}{L_{eq}}} = d_1^2\left[\sqrt{\frac{2gh_{f_1}}{f}}\ \sqrt{\frac{d_1}{L_1}}\right] + d_2^2\left[\sqrt{\frac{2gh_{f_2}}{f}}\ \sqrt{\frac{d_2}{L_2}}\right]$$

$$\boxed{\frac{d_{eq}^{5/2}}{\sqrt{L_{eq}}} = \frac{d_1^{5/2}}{\sqrt{L_1}} + \frac{d_2^{5/2}}{\sqrt{L_2}}}$$ Dupit equation

* Pipe Network *



Point D is known as stagnation pt.
Apply Bernoulli's theorem b/w A & D

$$Z_A = \frac{P_d}{w} + Z_D + h_{f_1}$$

Similarly b/w B & D

$$Z_B = \frac{P_d}{w} + Z_D + h_{f_2}$$

~~As~~ simi

Ques> A pipe line series network used to discharge water from tank A to B. having $l = (300m, 170m, 210m)$ respectively, $d = (300mm, 200mm, 400mm)$, having friction coeff. are 0.005, 0.0052, 0.0045 respectively. If the diff. b/w tank A & B is 12m. Calculate the vel. at pt ① pipe one (V_1) and also find the discharge. Consider minor losses.

$$Z_A - Z_B = 21/g$$
$$= 12$$

$$12 = \frac{0.5V_1^2}{2g} + \frac{4fL_1V_1^2}{2gd_1} + \frac{0.5V_2^2}{2gd_2} + \frac{4fL_2V_2^2}{2gd_2}$$
$$+ \frac{(V_2 - V_3)^2}{2g} + \frac{4fL_3V_3^2}{2gd_3} + \frac{V_3^2}{2g}$$

$$Q = V_1A_1 = A_2V_2 = A_3V_3$$

$$V_1A_1 = A_2V_2$$

$$\boxed{\frac{V_1A_1}{A_2} = V_2}$$

$$A_2V_2 = A_3V_3$$

$$\boxed{\frac{A_2V_2}{A_3} = V_3}$$

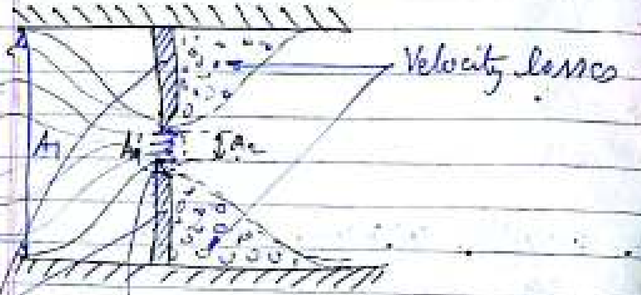
$$\frac{A_2\left(\frac{V_1A_1}{A_2}\right)}{A_3} = V_3$$

$$\boxed{\frac{V_1A_1}{A_3} = V_3}$$

$$12 = \frac{0.5V_1^2}{2g} + \frac{4fL_1V_1^2}{2gd_1} + \frac{0.5\left(\frac{V_1A_1}{A_2}\right)^2}{2gd_2} + \frac{4fL_2\left(\frac{V_1A_1}{A_2}\right)^2}{2gd_2}$$
$$+ \frac{\left(\frac{V_1A_1}{A_2} - \frac{V_1A_1}{A_3}\right)^2}{2g} + \frac{4fL_3\left(\frac{V_1A_1}{A_3}\right)^2}{2gd_3} + \frac{\left(\frac{A_1V_1}{A_3}\right)^2}{2g}$$

Vinacontractra (stream line parallel) 114

* Orifice meter *



Orifice plat — parallel streamlines (Vina contractra)

* Expression for Coefficient of friction in terms of shear stress *

Consider a pipe flow, having a wall shear stress τ const, at pt 1 pressure force is equal P_1A_1 and at pt 2 pressure force $= P_2A_2$

$\Sigma F_x = 0$

$P_1A_1 = P_2A_2 + F$ → shear force

$(P_1 - P_2)A = F$ — ① $\because A_1 = A_2 = A = \frac{\pi}{4}d^2$

We know that acc. to darcy equation

$$h_f = \frac{4fLV^2}{2gd} = \frac{P_1 - P_2}{\rho g}$$

$$P_1 - P_2 = \frac{4fLV^2\rho g}{2gd} \quad \text{--- ②}$$

$\tau = \frac{F}{A}$

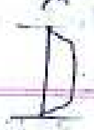
From ① & ②

$$\underbrace{\frac{4fLV^2\rho g}{2gd} \times \frac{\pi}{4}d^2}_{} = \overbrace{\tau \times \pi \times D \times L}^{F}$$

$$\frac{fV^2\rho}{2} = \tau$$

$$\boxed{f = \frac{2\tau}{\rho V^2}} \;**$$

$u' \to$ fluctuating factor



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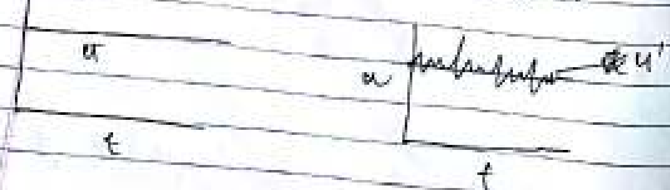
* Turbulent flow *

Mostly the flow through the pipe is always turbulent. Turbulent flow is a random, disordered, bulk mixing flow that occurs at a very higher vel. compare to laminar flow.

The vel. profile in turbulent flow is uniform and it becomes more uniform at increasing reynold's no. At any point, the vel. field in turbulent flow can be written as the combination of $\bar{u} + u'$. However $\bar{u} + u'$ is a long range cycle time interval is considered then the turbulent flow can be considered as steady flow.

In case of laminar vel. profile is non uniform but in turbulent flow vel. distribution is approximately uniform (due to higher molecular momentum exchange)



Total vel. of turbulent flow $= u' + \bar{u}$

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$\bar{u} =$ average vel.

$u' =$ fluctuating component (function of time)
$u' = f(t)$

① **SHEAR STRESS IN TURBULENT FLOW**

⇒ In case of turbulent flow the total shear stress

$$\tau_{total} = \tau_{viscous} + \tau_{random} \quad [1814] \text{ Ist. Theory}$$

$$= \mu\left(\frac{d\bar{u}}{dy}\right) + \eta\left(\frac{d\bar{u}}{dy}\right)$$

$$= \rho\nu\left(\frac{\partial\bar{u}}{\partial y}\right) + \rho\epsilon\left(\frac{d\bar{u}}{dy}\right)$$

where

$\mu \to$ dynamic viscosity

$\nu \to$ kinematic viscosity

$\eta \to$ eddy viscosity

$\epsilon \to$ eddy kinematic viscosity $= \dfrac{\eta}{\rho \text{(mass density)}}$

⇒ Reynolds Shear Stress theory (1886) $[\bar{\tau} = \rho u' v']$

$$\tau_{turbulent} = \rho \bar{V_y} . \bar{V_x}$$

where

$\bar{V_y}, \bar{V_x}$ are the fluctuating component in x, y direction due to turbulence

⇒ Prandtl mixing length theory (1925)

Acc. to prandtl mixing length theory –

$$V_x = l\left(\frac{d\bar{u}}{dy}\right)$$

$$V_y = l\left(\frac{d\bar{u}}{dy}\right)$$

Shear velocity (u_*) = $\sqrt{\frac{\tau_0}{\rho}}$

Now

$\overline{u'\times v'}$ becomes as $\overline{u'v'} = l^2\left(\frac{du}{dy}\right)^2$ --- (1)

Acc. to Reynold stress

$$\bar{\tau} = \rho\overline{u'v'}$$

From (1)

$$\bar{\tau} = \rho\, l^2\left(\frac{du}{dy}\right)^2$$

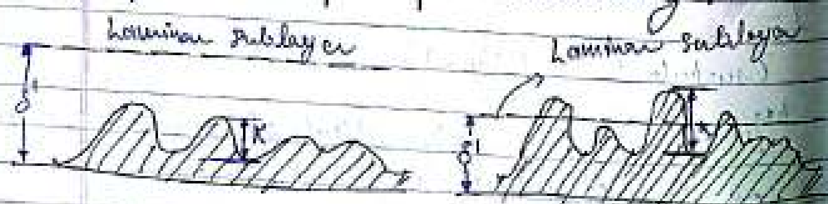
expression for shear stress in turbulent flow due to Prandtl

Total shear stress at any point in turbulent flow

$$\bar{\tau} = \mu\frac{du}{dy} + \rho l^2\left(\frac{du}{dy}\right)^2$$

(2) Hydrodynamically Smooth and Rough Boundaries

Let, k is the average height of the irregularities projecting from surface of a boundary.



(a) Smooth boundary (b) Rough boundary

From Nikuradse's experiment:

If $\frac{K}{\delta'} < 0.25$ the boundary is called smooth boundary.

$\frac{K}{\delta'} > 6.0$, the boundary is rough.

$0.25 < \frac{K}{\delta'} < 6.0$, the boundary is in transition.

In terms of roughness Reynolds no. $\frac{u_* k}{\nu}$:

where, $u_* \rightarrow$ Shear velocity

$k \rightarrow$ Karman constant $= 0.4$

$\frac{u_* k}{\nu} < 4$, boundary is considered smooth.

$\frac{u_* k}{\nu}$ lies b/w 4 & 100, boundary is in transition stage.

$\frac{u_* k}{\nu} > 100$, the boundary is rough.

(3) Velocity Distribution for Turbulent Flow in smooth pipes.

21/3/2017

* Prandtl mixing length theory *

Acc. to Prandtl length theory

$$\left[V_x = l\frac{d\bar{u}}{dy} \;,\; V_y = l\frac{d\bar{u}}{dy} \right] \quad \text{--- (i)}$$

we know that the turbulent shear stress

$$\tau_{turbulent} = \rho \, V_x . V_y$$

where, $l \rightarrow$ Prandtl mixing length

$\therefore$ from (i)

$$\boxed{\tau_{turbulent} = \rho \, l^2 \left(\frac{d\bar{u}}{dy}\right)^2}$$

Acc. to Karman,

$$l \propto y$$

$$l = ky \quad \text{--- (ii)}$$

where,

k = universal karman const. or kappa factor

As per Nikuradse –

$$l = 0.4y - 0.44 \, y^2/R$$

But, $y <<< R$

$$\therefore \; l = 0.4y \quad \text{--- (iii)}$$

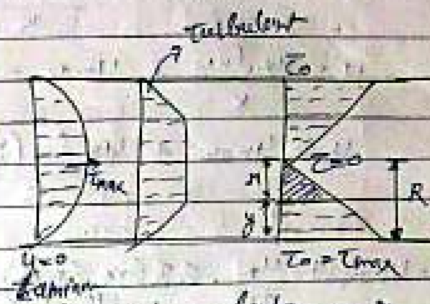
From (ii) & (iii)

$$k = 0.4$$

Velocity distribution equation in turbulent flow

$$\boxed{\tau_{total} = \mu \frac{du}{dy} + \tau_{turb.}}$$

$$\boxed{\tau_{total} = \underbrace{\mu \frac{du}{dy}}_{\text{laminar}} + \underbrace{\rho K^2 y^2 \left(\frac{du}{dy}\right)^2}_{\text{fluctuating factor}}}$$



Acc. to similar Δ rule –

$$\frac{\tau}{\tau_0} = \frac{r}{R}$$

$$\tau = \tau_0 . \frac{r}{R}$$

Where,

τ = shear stress acting at a distance r.

τ_0 = wall shear stress at a distance R

y = distance b/w max. shear stress at 'R' and shear stress at distance r.

$$\boxed{r = R - y}$$

$$\tau = \tau_0 . \frac{R-y}{R}$$

$$\tau = \tau_0 \left(1 - y/R\right)$$

Assume :- $y <<< R$

$$\boxed{\tau = \tau_0}$$

$$\tau_0 = \rho K^2 y^2 \left(\frac{du}{dy}\right)^2$$

$$\sqrt{\frac{\tau_0}{\rho}} = ky \left(\frac{du}{dy}\right)$$

$$u_* = ky \left(\frac{du}{dy}\right) \quad \text{----} \quad \boxed{u_* = \sqrt{\frac{\tau_0}{\rho}}}$$

$$du = \frac{u_* \, dy}{ky}$$

where,

K = kapa factor.

By integrating, we get –

$$u = \frac{u_* \ln y}{K} + C \rightarrow C = u - \frac{u_* \ln y}{K}$$

C = karman const.

Applying boundary condition -
at $y = R$, $u = u_{max}$

$$c = u_{max} - \frac{u_* \ln R}{K}$$

$$u = \frac{u_* \ln y}{K} + u_{max} - \frac{u_* \ln R}{K}$$

$$u = u_{max} + \frac{u_* \ln (y/R)}{K}$$

$$u = u_{max} + \frac{u_*}{0.4} \cdot 2.3 \log_{10}(y/R) \quad \text{--- } [\ln_e = 2.3 \log_{10}]$$

$$\boxed{u = u_{max} + 5.75 u_* \log_{10}(y/R)}$$

This eqn. is known as vel. distribution eqn. in turbulent flow

$$\boxed{u \propto \log(y)}$$

$$u - u_{max} = 5.75 \log_{10}(y/R) \cdot u_*$$

$$\frac{u - u_{max}}{u_*} = 5.75 \log(y/R)$$

$$\frac{u_{max} - u}{u_*} = 5.75 \log(R/y)$$

where,

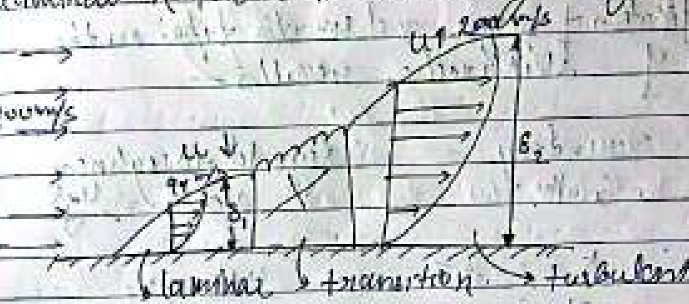
$\frac{u_{max} - u}{u_*}$ = vel. dimension factor and dimension less factor vel.

$u_{max} - u$ = velocity deffects

Hydrodynamically smooth and rough pipes

if - $Re < 2000$ - (laminar)
$Re > 4000$ - Turbulent
$4000 > Re > 2000$ - (transition flow)

laminar & turbulent boundary layer -



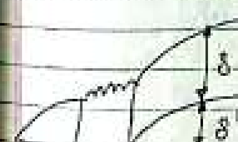
δ_1 = laminar boundary layer thickness
δ_2 = turbulent boundary layer thickness.

Condition for hydrodynamically smooth & rough pipe.

Smooth pipe:-

laminar ~~boundary~~ sub layer

where,
K = average height of pipe roughness
δ' = thick of laminar sub layer



In case of laminar boundary sub layer thickness is very small as compare to the turbulent boundary layer thickness

Smooth pipe:-

for smooth pipe, $\delta' > K$

Laminar Sublayer

⇒ Classification of rough and smooth pipe on the basis of Nikuradse results –

Pipe	Boundary Cond'n.	Reynold roughness no. $= \frac{u_* k}{\nu}$
Smooth pipe	$\frac{k}{\delta'} < 0.25$	$\frac{u_* k}{\nu} < 3$
Rough pipe	$\frac{k}{\delta'} > 6$	$\frac{u_* k}{\nu} > 70$

⇒ Velocity distri. eqn for rough and smooth pipe as per Prandtl velocity distribution eqn.

① Local velocity (u)

Smooth pipe $\left\{ \frac{u}{u_*} = 5.75 \log_{10}\left(\frac{u_* y}{\nu}\right) + 5.5 \right.$

Rough pipe. $\left\{ \frac{u}{u_*} = 5.75 \log_{10}\left(y/k\right) + 8.5 \right.$

② Average Velocity :- ($\bar{u}$)

for smooth pipe, $\frac{\bar{u}}{u_*} = 5.75 \log_{10}\left(\frac{u_* R}{\nu}\right) + 1.7$

for rough pipe, $\frac{\bar{u}}{u_*} = 5.75 \log_{10}\left(R/k\right) + 4.75$

⇒ Variation of friction factor for laminar flow –

In case of laminar flow the pressure loss is given by the following equation.

$$h_f = \frac{\Delta P}{w} = \frac{32 \mu \bar{u} L}{\rho g d^2}$$

As per darcy-

$$h_f = \frac{f L u^2}{2gd} \quad \text{---} ①$$

Multiply & divide by 2

$$h_f = 2 \times 32 . \mu \bar{u} L \times \frac{1}{2 \times \rho g d^2}$$

$$h_f = \frac{64 u^2 L}{\left(\frac{\bar{u} \rho d}{\mu}\right)} \times \frac{1}{2gd}$$

$$h_f = \frac{64}{Re} \times \frac{L u^2}{2gd} \quad \text{---} ②$$

From ① & ②

$$\boxed{f = \frac{64}{Re}}$$ friction factor

$4 f = h_f$ ⇒ friction of coef.

* Vel. distri. eqn for turbulent flow in smooth pipe :

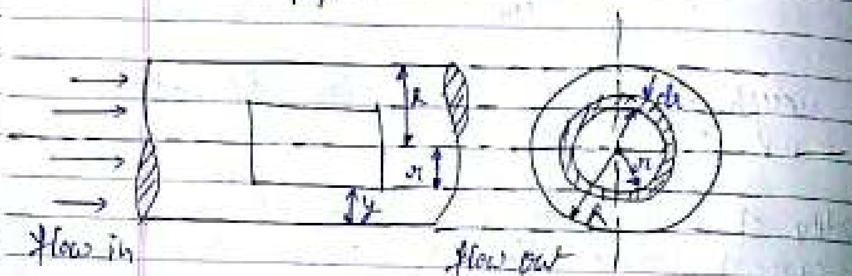


fig: turbulent flow for smooth pipe

Consider a turbulent flow which is flowing through a circular pipe having radius 'R' and small elimentry thickness dr at radius r.

We know that, acc. to Prandtl theory.

local velocity, $\left(\frac{u}{u_*}\right) = 5.75 \log_{10}\left(\frac{u_* y}{\nu}\right) + 5.5$

Elimentary discharge through elimentary area

$dQ = dA \times u$

$dA = 2\pi r dr$

$dQ = 2\pi r dr \left[5.75 \log_{10}\left(\frac{u_* y}{\nu}\right) + 5.5 \right] u_*$

⇒ Total discharge –

$Q = \int_0^R dQ = \int_0^R 2\pi r dr \left[5.75 \log_{10}\left(\frac{u_* y}{\nu}\right) + 5.5\right] u_*$

putting $y = R - r$

$Q = \int_0^R dQ = \int_0^R 2\pi r dr \left[5.75 \log_{10}\left(\frac{u_*(R-r)}{\nu}\right) + 5.5\right] u_*$

After integrating we get –

$Q = 5.75 \log_{10}\left(\frac{u_* R}{\nu}\right) + 1.75$

we know

$Q = A\bar{u}$

$\bar{u} = \frac{Q}{A} = \frac{Q}{\pi R^2}$

$\therefore \quad \bar{u} = \frac{5.75 \log_{10}\left(\frac{u_* R}{\nu}\right) + 1.75}{\pi R^2}$

* Variation of friction factor for various types of flows *

(a) Laminar flow $\quad Re < 2000 \quad f = \frac{64}{Re}$

(b) transition ~~turbulent flow~~ $\quad 2000 < Re < 4000$

(c) Turbulent flow $\quad Re > 4000 \quad$ No specific formula

(i) Smooth pipe $\quad 4000 < Re < 10^5 \quad \boxed{f = \frac{0.316}{Re^{1/4}}}$ (Blasius formula)

$f = 0.0032 + \frac{0.221}{Re^{0.237}}$ (Nikuradse)

~~(ii) Rough pipe~~

(ii) Rough pipe $\quad Re > 10^5$

$\boxed{\frac{1}{\sqrt{f}} = 2 \log_{10}(R/k) + 1.74}$

(Karman formula)

Variation of friction factor

$$f = \phi\{Re, K/D\}$$

where,

$\frac{K}{D}$ = relative roughness and

$\frac{R}{D}$ = relative smoothness

Q → A turbulent flow is flowing through a circular pipe having dia 400 mm, length 800 m. If the kinematic viscosity of the fuel is 0.018×10^{-4} m²/s, Q = 0.04. find
(i) friction factor
(ii) head loss
(iii) wall shear stress (τ_0)
(iv) middle line velocity ($\bar{u}$ u_{max})

$d = 400\,mm \times 10^{-3}$ m
$l = 800$ m

$Re = \frac{\bar{u} d}{\nu}$

$Q = A u$

$u = \frac{Q}{A} = \frac{0.4}{\pi/4 \times (0.400)^2}$

$u = 0.318$ m/s

$Re = \frac{0.318 \times 0.4 \times 10^4}{0.018}$

$= 7.06 \times 10^4$

$400 < Re < 10^5$

$f = \frac{0.316}{Re^{1/4}}$

$f = 0.193$

$h_f = \frac{f L V^2}{2 g d}$

$= 1.49 \times 10^3$ m

f, u_*, τ_0, u_{max}

$\tau_0 = \frac{\rho V^2 f}{2}$

$= \frac{1000 \times (0.318)^2 \times 0.193}{2}$

$= 0.9$ N/m²

$u_* = \sqrt{\frac{\tau_0}{\rho}}$

(iv) $\frac{u}{u_*} = 5.75 \log \frac{u_* y}{\nu} + 5.5$

$\frac{u_{max}}{u_*} = 5.75 \log_{10} \frac{u_* R}{\nu} + 5.5$

$u_{max} = \left(5.75 \log_{10} \frac{0.03 \times 0.2}{0.018 \times 10^{-4}} + 5.5\right) u_*$

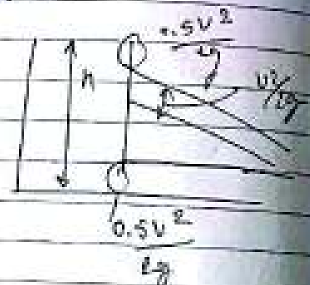
$u_{max} = 0.8$ m/s

* Water Hammer *

Consider a tank, a pipe line is attached at the bottom and a ~~wall~~ valve provided at end of the pipe as shown in fig. If length and dia of pipe are l, d having velo V, friction coeff. f and H = total height of water.

Power required

$P = \rho g Q H_{net}$



$P = \rho g Q [H - \text{all the losses}]$

$= \rho g Q [H - \text{major + minor}]$

$= \rho g Q \left[H - \frac{4fLV^2}{2gd} + \underbrace{\frac{0.5V^2}{2g}}_{\approx 0}\right]$

$P = \rho g Q \left[H - \frac{fLV^2}{2gd}\right]$

$P = \rho g (AV) \left[H - \frac{fLV^2}{2gd}\right]$

$P = \rho g A \left[HV - \frac{fLV^3}{2gd}\right]$

For max. power

$\frac{\partial P}{\partial V} = 0$

$\rho g A \left[H - \frac{3fLV^2}{2gd}\right] = 0$

$\rho g A \left[H - 3\left(\frac{fLV^2}{2gd}\right)\right] = 0$

$\underbrace{\rho g A}_{\neq 0} \left[H - 3h_f\right] = 0$

$H - 3h_f = 0$

$\boxed{H = 3h_f}$

$\boxed{h_f = \frac{H}{3}}$

$\eta = \frac{H - h_f}{H} = 66\%$

$= \frac{H - \frac{H}{3}}{H} = \frac{3H - H}{3H} = \frac{2H}{3H} = \frac{2}{3}$

Q. A turbulent flow is flowing through a circular pipe having length = 1000m, d = 500mm. Calculate the τ of water, assume:

① $\nu = 0.016 \times 10^{-4}\ m^2/s$, $Q_T = 0.0025\ m^3/s$. Also calculate mean line velocity, shear vel., wall shear stress.

~~$l = 1000 \times 10^{-3}$~~ $l = 1000m$ $d = 500 \times 10^{-3}\ m$	$\nu = 0.016 \times 10^{-4}\ m^2/s$ $Q_T = 0.0025\ m^3/s$	$u_{max} = ?$ $\tau_0 = ?$

$Q = AU$

$0.0025 = \frac{\pi}{4}(500 \times 10^{-3})^2 \times u$

$\boxed{u = 0.0127}\ m/s$

$Re = \frac{ud}{\nu} = \frac{0.0127 \times 0.5}{0.016 \times 10^{-4}}$

$\boxed{Re = 3968.75}$

$f = \frac{0.316}{(3968.75)^{1/4}}$

$\boxed{f = 0.04}$

$\tau_0 = \frac{\rho u^2 f}{2}$

$= \frac{1000 \times (0.0127)^2 \times 0.04}{2}$

$= 3.2258 \times 10^{-3}\ N/m^2$

$u_{max} = \left(5.75 \log_{10} \frac{u_* R}{\nu} + 5.5\right) u_*$

$= \left(5.75 \log_{10} \right($

$u_* = \sqrt{\frac{\tau_0}{\rho}}$

$u_* =$

Date
23/3/2017

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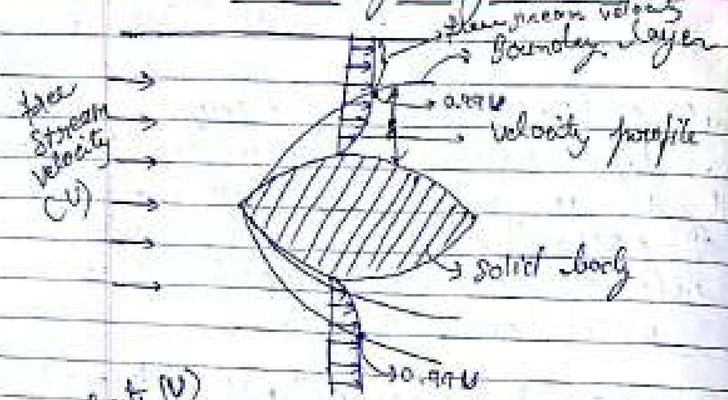
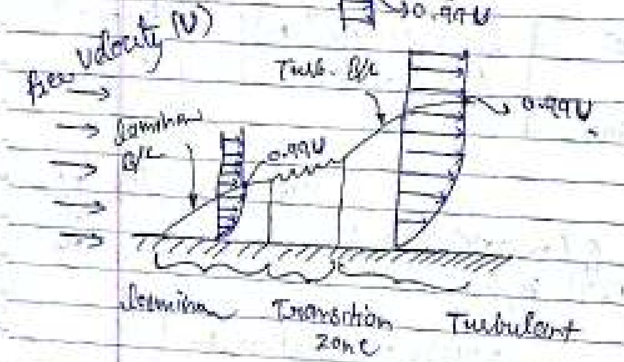
* Boundary layer flow *





When a fluid flows over solid body or a solid wall, the fluid particle vel. will be zero near the boundary (no slip cond^n). This means the that the vel of fluid close to the boundary will be same as that of the boundary. If the boundary is fixed, the vel. of fluid at that the boundary will be zero. The variation of vel. from zero to free stream vel. (U) in the direction normal to the boundary take place in a narrow region. This region of fluid is called boundary layer & The theory dealing with boundary layer flow is known boundary layer theory.

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* Different types of boundary layer :-

(1) Displacement thickness, denoted by δ^*

It is defined as a distance measured ⊥ to the boundary of the solid body by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation. denoted by δ^*

(2) Momentum thickness (θ) :-

It is defined as the distance measured ⊥ to the boundary of the solid body by which the boundary should displaced to compensate for the reduction in momentum on account of boundary layer formation.

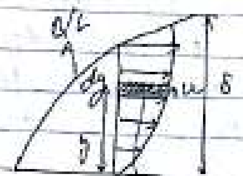
(3) Energy thickness (δ^{**}) :-

It is defined as the distance measured ⊥ to the boundary of the solid body by which the boundary should be displaced to compensate for the reduction of kinetic energy on account of boundary layer formation.

* Displacement thickness *

Consider a fluid when flowing over of flat plate having free stream vel. U.

Assume of a elementary stream 'dy' having velocity du.



$\rightarrow da = b\,dy$

⇒ Mass flow rate through the elementary strip –

$\dot{m} = \rho\, da\, u$

$\dot{m}_1 = \rho\, b\, dy\, u$ → width, → local vel.

⇒ If there is no plate then mass flow rate

$\dot{m}_2 = \rho\, b\, dy\, U$ → free stream velo.

⇒ ∴ loss of mass flow rate –

$\dot{m} = \rho b dy. U - \rho b dy\, u$

$= \rho b dy (U-u)$

⇒ Net mass flow rate ($\dot{m}$)

$\dot{m} = \int_0^\delta \rho b dy (U-u)$ ------ ①

loss of mass, Net $= \rho b \delta^* U$ ---- ②

$\rho b \delta^* U = \int_0^\delta \rho b dy (U-u)$

$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy}$$

* Momentum thickness *

Initial momentum, $m_1 = \dot{m}.v$

$= \rho b dy\, u.u$

If there is no plate, then –

$m_2 = (\rho b dy\, u) U$

loss of momentum $= (\rho b dy\, u) U - \rho b dy\, u.u$

$= \rho b dy (U-u) u$

Total losses of momentum $= \int_0^\delta \rho b dy (U-u) u$ --- ①

Net momentum $= \rho \theta b U^2$ --- ②

$\int_0^\delta \rho b dy (U-u) u = \rho \theta b U U$

$$\boxed{\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}$$ ⁂ –

* Energy thickness *

Initial kinetic energy

K.E. of the fluid when considering small elementary dy.

$$KE_1 = \frac{1}{2} m u^2$$

$$= \frac{1}{2} \rho\, b\, dy\, u\, u^2$$

If there is no plate –

$$KE_2 = \frac{1}{2}(\rho\, b\, dy\, u) U^2$$

Loss of $KE = \frac{1}{2}(U^2 - u^2)\rho\, b\, dy\, u$

Total loss of $KE = \int_0^\delta \frac{1}{2}(U^2-u^2)\rho\, b\, dy\, u$ ---- (i)

Net loss $= \frac{1}{2}\rho\, b\, \delta^{**}\, U \cdot U^2$ ---- (2)

from (i) & (ii)

$$\int_0^\delta \frac{1}{2}(U^2-u^2)\rho\, b\, dy\, u = \frac{1}{2}\rho\, b\, \delta^{**}\, U\, U^2$$

$$\boxed{\delta^{**} = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy}$$

24/3/2017

Ques: Calculate the displacement thickness, momentum thickness, energy thickness if the vel. distribution is given by the following eqn.

$$\frac{u}{U} = 2(y/\delta) - (y/\delta)^2$$

$\delta^* \rightarrow \frac{\delta}{3}$, $\theta \rightarrow \frac{2\delta}{15}$, $\delta^{**} \rightarrow \frac{22\delta}{105}$

Find 1) δ^*, (ii) θ, (iii) δ^{**}

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - 2(y/\delta) + (y/\delta)^2\right) dy$$

$$= \delta - \cancel{2}\frac{1}{\delta} \times \frac{y^2}{\cancel{2}} + \frac{1}{\delta^2} \times \frac{y^3}{3}$$

$$= \left[-\frac{y^2}{\delta} + \frac{y^3}{3\delta^2}\right]_0^\delta$$

$$= -\frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2}$$

$$= \cancel{\delta} - \cancel{\delta} + \frac{\delta}{3} = \frac{-2\delta + \delta}{3} = \frac{-2\delta}{3}$$

$$= \cancel{3\delta}$$

$$\theta = \int_0^\delta \left(\frac{2y}{\delta} - \left(\frac{y^2}{\delta}\right)^2\right)\left[1 - \frac{2y}{\delta} + (y/\delta)^2\right] dy$$

$$\theta = \int_0^\delta \frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^2} + \frac{2y^3}{\delta^2} + \frac{y^8}{\delta^6}$$

$$\theta = \int_0^\delta \left[\frac{2}{\delta} \times \frac{y^2}{2} - \frac{4}{\delta^2} \times \frac{y^3}{3} + \frac{2}{\delta^3} \times \frac{y^4}{4} - \frac{1}{\delta^2} \times \frac{y^5}{5} + \frac{2y^4}{\delta^4} + \frac{y^7}{\delta^7 \times 7}\right]_0^\delta$$

$$\frac{\cancel{2}}{\delta} \times \frac{\delta^2}{\cancel{2}} - \frac{4}{\delta^2} \times \frac{\delta^3}{3} + \frac{\cancel{2}}{\delta^3} \times \frac{\delta^4}{\cancel{4}\,2} - \frac{1}{\delta^2} \times \frac{\delta^5}{5} + \frac{2\delta^4}{\delta^4} + \frac{\delta^7}{\delta^7 \times 7}$$

$$\theta \Rightarrow \delta - \frac{4}{3} + \frac{\delta}{2} - \frac{\delta^3}{5} + 2 + \frac{1}{7}$$

$$= \cancel{0.809 + 1.38}$$

$$\theta = \delta\left(\frac{1}{1} - \frac{4}{3} + \frac{1}{2} - \frac{1}{3} + \frac{1}{2} - \frac{1}{5}\right)$$

$$= \frac{2\delta}{15}$$

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$$q=\int_0^\delta\left(\frac{2y}{\delta}-\left(\frac{y}{\delta}\right)^2\right)\left[1-\frac{2y}{\delta}+\left(\frac{y}{\delta}\right)^2\right]dy$$

$$=\int_0^\delta\left(\frac{2y}{\delta}-\frac{4y^2}{\delta^2}+\frac{2y^3}{\delta^3}-\frac{y^2}{\delta^2}+\frac{2y^5}{\delta^3}-\frac{y^6}{\delta^3}\right)dy$$

$$=\left(\frac{2}{\delta}\times\frac{y^2}{3}-\frac{4}{\delta^2}\times\frac{y^3}{3}+\frac{2}{\delta^3}\times\frac{y^4}{4}-\frac{y^3}{3\delta^2}+\frac{2y^6}{6\delta^3}-\frac{y^7}{7\delta^3}\right)_0^\delta$$

$$=\frac{2\delta^3}{3\delta}-\frac{4\delta^3}{3\delta^2}+\frac{2\delta^4}{4\delta^3}-\frac{\delta^3}{3\delta^2}+\frac{2\delta^6}{6\delta^3}-\frac{\delta^7}{7\delta^3}$$

$$=\frac{2\delta}{3}-\frac{4\delta}{3}+\frac{2\delta}{4}-\frac{\delta}{3}+\frac{2\delta^3}{6}+\frac{\delta^4}{7}$$

$= \delta($

$$Q=\int_0^\delta\left(\frac{2y}{\delta}-\frac{4y^2}{\delta^2}+\frac{2y^3}{\delta^3}-\frac{y^2}{\delta^2}+\frac{2y^3}{\delta^3}+\frac{y^4}{\delta^4}\right)dy$$

$$=\left[\left(\frac{2}{\delta}\times\frac{y^2}{2}\right)-\left(\frac{4}{\delta^2}\times\frac{y^3}{3}\right)+\left(\frac{2}{\delta^3}\times\frac{y^4}{4}\right)-\left(\frac{y^3}{3\delta^2}\right)+\left(\frac{2}{\delta^3}\times\frac{y^4}{4}\right)-\left(\frac{1}{\delta^4}\times\frac{y^5}{5}\right)\right]_0^\delta$$

$$=\left[\frac{y^2}{\delta}-\frac{4y^3}{3\delta^2}+\frac{y^4}{2\delta^3}-\frac{y^3}{3\delta^2}+\frac{y^4}{2\delta^3}-\frac{y^5}{5\delta^4}\right]_0^\delta$$

$$=\frac{\delta^2}{\delta}-\frac{4\delta^3}{3\delta^2}+\frac{\delta^4}{2\delta^3}-\frac{\delta^3}{3\delta^2}+\frac{\delta^4}{2\delta^3}-\frac{\delta^5}{5\delta^4}$$

$$=\delta-\frac{4\delta}{3}+\frac{\delta}{2}-\frac{\delta}{3}+\frac{\delta}{2}-\frac{\delta}{5}$$

$$=\delta\left(1-\frac{4}{3}+\frac{1}{2}-\frac{1}{3}+\frac{1}{2}-\frac{1}{5}\right)$$

$$=\frac{2\delta}{15}$$

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Q. A rough pipe of diameter 400mm and 1000 length carries water at the rate of 0.4 m³/s. The wall roughness is 0.012 mm. Find (i) coeff. of friction, (ii) wall shear stress (τ_0), (iii) centre line velo. & vel. at a distance 150 mm from the pipe wall.

$d = 400\,mm$, $l = 1000\,m$,

$= 400\times10^{-3}\,m$

$k = 0.012\,mm$ $\qquad \frac{1}{\sqrt{f}} = 2\log(R/k)+1.75$

$= 0.012\times10^{-3}$ $\qquad \frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{200}{0.012}\right)+1.75$

$Q = 0.4\,m^3/s$ $\qquad f = 9.63\times10^{-3}$

f coeff. $= \frac{f}{4} = 2.4\times10^{-3}$

$$\frac{u_{max}}{u_*} = 5.75\log_{10}(y/k)+8.5$$

$$Q = Au$$

$$u = \frac{Q}{A} = \frac{0.4}{\frac{\pi}{4}(0.4)^2} = 3.18\,m/s$$

$$\tau_0 = \frac{\rho V^2 f}{2} = \cancel{12.13}\ 48.6\,N/m^2$$

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = 0.1105\,m/s$$

$$u_{max} = u_*\left(5.75\log_{10}(y/k)+8.5\right)$$

$$= 0.1105\left(5.75\log_{10}\left(\frac{150\times10^{-3}}{0.012\times10^{-3}}\right)+8.5\right)$$

$= \cancel{5.4\ m/s}\ 3.542\,m/s$

A three pipe of 400mm, 200mm, 300mm dia have length of 400m, 200m, 300m. They are connected in series to make a compound pipe. The end of the this compound pipe are connected of two tank having diff. 16mm. If the f is same and equal to 0.005. Determine the Q through the compound pipe neglecting minor losses & also including them.

$z_A - z_B = \Sigma h_f$

$z_A - z_B = 16$

$16 = \frac{4fL_1V_1^2}{2gd_1} + \frac{4fL_2V_2^2}{2gd_2} + \cancel{\frac{(V_2-V_3)^2}{2g}} + \frac{4fL_3V_3^2}{2gd_3}$

$A_1V_1 = A_2V_2$ $\quad$ $A_1V_1 = A_3V_3$

$\frac{A_1V_1}{A_2} = V_2$ $\quad$ $\frac{A_1V_1}{A_3} = V_3$

$\boxed{V_1 = 0.8 \quad Q = 0.18\ m^3/s}$

~~2×F×200×10^-3×~~

$\frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2$ $\quad$ $\frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_3^2 \times V_3$

$d_1^2V_1 = d_2^2V_2$ $\quad$ $d_1^2V_1 = d_3^2 \times V_3$

$\frac{d_1^2V_1}{d_2^2} = V_2$ $\quad$ $\frac{d_1^2V_1}{d_3^2} = V_3$

$\frac{(400\times10^{-3})^2V_1}{200\times10^{-3}} = V_2$ $\quad$ $\boxed{V_3 = 1.77V_1}$

$1.33V_1 = V_2$

$\boxed{1.33V_1 = V_2}$

$16 = \frac{4fL_1V_1^2}{2gd_1} + \frac{4fL_2(2V_1)^2}{2gd_2} + \frac{4fL_3(1.33V_1)^2}{2gd_3}$

$$16 = \frac{4(0.05)\times 400\times V_1^2}{2\times 9000\times 400\times 10^{-3}} + \frac{4(0.05)\times 200\times (\ldots V_1)^2}{2\times 1000\times 200\times 10^{-3}}$$

$$+ \frac{4(0.05)\times 700\times (1.77V_1)^2}{2\times 1000\times 300\times 10^{-3}}$$

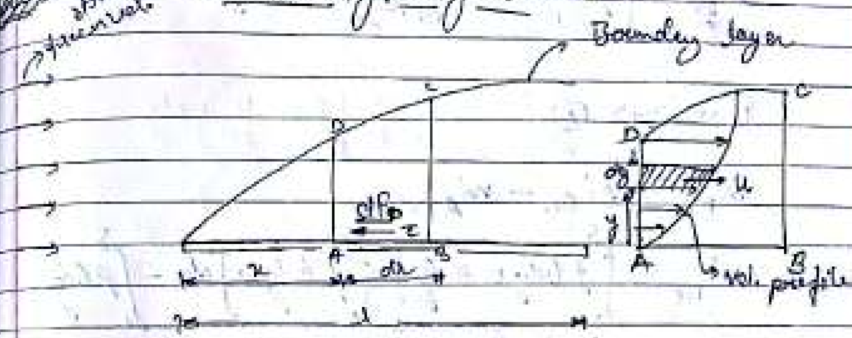
$16 = 0.1 V_1^2 + 0.4 V_1^2 + 0.17689 V_1^2$

$16 = 0.67689 V_1^2$

$\boxed{V_1 = 4.88\ m/s}$

tangential force due to shear stress

Drag force on flat-plate due to Boundary layer



dF_D = drag force on small elementary area ABCD.

l = total length of the plate.

F_D = total drag force

$= \int_0^l dF_D$

Acc to conservation of mass ⇒ ~~mass flow rate through AD + mass flow~~
mass flow rate AD +
mass flow rate DC = mass flow rate BC

$\dot m_{AD} + \dot m_{DC} = \dot m_{BC}$ ---- (1)

$\dot m_{DC} = \dot m_{BC} - \dot m_{AD}$ ---- (2)

$dm_{AD} = \rho\, b\, dy\, u$

$\dot m_{AD} = \int_0^\delta \rho\, b\, dy\, u$

Mass flow rate from DC

$\dot m_{BC} = \dot m_{AD} + \frac{\partial}{\partial x}\dot m_{AD}\times dx$

$$\dot{m}_{BC} = \int_0^\delta \rho b dy\, u + \frac{\partial}{\partial x}\left[\int_0^\delta \rho b dy\, u\right] dx$$

From eqn ②

$$\dot{m}_{DC} = \dot{m}_{BC} - \dot{m}_{AD}$$

$$\dot{m}_{DC} = \int_0^\delta \rho b dy\, u + \frac{\partial}{\partial x}\left[\int_0^\delta \rho b dy\, u\right] dx - \int_0^\delta \rho b dy\, u$$

$$\boxed{\dot{m}_{DC} = \frac{\partial}{\partial x}\left[\int_0^\delta \rho b dy\, u\right] dx}$$

we know the acc^n to Newton's law of motion

$$F_D = \int_0^L dF_D$$

F = Rate of change of momentum

$$\boxed{F = \dot{m}V}$$

Entry momentum from AD

$$(\text{momentum})_{AD} = \int_0^\delta \rho b dy\, u \times u$$

$$= \int_0^\delta \rho b dy\, u^2$$

And similarly

$$\boxed{(\text{momentum})_{DC} = \frac{\partial}{\partial x}\int_0^\delta \rho b dy\, u U dx}$$

$$(\text{momentum})_{BC} = \int_0^\delta \rho b dy\, u^2 + \frac{\partial}{\partial x}\int_0^\delta \rho b dy\, u^2 dx$$

Rate of change of momentum = final − initial

$$= M_{Final} - M_{Initial}$$

$$= \cancel{\int_0^\delta \rho b dy\, u^2} + \frac{\partial}{\partial x}\int_0^\delta \rho b dy\, u^2 dx - \cancel{\int_0^\delta \rho b dy\, u^2} - \frac{\partial}{\partial x}\int_0^\delta \rho b dy\, u U dx$$

$$= \frac{\partial}{\partial x}\int_0^\delta \rho b dy\, u^2 dx - \frac{\partial}{\partial x}\int_0^\delta \rho b dy\, u U dx$$

$$= \cancel{\frac{\partial}{\partial x}\int_0^\delta \rho b dy\, u dx \left[u dx - U dy\right]}$$

$$= (\rho b dx) \times \frac{\partial}{\partial x}\left[\int_0^\delta u^2 dy - \int_0^\delta u U dy\right] \quad \text{---- ③}$$

We know

$$F_D = \int_0^L dF_D$$

$$\text{but } dF_D = \tau_0 \times A \quad \text{---} \quad \left[\tau = F/A\right]$$

$$dF_D = \tau_0 \times b dx \quad \text{---- ④}$$

④ = ③

$$dF_D = -\tau_0 \times b dx = (\rho b dx)\frac{\partial}{\partial x}\int_0^\delta u^2 dy - \int_0^\delta u U dy$$

$$\tau_0 = \rho \frac{\partial}{\partial x}\int_0^\delta u U dy - \int_0^\delta u^2 dy$$

$$\tau_0 = \rho \cdot \frac{\partial}{\partial x} \times U^2 \int_0^\delta \frac{u}{U} dy - \int_0^\delta \frac{u}{U^2} dy$$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\int_0^\delta \frac{u}{U}\,dy - \int_0^\delta \frac{u}{U^2}\,dy$$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\int_0^\delta \frac{\partial}{\partial x}\frac{u}{U}\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U^2}\right)dy \rightarrow 0$$

$$\boxed{\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}}\;\frac{u}{U}\int_0^\delta \theta\,dy$$

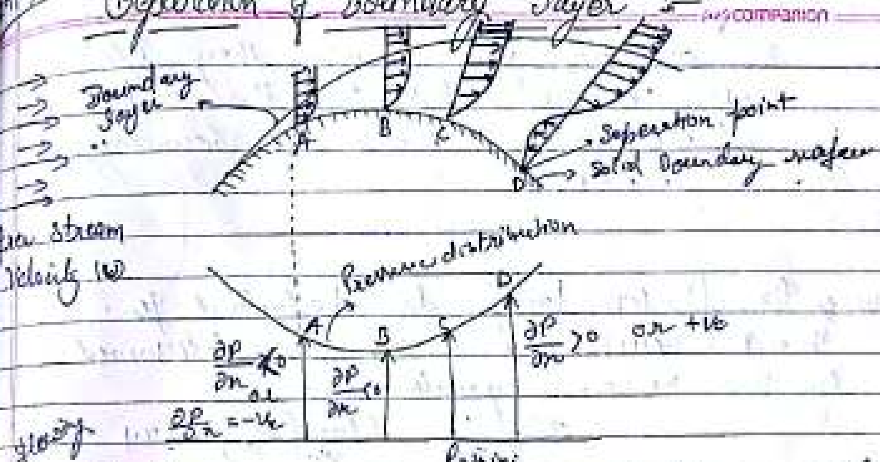
Von-Kerman Integral momentum equation

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\frac{x u}{U}$$

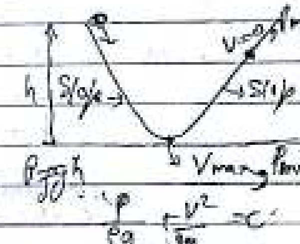
(This equation is valid for laminar flow, turb. flow & transition flow)

Reversable pressure gradient $\frac{\partial p}{\partial x}$ = +ve

Separation of Boundary layer :-



When a solid body is immersed in a flowing flow fluid, a thin layer of fluid known as boundary layer is formed adjacent to the solid body.



In this thin layer of fluid, the vel. varies from zero to free stream velocity. Along the length of the solid body the thickness of boundary layer increases. The fluid layer adjacent to the solid surface has to do work against the surface friction at the cost of its kinetic energy. This loss of K.E is recovered from the immediate fluid layer in contact with adjacent fluid layer. Along the length of the solid body at a certain pt, a stage may come when the boundary layer may not be able to keep overcome its frictional resistance. If it cannot provide K.E most [molecular momentum exchange]

to overcome this resistance they B/L will be separated from the surface this phenomenon is known as boundary layer separation.

Ques) The friction factor for turbulent flow through rough pipe can be determined by von-karman equation.

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{R}{k}\right) + 1.74$$

where k = average roughness height
R = pipe radius
f = friction factor

Two ~~sewers~~ reservoir with a surface level difference of 20m. are to be connected by 1m dia pipe, ~~6k~~ 6km long. What will be the discharge when a cast iron pipe of roughness = 0.3mm. What will be % increase in the discharge if a cast iron pipe replaced by steel pipe having roughness = 0.1mm. of $\frac{1}{\sqrt{f}}$

for cast iron

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{R}{k}\right) + 1.74$$

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{0.5}{0.3\times10^{-3}}\right) + 1.74$$

neglecting minor losses $f = 0.01493$

$$H = \frac{fLV^2}{2gd} \Rightarrow 20 = \frac{0.014 \times 6\times10^{3} \times V_1^2}{2\times9.81\times1}$$

$\boxed{V = 2\,m/s \;\; 2.16\,m/s}$

$$Q = A\times V = \frac{\pi}{4}(1)^2 \times 2.16$$

$\boxed{Q = 1.6 \;\; 1.7}$

for steel pipe

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{0.5}{0.1\times10^{-3}}\right) + 1.74$$

$\boxed{f = 0.0119}$

$$H = \frac{fLV^2}{2gd} \Rightarrow 20 = \frac{0.0119 \times 6\times10^{3} \times V_2^2}{2\times9.81\times1}$$

$\boxed{V_2 = 2.34\,m/s}$

$Q = AV$
$= \frac{\pi}{4}\times1^2\times2.34$

$Q = 1.84$

$$\frac{1.84-1.7}{1.84}\times100 =$$

$$\frac{Q_2 - Q_1}{Q_2}\times100 = 11\%$$

Q-> A pipe line AB of dia 300mm & length 400m carries water at a rate of 50 l/s. The flow takes place A to B and the pt B is 30 m above A. Find pressure at A if the pressure at B = 19.62 N/cm². Take f = 0.008 coeff.

$d = 300\times10^{-3}\,m$

$l = 400\,m$

$q = 50\ l/s = 50\times10^{-3}\ m^3 s^{-1}$

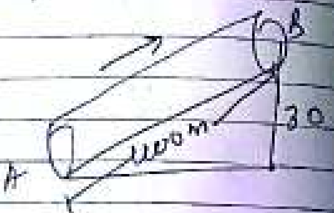
$P_B = 19.62\ N/cm^2$

$A = \frac{\pi}{4}\times0.3^2 = 0.0707\,m^2$

$Q = A_1V_1$

$\cancel{50 = \frac{\pi}{4}(300\times10^{-3})^2\times V_1}$

$\boxed{V_1 = 707.35}$



$\cancel{P_1 - P_2 = \frac{4fLV^2}{}}$

$\cancel{P_1 - P_2 = \frac{32\,\mu\,\bar{u}\,l}{D^2}}$

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$

$Q = AV_A = AV_B$

$V_A = V_B$

$\frac{P_A}{\rho g} = \frac{P_B}{\rho g} + z_B + \frac{4fLV^2}{2gd}$

$= \frac{19.62\times10^{4}}{9810} + 30 + \frac{4\times0.008\times400\times0.707^2}{2\times9.81\times0.3}$

$P_A = 51.09\times \rho g$

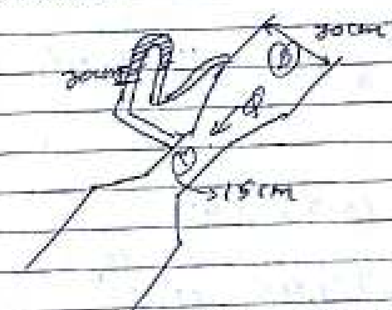
$\boxed{P_A = 50.12\ N/cm^2}$

Find the discharge of water flowing through a pipe of 30cm dia placed in inclined position where a venturimeter is fitted, having a throat diameter of 15cm. The diff. of pressure b/w the main & the throat is measured by a liquid of sp.gr 0.6 in an inverted U-tube which gives a reading of 30cm. The loss of head b/w main & throat is 0.2 times the kinetic head of the fluid. Calculate Q the venturi meter.

$d_1 = 30\,cm$

$d_2 = 15\,cm$

$x = 30\,cm$



$Q = A_1V_1 = A_2V_2$

$h = x\left(1 - \frac{S_m}{S}\right)$

$= 30\left(1 - \frac{0.6}{1}\right)$

$\boxed{h = 12\,cm}$

$h = \frac{P_1 - P_2}{w}$

$h_f = 0.2\frac{V_1^2}{2g}$

Apply Bernoulli's eqn b/w ① & ②

$\frac{P_1}{w} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{w} + \frac{V_2^2}{2g} + \cancel{Z_2} + h_f$

$\frac{P_1 - P_2}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_f$

$A_1V_1 = A_2V_2$

$\frac{\pi}{4}(30)^2\times V_1 = \frac{\pi}{4}(15)^2\times V_2$

$\boxed{V_1 = V_2/4}$

$V_1 4 = V_2$

$$12 = \frac{(4V_1)^2}{2g} - \frac{V_1^2}{2g} + \frac{0.24V_1^2}{2g}$$

$$12 = \frac{16V_1^2}{2g} - \frac{V_1^2}{2g} + \frac{0.24V_1^2}{2g}$$

$$12 = \frac{1}{2g}\left(16V_1^2 - V_1^2 + 0.24V_1^2\right)$$

$$= \frac{1}{2g}\left(17.24V_1^2\right)$$

$$12 = \frac{1}{2\times 9.81}\left(17.24V_1^2\right)$$

$$\frac{12\times 2\times 9.81}{17.2} = V_1^2$$

$$\boxed{V_1 = 3.93 \text{ cm/s}}$$

$$\boxed{V_2 = 15.73 \text{ cm/s}}$$

$$Q = A_1V_1$$

$$= \frac{\pi}{4}(30)^2 \times 3.93$$

$$= 2777.95 \text{ cm}^3\text{s}^{-1}$$

Q) Find the vel. of flow of an oil through the pipe when diff. of Hg level in differential U-tube manometer connected to the pipe & pitot tube is 100 mm. Take the coeff. of pitot tube is 0.98 & rel. sp. gr. of oil as 0.8.

$$V = C_v\sqrt{2gh} \Rightarrow 0.98\sqrt{2\times 9.81\times h}$$

$$h = x\left(\frac{S_m}{S} - 1\right) \qquad V = 0.98\sqrt{2\times 9.81\times 1.600}$$

$$= 100\left(\frac{13.6}{0.8} - 1\right) \quad \boxed{V = 5.4 \text{ m/s}}$$

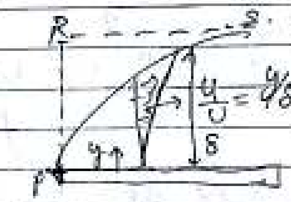
$$= 1600 = 1.600 \text{ m}$$

Q10 ⇒ A smooth plate with a sharp leading edge is placed along a gas stream flowing at a vel. of 10 m/s. The thickness of boundary layer at section RS is 10 cm and width of the plate is 1 cm ⊥ to the paper. And the density of gas $\rho = 1$ kg/m³. Assume the B/L is thin two dimensional follows a linear vel. distr. is given by the following equation $\frac{u}{U} = y/\delta$

$\delta \Rightarrow$ B/L thickness
$U =$ free gas stream vel.
$u =$ local vel.

The mass flow rate across section RS is given

RS = 0, 0.05, 0.10, 0.15



$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(1 - y/\delta\right) dy$$

$$= \left[y - \frac{1}{\delta}\times\frac{y^2}{2}\right]_0^\delta$$

$$= \delta - \frac{1}{\delta}\times\frac{\delta^2}{2}$$

$$= \delta - \frac{\delta}{2}$$

$$= \frac{2\delta - \delta}{2}$$

$$= \frac{\delta}{2}$$

$$= 0.5\delta$$

$$\tau = F/b$$

$$\tau_0$$

$$\dot{m}_{bc} = \frac{\partial}{\partial x}\int_0^\delta \rho\, b\, dy\, u.\text{width}$$

$$= \int_0^\delta \rho \frac{y}{\delta} U\, dy$$

$$\boxed{= 0.07}$$

$$\int_0^\delta 1\alpha$$

Scanned by CamScanner

Ques. Air is flowing over a smooth plate with the vel. of 10 m/s. The length of the plate is 1.2 m & width 0.8 m. If the laminar boundary layer exists upto a value of Reynold no. 2×10^5. Find the max. distance from the leading edge upto which laminar boundary layer exist. And the max. thickness of laminar B/L in the if vel. profile is given the the eqn. If the K.V. of air $= 0.15\times10^{-4}$ m²/s

$$\frac{u}{U} = 2(y/\delta) - (y/\delta)^2$$

Note :-

① due ↑ $\delta = \dfrac{5.48 \times x}{\sqrt{Re_x}}$

② If $\dfrac{u}{U} = \dfrac{3}{2}(y/\delta) - (y/\delta)^3$

$$\delta = \frac{4.64x}{\sqrt{Re_x}}$$

$Re = 2\times10^5 = \dfrac{Vx}{\nu}$

$\therefore \dfrac{2\times10^5 \times \nu}{V} \Rightarrow x = 0.3$ mm

$\delta = \dfrac{5.48\,x}{\sqrt{Re_x}} \Rightarrow \dfrac{5.48\times0.3}{\sqrt{2\times10^5}} \Rightarrow 3.67$ mm

$\sqrt{\dfrac{Vx}{\nu}}$ $= 3.67$ mm

UNIT-IV

Dimension And Model Analysis

* Buckingham's π Theory :-

Dimension analysis is a method of dimensions. It is a mathematical technique used in per research work for design & conducting model test. It

It deal with the dimension of the physical quantity involve in the involve in the phenomenon. This π theory is generally used for calculating the dimension analysis

* Statement of Buckingham's π theory :-

If there are n-variables (independent & dependent variable) in a physical phenomenon and if these variable contains m-fundamental dim dimension (M, L, T) then the variable are arranged into (n-m) dimension less term. each term is known as π term.

Method selecting repeating variables

The no. repeating variables are equal to no. of fundamental dimension of problem. the choice of repeating variable

The repeating variable should be chosen in such a way that one variable contain geometrical property, 2nd variable contains flow property, third variable contain fluid property. ⇒ The variable with geometrical property are length, area, height, volume, diameter.

⇒ Variable with flow property are velocity, pressure, acc^n.

⇒ Variable with fluid property are kinematic viscosity, dynamic viscosity, sp. gravity etc.

Q) The eff. η for a fan depends upon density, D, μ, ω (angular), diameter of rotor, discharge Q. Express η in terms of dimensional parameter.

$\eta = f[D, \rho, \omega, Q, \mu]$ ---- (1)

Step:2

A given η is a function of ρ, μ, ω, d, Q

$\eta = f[D, \rho, \omega, Q, \mu]$ or

or

$\eta = f_1[\eta, \mu, \omega, D, Q] = 0$

Here the value of $n = 6$

$\rho = kg/m^3 = ML^{-3}$

$\mu = Ns/m^2 = \frac{m \times a \times T}{L^2} = \cancel{MLT}$

$= ML^{-1}T^{-1}$

$\omega = T^{-1}$

$D = L$

$Q = ML^{-3}$

In this case eq^n the total no. of fundamental dimension ⇒ $M = 3$

Total no. of π term $= n - m$

$= 6 - 3$

$= 3$

Step 12 Total no. of π term ⇒ $f[\pi_1, \pi_2, \pi_3] = 0$

geometrical property $= D$

fluid " $= \rho$

flow " $= \omega$

$\pi_1 = D^{a_1} \cdot \rho^{b_1} \cdot \omega^{c_1} \eta$

$\pi_2 = D^{a_2} \cdot \rho^{b_2} \cdot \omega^{c_2} \cdot \mu$

$\pi_3 = D^{a_3} \cdot \rho^{b_3} \cdot \omega^{c_3} \cdot Q$

Calculating the value of a_1, b_1, c_1

$M^0L^0T^0 = L^{a_1} (ML^{-3})^{b_1} \cdot (T^{-1})^{c_1} \cdot M^0L^0T^0$

For π_1

$$M^0L^0T^0 = (L^{a_1}L^0)(M^{-b_1}M^0)(L^{-3c_1}L^0)(T^{-c_1}T^0)$$

$$a_1=0,\quad b_1=0,\quad c_1=0$$

Now π_2

$$M^0L^0T^0 = (L)^{a_2}(ML^{-3})^{b_2}(T^{-1})^{c_2}\,[ML^{-1}T^{-1}]$$

$$\cancel{M^0T^0L^0 = (L^{a_2}L^{-3b_2}L^{-1})(M^{b_2}M^{1})(T^{-1}T^{-1})^{c_2}}$$

$$M^0L^0T^0 = (L^{a_2})(M^{b_2}L^{-3b_2})(T^{-c_2})(ML^{-1}T^{-1})$$

$$= (L^{a_2}\cdot L^{-3b_2}L^{-1})(M^{b_2}M^{1})(T^{-3b_2}T^{-3})$$

$a_2-3b_2-1=0$, $\quad b_2+1=0$, $\quad -3b_2-c_2=0$

$a_2-3(-1)-1=0$, $\quad \boxed{b_2=-1}$, $\quad -3(-1)-c_2=0$

$a_2+3-1=0$, $\quad 3-c_2=0$

$\boxed{a=-2}$ $\quad \boxed{c_2=3}$

$$\pi_1 = D^0\rho^0\omega^0\eta = \eta$$

$$\pi_2 = D^{-2}\rho^{-1}\omega^{-1}\mu = \frac{\mu}{D^2\rho\omega}$$

$$\pi_3 = D^{-3}\rho^{-1}\omega^0 Q = \frac{Q}{D^3\rho}$$

$$f(\pi_1,\pi_2,\pi_3)=0$$

$$f\left(\eta,\ \frac{\mu}{D^2\rho\omega},\ \frac{Q}{D^3\rho}\right)=0$$

Finally

$$\eta = \phi\left\{\frac{\mu}{D^2\rho\omega},\ \frac{Q}{\rho}\right\}$$

Ans

Ques→ The frictional torque T of a disc of dia. D rotating at a speed N rpm in a fluid of viscosity μ, density ρ in a turbulent flow is given by

$$T = D^5N^2\rho\,\phi\left[\frac{\mu}{D^2N\rho}\right]$$

$T = f[D, N, \rho, \mu]$ or

$$f_1[T\,D\,N\,\rho\,\mu]=0$$

$\therefore n=5$

Total no. of π term $= 5-3 = 2$

Total no. of π term

$$f[\pi_1, \pi_2]=0 \qquad a_2=-2$$

$$\pi_1 = D^{a_1}N^{b_1}\rho^{c_1}T \qquad b_2=-1$$

$$\pi_2 = D^{a_2}N^{b_2}\rho^{c_2}\mu \qquad c_2=-1$$

Calculating the value of a_1, b_1, c_1

$$\cancel{M^0L^0T^0 = L^{a_1}(T^{-1})^{b_1}(ML^{-3})^{c_1}}$$

For π_1

$$M^0L^0T^0 = [L^{a_1}][MLT^{-2}]^{b_1}[ML^{-3}]^{c_1}[ML^2T^{-2}]$$

$$= (L^{a_1})(M^{b_1})(L^{b_1})(T^{-2b_1})(M^{c_1})(L^{-3c_1})(M)(L^2)(T^{-2})$$

$$= (L^{a_1}L^{b_1}L^{-3c_1}L^2)(M^{b_1}M^{c_1}M)(T^{-2b_1}T^{-2})$$

$0 = a_1+b_1-3c_1+2$, $\quad b_1+c_1+1=0$, $\quad -2b_1-2=0$

$0 = a_1-1-3(0)+2$, $\quad \cancel{-1}+c_1+\cancel{1}=0$, $\quad -2b_1 = 2$

$\boxed{a_1=-1}$ $\quad \boxed{c_1=0}$ $\quad \boxed{b_1=-1}$

Now for π_2, $\pi_2 = D^{a_2} . N^{b_2} . \rho^{c_2} . \mu$

$[M^0L^0T^0] = [L^{a_2}] [\cancel{ML^2T^{-2}}]^{b_2} [ML^{-3}]^{c_2} [ML^{-1}T^{-1}]$

$= (L^{a_2}) (M^{b_2}) (L^{2b_2}) (T^{-2b_2}) (M^{c_2}) (L^{-3c_2})$

$(M) (L^{-1}) (T^{-1})$

$= (L^{a_2} . L^{2b_2} . L^{-3c_2} . L^{-1}) (M^{b_2} . M^{c_2} . M) (T^{-2b_2} . T^{-1})$

$0 = \cancel{a_2 + b_2}$

$0 = a_2 + b_2 - 3c_2 - 1$, $b_2 + c_2 + 1 = 0$, $-2b_2 - 1 = 0$

$-1 + c_2 + 1 = 0$, $-2b_2 = 1$

$b_2 = -1/2$

$b_2 = 0.5$

W.K.T

$T = ML^2T^{-2}$

$D = L$

$N = T^{-1}$

$\mu = ML^{-1}T^{-1}$

$\rho = ML^{-3}$

$f_1(\pi_1, \pi_2) = 0$

$\pi_1 = D^{a_1} . N^{b_1} . \rho^{c_1} . T$

$\pi_2 = D^{a_2} . N^{b_2} . \rho^{c_2} . \mu$

Dimension analysis of π_1

$M^0L^0T^0 = [L]^{a_1} . [T^{-1}]^{b_1} [ML^{-3}]^{c_1} [ML^2T^{-2}]$

$= L^{a_1} . T^{-b_1} . M^{c_1} . L^{-3c_1} . ML^2T^{-2}$

$= (L^{a_1} . L^{-3c_1} . L^2) (T^{-b_1} . T^{-2}) (M^{c_1} . M^1)$

$a_1 - 3c_1 + 2 = 0$, $-b_1 - 2 = 0$, $c_1 + 1 = 0$

$a_1 - 3(-1) + 2 = 0$

$a_1 = -3 - 2$, $-b_1 = 2$

$\boxed{a_1 = -5}$ $\boxed{b_1 = -2}$ $\boxed{c_1 = -1}$

Now for π_2

$M^0L^0T^0 = [L]^{a_2} [T^{-1}]^{b_2} [\cancel{ML^2T^{-2}}]^{c_2} [ML^{-1}T^{-1}]$ ML^{-3}

$= (L^{a_2}) (T^{-b_2}) (M^{c_2}) (L^{-3c_2}) (T^{-2c_2})$ $[ML^{-1}T^{-1}]$

$= (L^{a_2} . L^{-3c_2} . L^{-1}) (T^{-b_2} . \cancel{T^{-2c_2}} . T^{-1}) (M^{c_2} . M)$

$a_2 - 3c_2 - 1 = 0$, $-b_2 - 2c_2 - 1 = 0$, $c_2 + 1 = 0$

$a_2 - 3(-1) - 1 = 0$, $-b_2 - 2(-1) - 1 = 0$, $\boxed{c_2 = -1}$

$a_2 - 2 - 1 = 0$, $-b_2 + 2 - 1 = 0$

$\boxed{a_2 = -2}$ $-b_2 + 1 = 0$

$-b_2 = -1$

$\boxed{b_2 = 1}$

Now substituting the values of a_2, b_2, c_2 in π_2

$\pi_2 = D^{-2} . N^{-1} . \rho^{-1} \mu = \dfrac{\mu}{D^2 N \rho}$

$\pi_1 = D^{-5} N^{-2} \rho^{-1} T = \dfrac{T}{D^5 N^2 \rho}$

$f_1(\pi_1, \pi_2) = 0$

$f_1\left(\dfrac{T}{D^5 N^2 \rho}, \dfrac{\mu}{D^2 N \rho}\right) = 0$ or $\dfrac{T}{D^5 N^2 \rho} = \phi\left[\dfrac{\mu}{D^2 N \rho}\right]$

$\boxed{T = D^5 N^2 \rho \, \phi\left[\dfrac{\mu}{D^2 N \rho}\right]}$ proved

$\boxed{Q = L^3T^{-1}}$ $\boxed{\sigma = MT^{-2}}$, $\boxed{W = ML^{-2}T^{-2}}$