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Date
10/1/2016

1

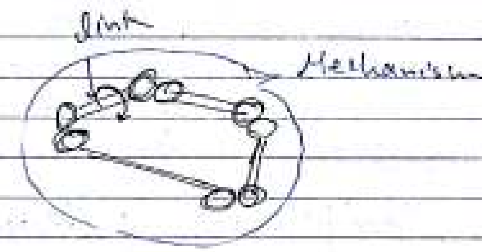
Kinematics of M/C

Theory of Machine

Kinematics of M/C | Dynamics of M/C

(it deals with relative motion without considering force effect) | (it deals with relative motion with considering force effect)

Machine :-
Mechanism :-
Structure :-
Link :-



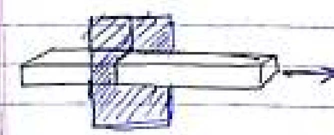
Types of link

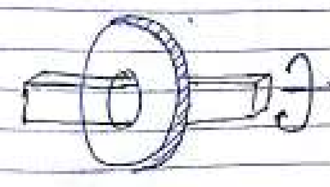
Rigid body, Resistant body

i) Rigid link
ii) Flexible link
iii) Fluid link

UNIT-I

* Constrained Motion *





Completely Constrained motion   Incompletely constrained motion.

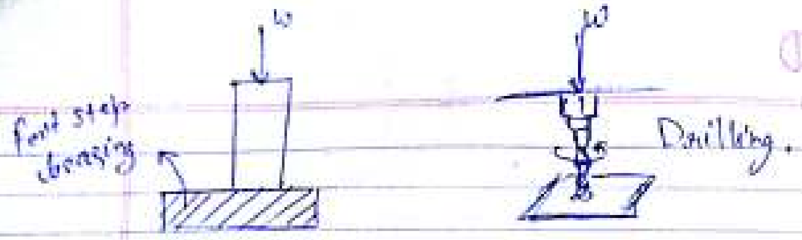


fig: Succesfully constrained motion

* Kinematics of link :-

* Kinematics pair:-

⇒ when a two link are connected to each other then such kind of link form pair. If the motion b/w them completely constraint or succesfully constraint. Then such kind of pair known as Kinematic pair

Classification of kinematic pair:-

i) According to nature of contact

(a) Sliding pair (d) Turning pair
(b) Rolling pair (e) Spherical pair
(c) Screw pair

ii) Acc. to surface contact :-
i) Lower pairs (surface contact)
ii) Higher pair → point or line contact.
eg:- gears

* Machine :- A machine is a device which transforms energy available in one form to another to do certain type of desired useful work.

* Mechanisms :- A mechanism is a set of machine elements or components or parts arranged in a specific order to produce a specified motion. The terms related to the study of mechanisms are - Machine, Kinematics, Resistant Bodies; Rigid bodies, & structure & frame.

* Kinematics : It is a subject which deal with the study of relative motion of parts constituting a machine, neglecting forces producing the motion.

* Resistant Bodies:- These are those bodies which do not suffer appreciable distortion or change in physical form due to forces acting on them, eg, spring, belts and fluids etc.

Elastic bodies are also resistant bodies if the deformation is not appreciable. It is capable of transmitting the required forces with negligible deformation.

* Rigid Bodies:- These are those bodies which do not deform under the action of forces. All resistant bodies are considered rigid bodies for purpose of transmitting motion.

* <u>Structure</u> :- It is an assemblage of a no. of resistant bodies meant to take up load or subjected to forces having straining action, but having no relative motion b/w its members, e.g. a roof truss

* <u>Frame</u> :- It is a structure which supports moving parts of a machine.

o <u>Kinematic Joint</u> *

A kinematic joint is the connection b/w two links by a pin. There is clearance b/w the pin and the hole in the ends of the link being connected so that there is free motion of the links.

* Types of joint :-

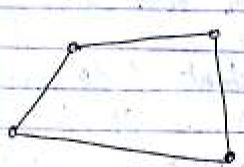


fig: Binary joints

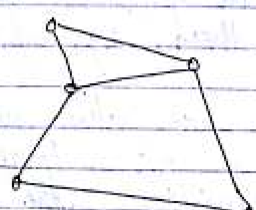


Fig: Ternary joint

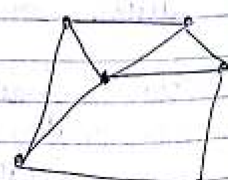
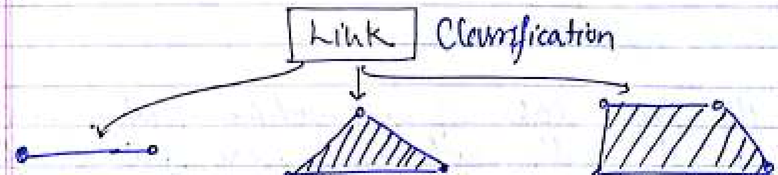


Fig. Quaternary joint

o <u>Kinematic Link</u> o

A link is a resistant body, which do not deform under the action of forces, and

constitute part of a machine, connecting other parts, which have motion relative to it.



(a) Binary link (two vertices)
(b) Ternary link (has three vertices).
(c) Quaternary link (Four Vertices)

* <u>Types of links</u> :-

(a) <u>Rigid link</u> :- A rigid link is one which does not undergo any deformation while transmitting motion. Links in general are elastic in nature. They are considered rigid if they do not undergo appreciable deformation while transmitting motion. eg. connecting rod, crank, tappet rod etc.

(b) <u>Flexible link</u> :- A flexible link is one which while transmitting motion is partly deformed in a manner not to affect the transmission of motion. eg. belts, ropes, chains, and springs etc.

(c) <u>Fluid link</u> :- A fluid link is one which is deformed by having fluid in a closed vessel and the motion is transmitted through the fluid by pressure, as in the case of a hydraulic press, hydraulic jack, and fluid brake.

(d) **Floating link :-** It is a link which is not connected to the frame.

Kinematic Pair :

The two links of a machine, when in contact with each other, are said to form a pair. A kinematic pair consists of two ~~lnk~~ link which have relative motion b/w them.

* Classification of Kinematic Pairs *

Kinematic Pairs

- Types of Relative motion
  - i) Sliding Pair
  - ii) Turning Pair
  - iii) Rolling Pair
  - iv) Screw or Helical pair
  - v) Spherical Pair
- Types of Contact
  - i) Lower Pair
  - ii) Higher Pair
  - ~~iii)~~
- Types of mechanical constraint
  - i) Closed Pair
  - ii) Unclosed Pair

* Sliding Pair :-

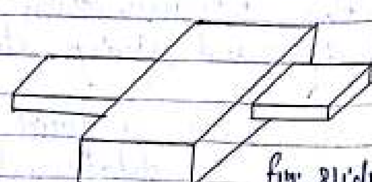
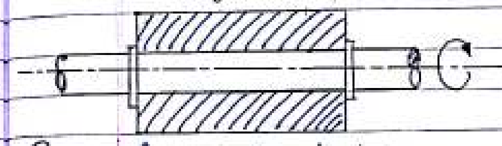


fig: Sliding bar : rectangular bar in a rectangular hole.

It consist of two elements connected in such a manner that one is constrained to have sliding motion relative to another.

e.g: rectangular bar in rectangular hole, piston & cylinder of an engine, cross-head and guides of a steam engine, ram and its guides in a shaper, tailstock on the lathe bed etc, all constitute sliding pair.

* Turning Pair :-

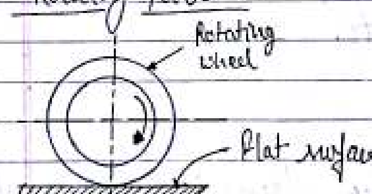


Turning Pair : Collared shaft revolving in a circular hole. ~~e.g: Crankshaft to~~

It consist of two elements connected in such a manner that one is constrained to turn or revolve about a fixed axis of another element.

e.g: crankshaft turning in a bearing, cycle wheel revolving over their axles etc.

* Rolling Pair :-



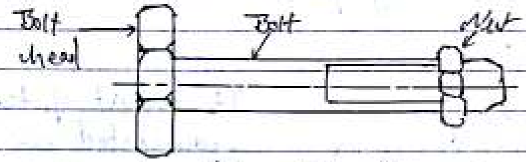
When two elements are so connected that one is constrained to roll on another elements which is fixed, forms a rolling pair.

e.g:. Ball and Roller bearing, a wheel rolling on a flat surface etc.

* Screw (or Helical) Pair :-

When one element turns about the other elements by means of threads, forms a screw pair. The motion in this case is e combination of sliding and turning.

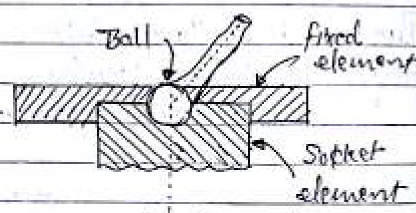
eg: The lead screw of a lathe with nut, bolt with a nut, screw with nut of a jack.



Screw Pair: Bolt with nut.

* Spherical pair :-

When one element in the form of a sphere turns about the other fixed element, it forms a spherical pair.
eg mirror attachment of vehicles, The ball and socket joint etc.



Ball and socket joint

* Lower Pair :-

When the two elements have surface (or area) contact while in motion and the relative motion being purely turning or sliding, is called a lower pair.

All sliding pairs, turning pairs and screw pairs form lower pair.

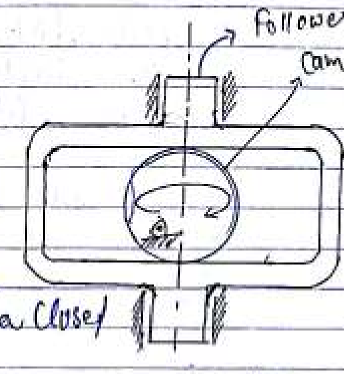
e.g: nut turning on a screw, shaft rotating in a bearing, universal joint etc.

* Higher pairs :- When two elements have point or line contact while in motion & the relative motion being the combination of sliding and turning, then the pair is known as a higher pair.

e.g: Belts, ropes, and chain drive, gears etc

* Closed Pair :-

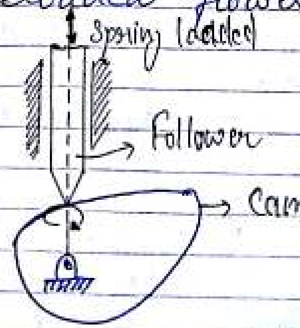
When two elements of a pair are held together mechanically in such a manner that only required type of relative motion occurs, is called a Closed pair.



enclosed cam and follower are closed pairs

* Unclosed Pairs :-

When the two elements of a pair are not held mechanically and are held in contact by the action of external forces, are called unclosed pair. e.g: cam and spring loaded flower pair.



• Kinematic Chain •

When the kinematic ~~coupt~~ pair are coupled in a such a way that the last link in joint to the first link to transmit the definite motion (i.e. completely constrained or successfully constrained motion) it is called kinematic chain.

If each link is assume to form two pairs with adjacent link then the relation b/w the no. of pairs (p) forming a kinematic chain and the no. of links (l) may be expressed in the form of an equation(1)

$$l = 2p - 4 \quad ----(1)$$

Another relation b/w the no. of link & the no. of joint (j) which constitute a kinematic chain is given by the expression

$$j = \frac{3}{2}l - 2 \quad ----(2)$$

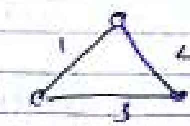
The equation (1) & (2) are applicable only to the kinematic chain in which lower pairs are used. It can also used when higher pair given but it may be taken as equivalent to two lower pair.

1. Locked Chain :- $\boxed{LHS > RHS}$

When the left hand side values of given equation is greater than right hand side

value then such kind of kinematic chain is called locked chain.

eg:-



$p = 3, \; l = 3, \; j = 3$

$3 = 2(3) - 4 \quad --(1)$

$3 = 6 - 4$

$\boxed{3 = 2}$

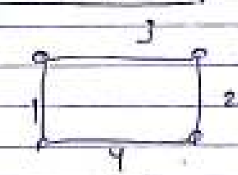
LHS > RHS

$3 = \frac{3}{2}(3) - 2$

$3 = \frac{9}{2} - 2$

LHS > RHS $---(2)$

2. Constrained Chain :- If $\boxed{LHS = RHS}$



$l = 2p - 4$

$4 = 2(4) - 4$

$4 = 8 - 4$

$4 = 4$

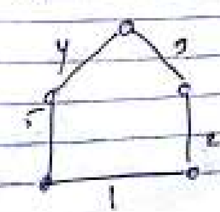
$\boxed{LHS = RHS}$

$4 = \frac{3}{2}(4) - 2$

$4 = 4$

$\boxed{LHS = RHS}$

3. Unconstrained Chain :- $\boxed{LHS < RHS.}$



Types of Joint in Chain

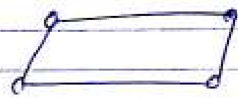
→ There are three kind joints are found in kinematic chain

1) **Binary joint** :–

When the two link are joint at the same connection the joint is known as binary joint

$$\boxed{j = \frac{3l}{2} - 2}$$

e.g:-



$4 = \frac{3}{2} \times 4 - 2$

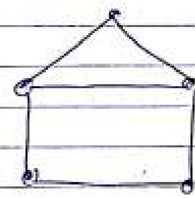
$\boxed{4 = 4}$

② **Ternary joint** :–

Three link are joint at the same connection the joint are called ternary joint. Here one ternary joint is equal to two binary joint in chain.

$l = 6$

$j = 3 + (2)2 = 7$ binary joint



$7 = \frac{3}{\cancel{2}}(\cancel{6}^{3}) - 2$

$7 = 9 - 2$

$\boxed{7 = 7}$

③ **Quaternary Joint** :–

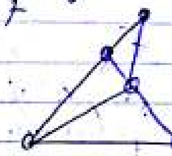
When the four links are connected at the same connection.

It is ~~called~~ equal to 3 binary joint.

In general, when (l) no. of links are

joint at the same connection the joint is equivalent to (l−1) binary joint

$j = \frac{3}{2} l - 2$



$9 = \frac{3}{2}(7) - 2$

$9 = \frac{21}{2} - 2$

$j = 2 + 2(2) + 3(1)$

$= 2 + 4 + 3$

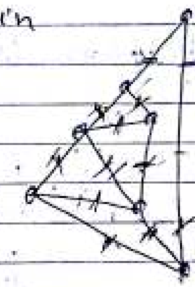
$= 9$

$9 = \frac{21 - 4}{2}$

$18 = 21 - 4$

$18 = 17$

L.H.S > RHS → lock chain



$j = 2 + 2(4) + 3(2)$

$= 2 + 8 + 6$

$= 16$

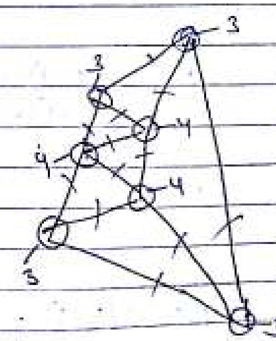
~~$16 = \frac{3}{}$~~ $15 = \frac{3}{\cancel{2}}(\cancel{12}^{6}) - 2$ $\frac{33}{2} - 2$

$15 = \cancel{18 - 2}$

$15 = \cancel{16}$

$l = 12$

$j = \frac{3(12)}{2} - 2$



$J = 4(2) + 3(3)$

$= 8 + 9$

$= 17$

$17 = \frac{13}{\cancel{2}}(\cancel{12}^{6}) - 2 \Rightarrow 17 = 76$

$\boxed{17 < 76}$

11/1/2017

14

• Degree of freedom •
(Movability)

It is defined as no. of input parameters (usually pair variables) which must be independently controll in order to bring the mechanisms into a use full engineering purpose, it is also called movability of a mechanisms. The relative motion in a mechanism is described by Grashof's law and degree of freedom can be shown by the given equation–

Kutzbach criterion

$$f = 3(N-1) - 2j - h \quad \text{---} \quad (1)$$

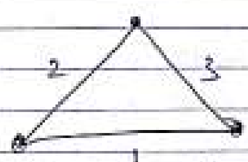
where $f \Rightarrow$ D.O.F

N = No. of link

j = no. of lower pair

j_h = No. of higher pair

Example:–



$f = 3(N-1) - 2j - h$

$= 3(3-1) - 2[3] - 0$

$= 6 - 6$

$\boxed{f = 0}$ relative motion not possible

$f = 3(4-1) - 2(4) - 0$

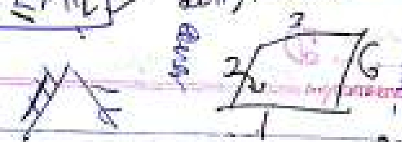
$= 9 - 8$

$\boxed{f = 1}$

Relative motion possible

15

Grashof's law $\Rightarrow$ $S + L \leq P + Q$ design prob-



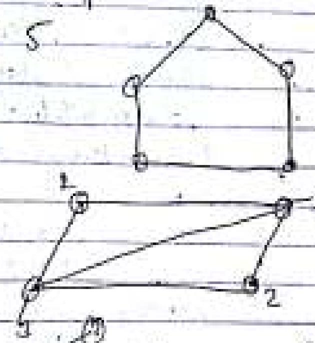
$f = 3(5-1) - 2(5) - 0$

$= 12 - 10$

$\boxed{f = 2}$

7 X 5

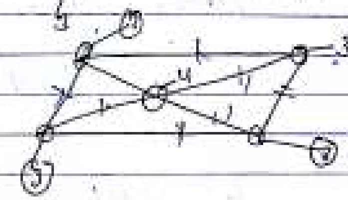
$f = 3(5-1) - 2(2 + 2(2)) + 0$



$= 12 - 2(6)$

$= 12 - 12$

$= 0$



$f = 3(8-1) - 2[4(2) + 4(3)]$

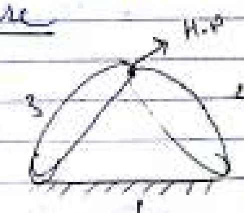
$= 21 - 2[8 + 12]$

$= 21 - 2[20]$

$= 21 - 40$

$= -19$

* When degree of freedom -1 or less than 0 then such kind of mechanism known supper structure



$N = 3$

$j = 2$

$h = 1$

$f = 3(3-1) - 2 \times 2 - 1$

$= 1$

Grubler Criterion for Plane

It is applied to the mechanism with only single degree of freedom joint when overall movability of the mechanism is unity. We know that from the equation : DOF

$F = 3(N-1) - 2j - h$

$1 = 3(N-1) - 2j - h$

putting, $h = 0$

$\boxed{3N - 4 - 2j = 0}$ — Grubler Criterion ---- (2)

In little consideration will show that a plain mechanism with a movability of one & only single DOF joint cannot have odd no. of links. The simplest possible mechanism of this type are - four bar chain mechanism & single slider chain mechanism, when no. of links is 4 & no. of joint is 4.

Mechanisms

When one of the link in a kinematic chain is fixed then such type of chain is known as mechanisms.

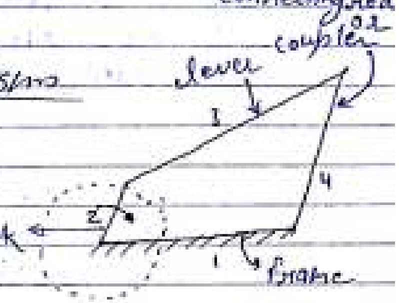
* Inversion of Mechanisms :-
we known that when one of the link is fixed in a kinematic chain is known as kinematic chain.

So, we can obtain as many mechanism as the no. of link in a kinematic chain by fixing and turn different link in a kinematic chain. This method of obtaining diff. mechanism by fixing diff. link in a kinematic chain is known as inversion of the mechanism.

17/1/17

* Inversion of Mechanisms *

(i) Four bar chain mechanisms



A kinematic chain is a combination of four kinematic pair such that the relative motion the links or element is completely constrained.

fig : 4 bar chain Mechanism

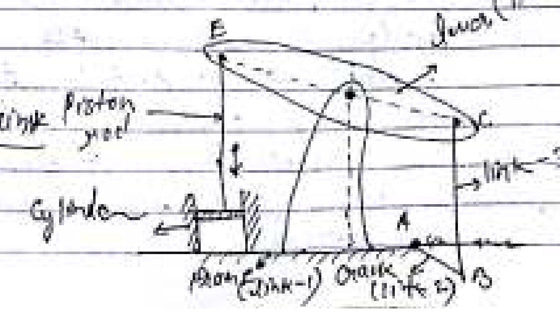
A four bar chain consist of 4 links, each of them forming a turning pair at point A, B, C, D.

~~(a) Inversion of four bar chain (crank & lever mechanism)~~

~~(a) Beam~~

* Inversion of 4 barchain

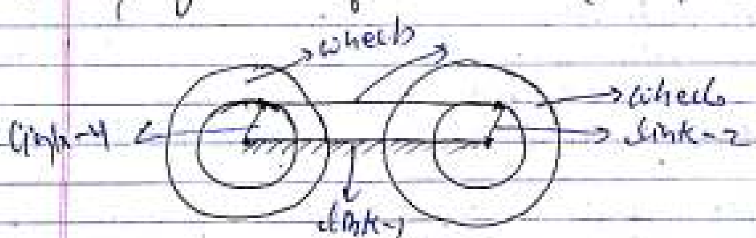
(a) Beam Engine or Crank & lever beam



A part of the mechanisms of a beam engine which consist of 4-links as shown in figure in this mechanisms when the crank rotates about the fixed centre A. The lever oscillate about the fixed centre 'D', the end 'E' of the lever CDE is connected to the piston rod which reciprocates ~~into rotary~~ due to rotation of the crank.

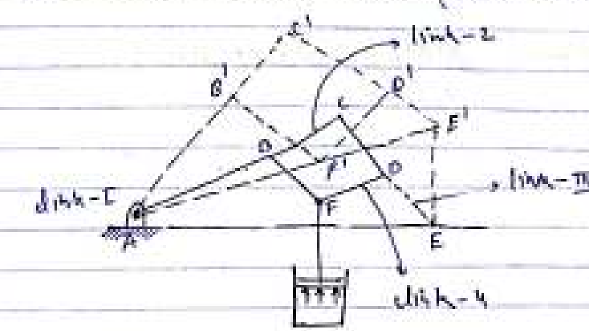
In other words the purpose of mechanism is to convert rotary motion into reciprocating motion.

(b) Coupling rod of locomotive (double crank Mech)

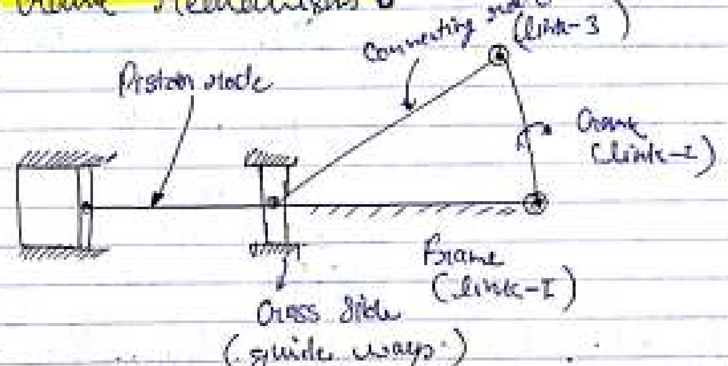


In this mech. the link AD & DC (having equal length) act as a crank and are connected to the respective wheel. The link CD act as coupling rod & link AB is fixed in order to maintain the constant centre to the centre distance them. This mech. is used for transmitting rotary motion from one wheel to another wheels.

(c) Watt's indicator mechanisms (double lever Mech)



Slider Crank Mechanisms

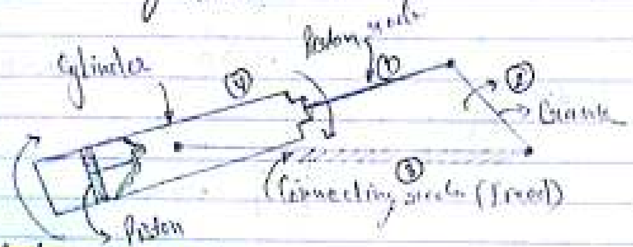


It is a modification of four bar chain mech. it consist of one sliding pair & three turning pair. It is basically used to convert rotary motion into reciprocating motion & vice-versa.

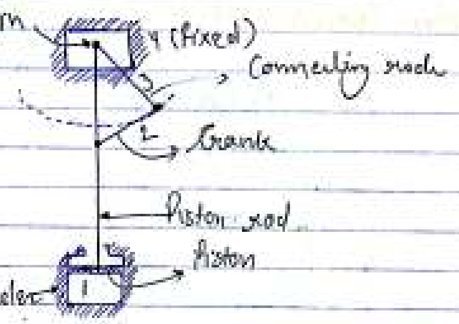
In an single slider crank chain as shown in figure the link one & two, link-2 & link-3, link (3-4) form the three turning pair while the link (4-1) form the a sliding pair

Inversion of Slider crank Mechanisms

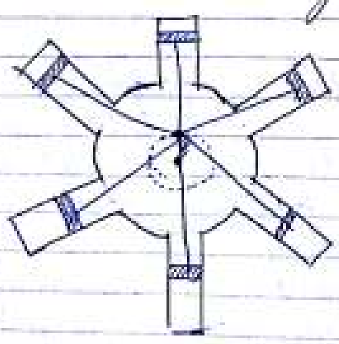
① Oscillating Slider Mechanisms :-
cylinder



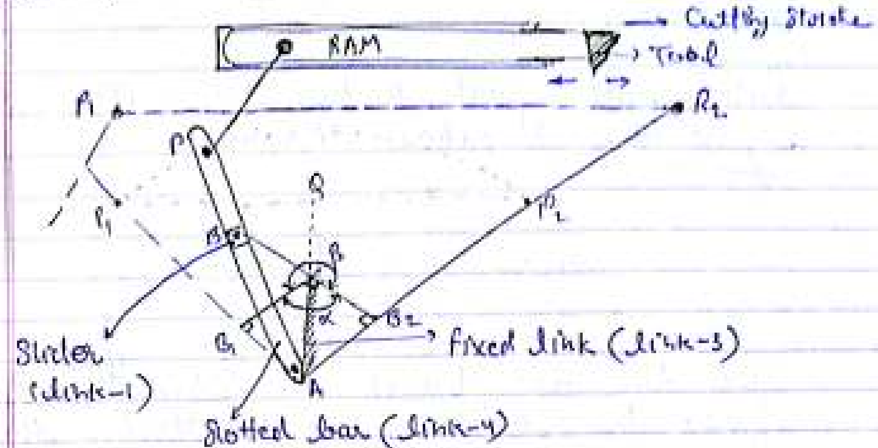
② Pendulum pump



③ Rotary Internal Combustion Engine (Gnome Engine)



④ Crank and Slotted lever Mechanisms



Cutting Ratio (C.R.) $= \frac{\beta}{\alpha} = \frac{\beta}{360-\beta} = \frac{360-\alpha}{\alpha}$

28/8/2017 In this mechanisms the link (AC) forming the turning pair is fixed is fixed as shown in figure the link three corresponds to the connecting rod of a reciprocating steam engine. The driving crank (CB) revolves with uniform angular velocity about the fixed center (C).

A sliding block attached to the crank pin at (B) & slides along the slotted bar (AP) and thus causes AP to oscillate about the pivoted pt (A) short link PR transmitted the motion from AP to the ram which carry the tool reciprocal along. The line of stroke of the work line of stroke R_1R_2

Ram (vi R_1, R_2) is ~~resiter~~ perpendicular to AC produces.

Cutting ratio equal to time of cutting stroke upon time of return stroke.

$$CR = \frac{\beta}{\alpha} = \frac{\beta}{360-\beta}$$

Since the tool travels i.e distance of R_1 & R_2 during the cutting & return stroke, there fore travel by the tool or length of stroke equal to –

$$R_1R_2 = P_1P_2 = 2P_1Q$$

$$= 2AP_1 \sin\angle P_1AQ$$

$$= 2AP_1 \sin(90-\alpha/2)$$

$$= 2AP_1 \cos(\alpha/2)$$

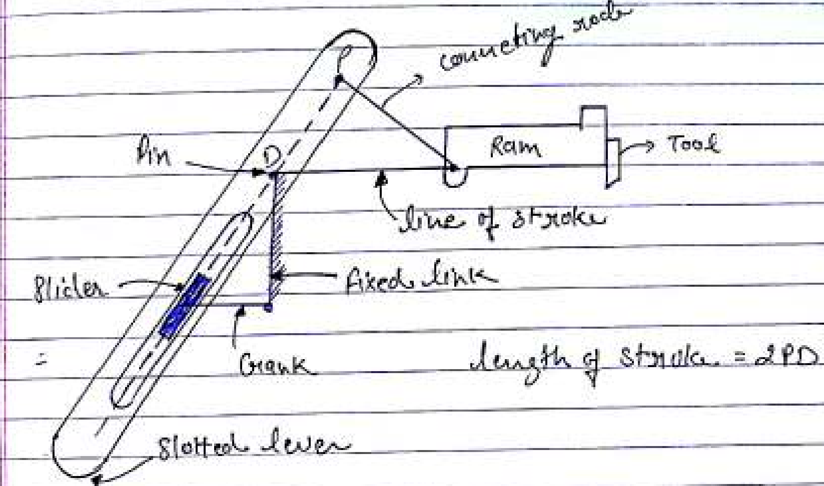
$AP_1 = AP$

$$= 2AP\cos(\alpha/2) \quad \triangle AQC$$

$$= 2AP \times \frac{CB_1}{AC}$$

$$= 2AP \times \frac{CB}{AC}$$

(5) Whitwork quick-return motion mechanisms.



length of stroke $= 2PD$

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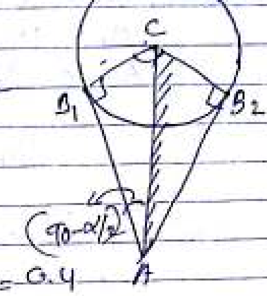
24

Q→ A crank and slotted lever mech. used in a shaper has a central distance of 300mm b/w the centre of oscillation of slotted lever & centre of rotation of the crank, the ~~stati~~ radius of crank 120 mm. find time of cutting to the time of return strock.

Solution→ $AC = 300mm$

$CB = 120\ mm = CB_1 = CB_2$



$$C.R = \frac{\beta}{\alpha} = \frac{\beta}{360-\alpha}$$

$$\sin(90-\alpha/2) = \frac{CB_1}{AC} = \frac{120}{300} = 0.4$$

$$90-\alpha/2 = \sin^{-1}(0.4)$$

$$\alpha/2 = 66.42$$

$$\boxed{\alpha = 132.84}$$

$$\beta = 360-\alpha = 360-132.8$$

$$\boxed{\beta = 227.2}$$

$$CR = \frac{\beta}{\alpha} = \frac{360-132.8}{132.8}$$

$$\boxed{C.R = 1.72}$$

Ques A crank and slotted lever quick return mech. the dist. b/w the fixed centre is 100mm. find the inclination of slotted bar with the vertical in the extreme position. at time ratio of the cutting ratio of the cutting stroke to the return strok. If the length of slotted lever is 400mm. find depth of stroke. If the line of stroke passes through the extreme position of the lever

The lever

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Ans ⇒ $\frac{BC}{AB} = 120\,mm$

$AP_1 = 450\,mm$

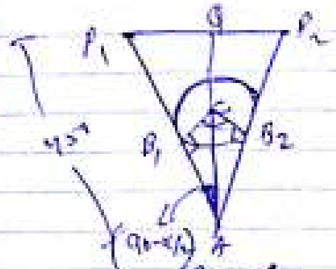
$\frac{P}{d} = \sin\alpha$

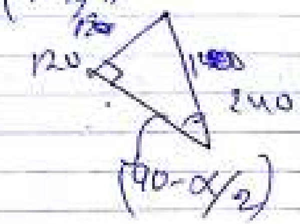
$\frac{120}{1?}$

$AC = \cancel{160}\ 240\,mm$

$CB_1 = 120\,mm$

$AP_1 = 450\,mm$





$\frac{120}{240} = \sin(90-\alpha/2)$

$\sin^{-1}(0.5) = 90-\alpha/2$

$\boxed{\therefore (90-\alpha/2) = 30}$

$\alpha/2 = 90-30$

$\boxed{\alpha = 120}$, $\boxed{\beta = 240}$

$P_1P_2 = P_1P_2 = 2AP_1 \times \frac{CB_1}{AC}$

$= 2 \times 450 \times \frac{120}{240}$

$= 450\,mm$

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Date 24/11/2016

Double - Slider Crank Chain

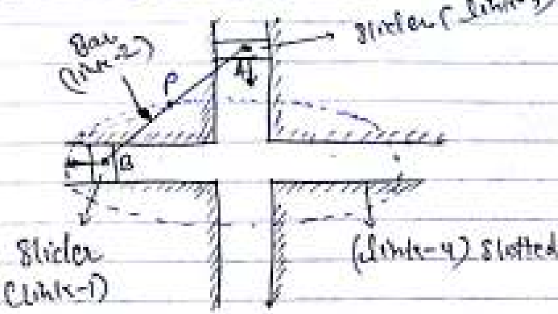


fig of Elliptical trammel

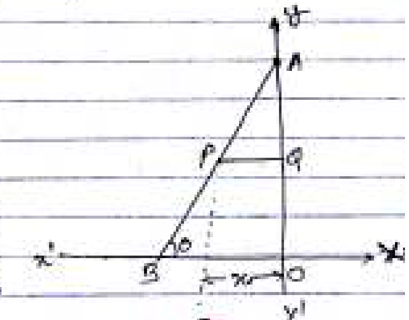


fig (b)

A kinematic chain which consist of two turning pair & two sliding pair is known as double - slider crank chain mechanism.

In figure (a) we see that the link-1 & 2 forming the turning pair. Link-2 & 3 forming another turning pair. The link-3 & 4 forming the sliding pair & link-1 & 4 forming the sliding pair.

Inversion of Double-Slider crank chain:

It is an inst. used for drawing ellipse. This inversion is obtained by slotted plate (link-4) as shown in figure (a) the fixed plate or link-4 has two straight grooves cut in it at right angle to each other when the link-1 & 3 slide along their respective grooves at any pt on the link-2 such as (P) traces out an ellipses as shown in figure (a).

A little consideration will show that AP & BP are semi-major axis & semi-minor axis of the ellipse respectively, this can be proved as follows –

Let us take OX & OY as horizontal & vertical axis & let the link-BA inclined at an angle (θ) with horizontal as shown in figure (B)

now, the coordinate of the point on the link (BA) will be –

$x = PQ$

x/x $\quad x = PQ = AP\cos\theta$

$\cos\theta = \dfrac{x}{AP}$ --- (1)

$y = BP\sin\theta$

$\sin\theta = \dfrac{y}{BP}$ --- (2)

Now, squaring ~~the~~ & adding the eqn (1) & (2)

$\cancel{\left(\dfrac{x}{AP}\right)^2 + \left(\dfrac{y}{BP}\right)^2 = 1}$

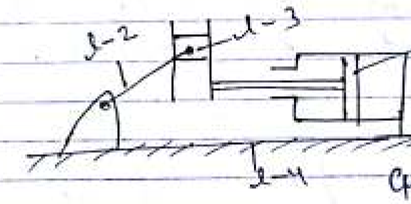
$\cos^2\theta + \sin^2\theta = (x/AP)^2 + (y/BP)^2$

$$\boxed{\left(\dfrac{x}{AP}\right)^2 + \left(\dfrac{y}{BP}\right)^2 = 1} \quad \text{--- (A)}$$

From eqn (A) we can prove that pt (P) traces out an ellipse on the surface.

Note:- If the value of $AP = BP$, pt (P) can traces out as circle on the given surface

* Scotch Yoke Mechanisms :-



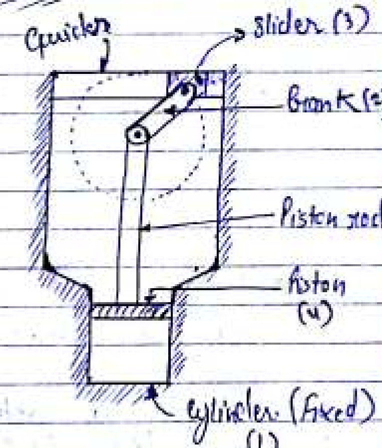


Fig:- Donkey pump.

* Oldham coupling :-

* Universal joint :-

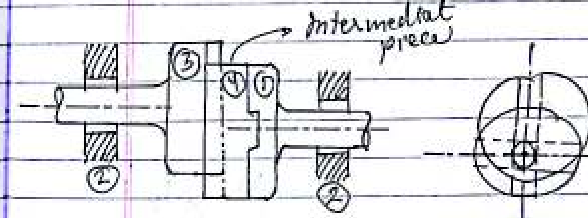


Fig:- Oldham's Coupling

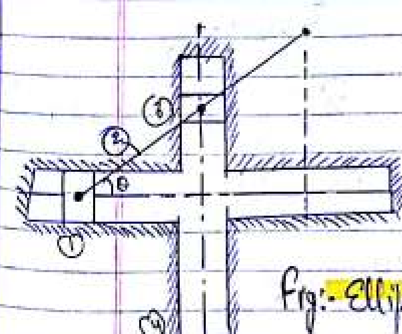


Fig:- Elliptical trammel

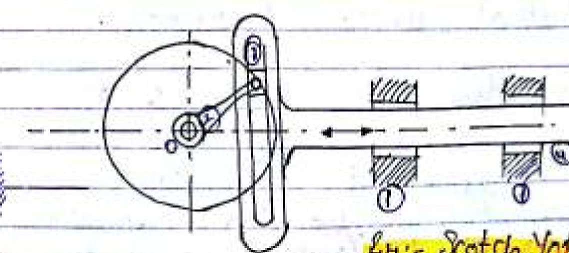
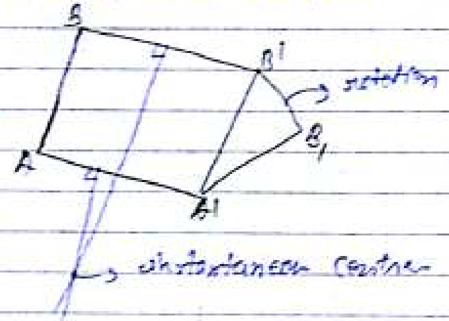


Fig:- Scotch Yoke

Velocity In Mechanisms

(Instantaneous Centre Method)

Some time a body has simultaneously a motion of rotation as well as translation such as wheel of a car on the ground such as motion will have the combined effect of rotation & translation.



- Velocity of a pt on a link by instantaneous centre method :

Consider a motion of a rigid link (AB) about some centre (I) known as instantaneous centre or virtual centre of rotation.

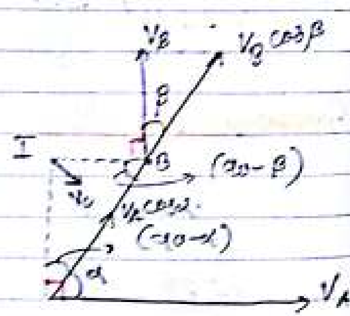


Fig:- Velocity of a point on link

Consider two point (A) & (B) on a rigid link, let V_A & V_B be the velocity of pt A & B whose direction are given by an angle α & β as shown in figure.

If V_A is known in magnitude & direction only & V_B in direction only.

Then magnitude of V_B can be determined by the instantaneous centre method.

Draw AI & BI perpendicular to the direction V_B & V_A respectively.

Let these lines intersect at (I) which is known as instantaneous centre of the link. The complete rigid link is rotate or turn about the centre (I).

Since, A & B are the points on a rigid link therefore, there cannot be any relative motion b/w them along the AB.

⇒ Now resolving the velocities along AB.

$$V_A \cos\alpha + V_B \cos\beta = 0$$
$$V_A \cos\alpha = - V_B \cos\beta$$

$$\boxed{V_A \cos\alpha = V_B \cos\beta}$$

Now,

$$\frac{V_A}{V_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90-\beta)}{\sin(90-\alpha)} \quad \text{---} (1)$$

Applying sin rule:-

$$\frac{AI}{\sin(90-\beta)} = \frac{BI}{\sin(90-\alpha)}$$

$$\frac{\sin(90-\alpha)}{\sin(90-\beta)} = \frac{BI}{AI} \Rightarrow \frac{\sin(90-\beta)}{\sin(90-\alpha)} = \frac{AI}{BI} \quad \text{--- (2)}$$

From (1) & (2)

$$\frac{V_A}{V_B} = \frac{AI}{BI}$$

$$\boxed{\frac{V_A}{AI} = \frac{V_B}{BI} = \frac{V_C}{CI}}$$

* Properties of Instantaneous Centre :-

* A rigid link rotates instantaneously relative to another link at the instantaneous centre for the configuration of the mechanism consider.

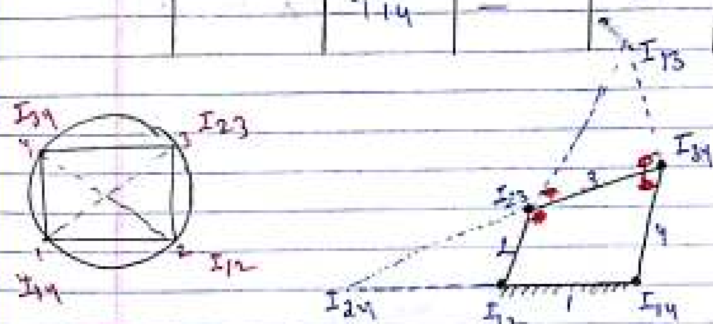
* The two rigid link have no linear velocity relative to each other at the instantaneous centre. At this point (instantaneous centre) the two rigid link have the same linear velocity. Relative to third rigid link in other words the velocity of instantaneous centre relative to any third rigid link will be same whether the instantaneous centre is regarded as a pt on first rigid link or the second rigid link

* No. of Instantaneous Centre in a Mechanisms :-

$\boxed{N = \frac{n(n-1)}{2}}$, where → n = no. of link

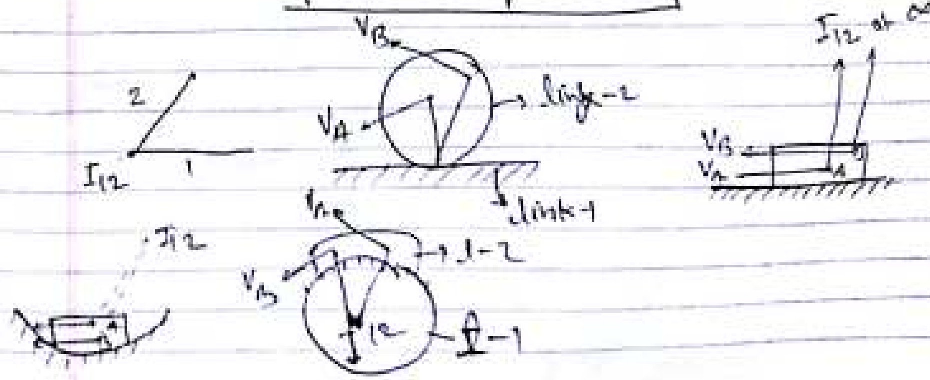
* Method of locating instantaneous centre :-

Link	1	2	3	4
I.C.	I_{12} I_{13} I_{14}	I_{23} I_{24} —	I_{34} — —	— — —



I_{14} I_{34}
I_{12} I_{23}

~~Loto~~ Location of I.C



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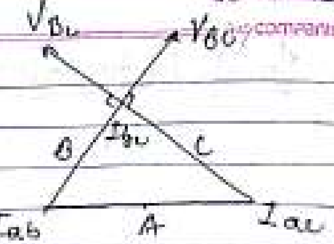
* Kennedy Theorem *

(Instantaneous centre ~~are~~ in a st. line)

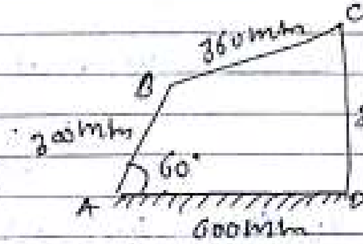


This theorem states that if three bodies moves relatively to each other & they have three inst. centre & ~~it~~ lie on a straight line.

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2}$$

$$\boxed{N = 3}$$

Q → In a pin jointed four-bar mech. as shown in fig. AB = 300m, BC = CD = 360mm & AD = 600mm. The angle BAD = 60°. The crank - AB uniformally at 100 rpm locat all rotates the I.C. And the angula velocity of link-BC



$N_{AB} = 100\,rpm$

$\omega_{AB} = \frac{2\pi N}{60}$

$= \frac{2\pi \times 100}{60} = 10.47$ rad./sec.

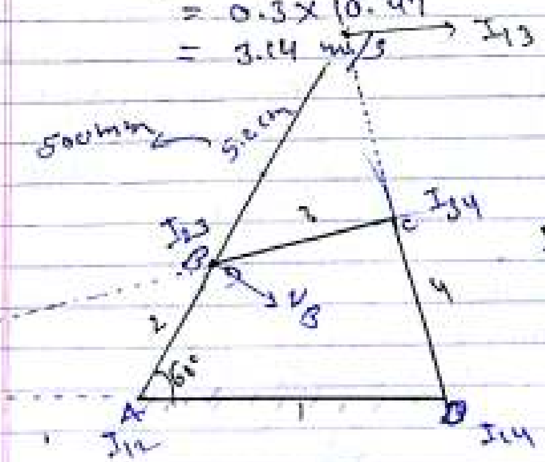
Since the length of crank 300mm = 0.3m therefore, velocity of pt B on link - AB

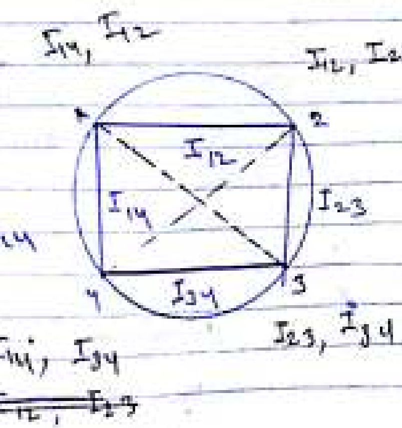
$V_B = AB \times \omega_{AB}$ --- $(r = 2100)$

$= 0.3 \times 10.47$

$= 3.14$ m/s

1cm = 10mm





$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

	1	2	3 4
Ic	I_{12}	I_{23}	I_{34}
	I_{13}	I_{24}	
	I_{14}		

By measurement,

$(I_{13})_B = 500\,mm = 0.5\,m$

$V_B = V_{BC} = \omega_{BC} \times r$

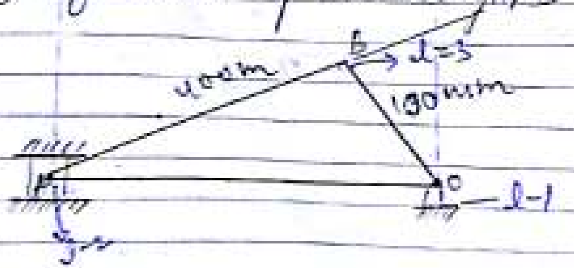
$3.14 = \omega_{BC} \times I_{13B}$

$3.14 = \omega_{BC} \times 0.5$

$\omega_{BC} = \frac{3.14}{0.5} = 6.28$ rad/sec.

Ques → Locate all the I.C. of the slider crank mech. as shown in fig. the length of a crank OB & connecting rod are 100 mm & 400 mm if the crank rotat with $\omega = 10$ rad./sec.

(i) Vel. of slider A
(ii) Ang. vel. of connecting rod AB



$(\omega)_{OB} = 10$ rad/sec.

$OB = 100\,mm$

$N = \frac{4(4-1)}{2} = \frac{12}{2}$

$\boxed{N = 6}$

$V_{OB} = V_B = \omega_{OB} \times AB$
$= 10 \times 0.1$
$= 1\,m/s$

By the measurement we find that –

$(I_{13})_A = 460\,mm = 0.46\,m$

$(I_{13})_B = 560\,mm = 0.56\,m$

(i) Vel. of slider A

Let V_A = vel. of slider A
we know that

$$\frac{V_A}{(I_{13})_A} = \frac{V_B}{(I_{13})_B}$$

$V_A = V_B \times \frac{I_{13}A}{I_{13}B} \Rightarrow 1 \times \frac{0.46}{0.56} = 0.82\,m/s$

(ii) Ang. vel. of connecting rod – AB

ω_{AB} = Ang. vel. of connecting rod

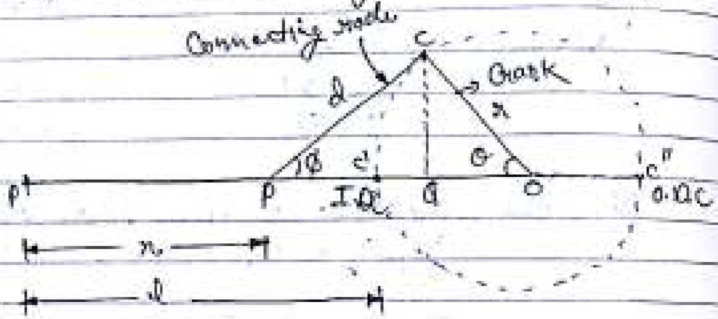
$V = R\omega$
$\omega = \frac{V}{r}$

$\frac{V_B}{(I_{13})_B} = \omega_{AB}$

$\frac{1}{0.56} = \omega_{AB}$

$\omega_{AB} = 1.78$ rad/sec

Analytical Method for Velocity and acc^n of piston



Consider the motion of crank and connecting rod of a reciprocating steam engine as shown in figure

Let OC be the crank and PC be the connecting rod.

Let the crank rotates with an angular velocity ω radian/sec, & crank turns through an angle θ from the inner dead centre.

Let, small x be the displacement of reciprocating body P from the IDC after time (t) second during which the crank has turn through an angle θ,

Let l = length of the connecting rod
r = radius of a crank
ϕ = inclination angle of connecting rod to the line of stroke PO

If n = ratio of length of connecting rod to the radius of crank = l/r

* Velocity of piston :-

From the geometry of the figure -

$$x = PP' = OP' - OP$$
$$= (P'C' + C'O) - (PQ + OQ)$$
$$= (l + r) - (l\cos\phi + r\cos\theta)$$

{From the Δ CQP & CQO}

$$x = l + r - l\cos\phi - r\cos\theta$$
$$= l(1 - \cos\phi) + r(1 - \cos\theta)$$
$$= r\left[(1-\cos\theta) + \frac{l}{r}(1-\cos\phi)\right]$$

$$x = r[(1-\cos\theta) + n(1-\cos\phi)] \quad \text{---- (1)} \quad [l/r = n]$$

From the Δ CPQ, & CQO

$$\frac{CQ}{l} = \sin\phi, \quad \boxed{CQ = l\sin\phi}$$

$$\frac{CQ}{r} = \sin\theta, \quad \boxed{CQ = r\sin\theta}$$

$$CQ = l\sin\phi = r\sin\theta$$

$$\boxed{\frac{l}{r} = \frac{\sin\theta}{\sin\phi}}$$

$$\therefore \quad n = \frac{\sin\theta}{\sin\phi}$$

$$\boxed{\sin\phi = \frac{\sin\theta}{n}} \quad \text{--- (2)}$$

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we know that

$$\cos\phi = \sqrt{1-\sin^2\phi} = \left(1 - \frac{\sin\theta}{n}\right)^{1/2}$$

$\Rightarrow$ Now expanding the above eqn. by Binomial expansion

$$\cancel{(1-x)^n = 1-nx}$$

$$\cos\phi = 1 - \frac{1}{2}\frac{\sin^2\theta}{n^2} + \ldots$$

$$1-\cos\phi = \frac{\sin^2\theta}{2n} \quad \text{--- (3)}$$

Put the value of $1-\cos\phi$ in eqn (1)

$$\cancel{x = r\left[\frac{\sin^2\theta}{2n} + \frac{1}{n}\left(\frac{\sin^2\theta}{2n}\right)\right]}$$

$$\cancel{x = \sin^2\theta\,(}$$

$$x = r\left[(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right] \quad \text{--- (4)}$$

Now, differentiating the eqn (4) w.r.t. θ.

$$\frac{dx}{d\theta} = r\left[(\sin\theta) + \frac{\cancel{2}\sin\theta \times \cos\theta}{\cancel{2}n}\right]$$

$$\frac{dx}{d\theta} = r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) \quad \text{--- (5)}$$

$$V_p = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$$

$$V_p = \frac{dx}{d\theta} \times \omega$$

$\therefore$

$$\boxed{V_p = r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)} \quad \text{--- (7)}$$

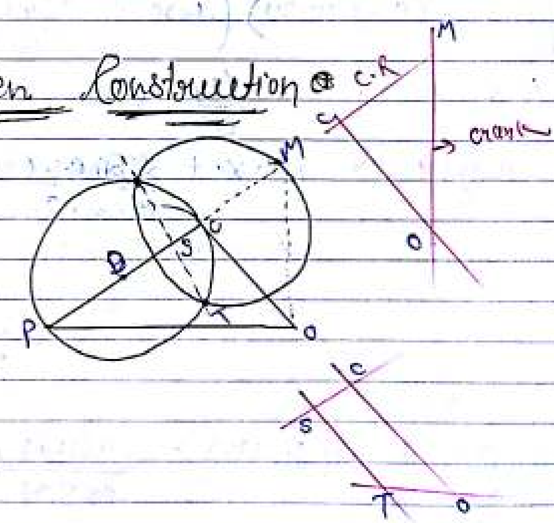
* **Acceleration of piston :—**

$$a_p = \frac{dV_p}{dt} = \frac{dV_p}{d\theta} \times \frac{d\theta}{dt} = \frac{dV_p}{d\theta} \times \omega$$

$$\frac{dV_p}{d\theta} = r\omega\left(\cos\theta + \frac{\cancel{2}\cos 2\theta}{\cancel{2}n}\right)$$

$$\therefore \boxed{a_p = r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)}$$

- Klein Construction



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100 cm = 1000 mm
100 cm = 1 m

Ques ⇒ If the crank and connecting rod are 300mm and 1 m long respectively. If the crank rotate at const. speed of 200 rpm determine the crank angle and which the max. velocity occurs, max. velocity of the piston.

Solution ⇒ Given data –

$r = 300\,mm = 0.3\,m$

$l = 1\,m$

$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 200}{60} = 20.94$ — $\boxed{N = 200}$

$n = \frac{l}{r} = \frac{1}{300\times10^{-3}} = 3.33$

$V_p = r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) = (0.3)(20.94)\left(\sin\theta + \frac{\sin 2\theta}{2\times 3.33}\right)$

$\frac{dv_p}{d\theta} = (0.3)(20.94)\left(\cos\theta + \frac{2\sin\theta\cdot\cos\theta}{2\times 3.33}\right)$

for max. , $\frac{dv_p}{d\theta} = 0$

$(0.3)(20.94)\left(\cos\theta + \frac{2\sin\theta\cdot\cos\theta}{2\times 3.33}\right) = 0$

$\cos\theta + \frac{2\cos^2\theta - 1}{3.33} = 0$

$3.33\cos\theta + 2\cos^2\theta - 1 = 0$

$\boxed{\theta = 75^\circ}$

$(V_p)_{max} = (0.3)(20.94)\left(\sin(75) + \frac{\sin(75\times2)}{2\times 3.33}\right)$

$V_p = 6.54\,m/s$

Ques The crank and connecting rod 0.3m & 1.5m in length the crank rotates at 180rpm in clock wise direction the vel. & acc. of the piston when the is at 40° from the IDC Also determine the position of the for zero acc. of the piston

$r = 0.3;\ l = 1.5\,m$ | $\theta = 40^\circ$

$n = \frac{l}{r} = \frac{1.5}{0.3} = 5$ | $\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times180}{60}$

$\omega = 18.85$

$V_p = (0.3)(18.85)\left(\sin(40^\circ) + \frac{\sin(2\times40)}{2\times 5}\right)$

$\cancel{\boxed{V_p = 5.74}}$ m/s $\boxed{V_p = 4.019}$ m/s

$a_p = (0.3)(18.85)^2\left(\cos(40) + \frac{\cos(2\times40)}{2\times5}\right)$

$= \cancel{78.24}\ \boxed{85.36\ m/sec}$

At zero acc.

$0 = r\omega^2\left(\cos\theta_1 + \frac{\cos 2\theta_1}{n}\right)$

$0 = (0.3)(18.85)^2\left(\cos\theta_1 + \frac{\cos 2\theta}{5}\right)$

$5\cos\theta_1 + \cos 2\theta_1 = 0$

$5\cos\theta_1 = -\cos 2\theta_1$

$5\cos\theta_1 = -(2\cos^2\theta - 1)$

$\boxed{\theta_1 = 79.27^\circ}$

Unit-2

Cams & Follower

Cam is a rotating m/c element which convert rotary motion into reciprocating motion.

A cam is rotating m/c element which gives reciprocating or oscilating motion to another element is known as follower.

The cam & follower have line contact or pt contact and constitute a higher pair. The cams are usually rotates (at) uniform speed by a shaft but followers motion is pre-determined & will be according the shape of cam.

The cams are widely use for operating the inlet & exhaust valve of a IC engine, automatic attachment of m/cries, paper cutting m/c, feed mechanisms of automatic lathe.

Classification of followers:

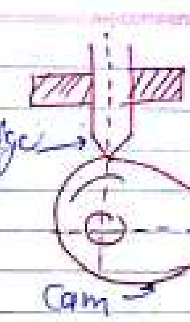
The followers may be classified as—

* According to the surface in contact :—

① Knife edge follower

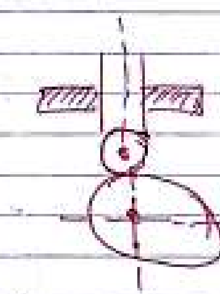
when the contacting end

of the follower has a sharp knife edge. It is called knife edge follower.



The sliding motion takes place b/w the contacting surface (i.e. cam & follower surfaces) In knife edge follower excessive wear occurs due to its shape.

② Roller follower.



when the contacting end is roller it is called roller follower. Since, the rolling motion takes place b/w the contacting surfaces. Therefore, ~~weight rate of~~ wear is greatly reduced. The roller follower is mostly used in gas turbine & air craft engine.

③ Flat faced :— when the contacting end of the follower is flat, plane it is called flat face follower. The side thrust b/w follower & the guide is much reduce in case of flat faced follower. The only side thrust due to friction b/w the contact surfaces of the follower & the cam.

The relative motion b/w these surfaces is largely of sliding nature but wear may

may be reduced by offsetting the axis.

The flat face follower are generally used when space is limited such as in cam which operates the valve of automobile engine.

Note :- when the flat faced follower is circular then it is mushroom follower.

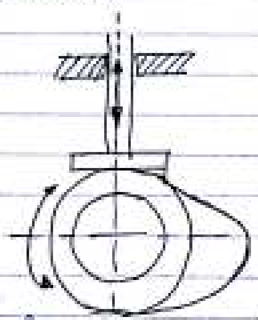
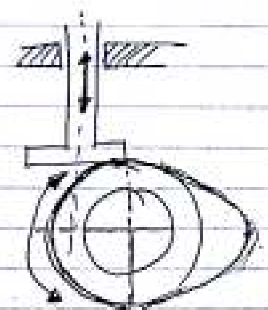
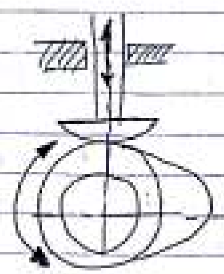


Fig:- Plat faced guide



Cam with offset



* According to the motion of the follower *

① Reciprocating follower :-

When the follower reciprocate in guide

as the cam rotates uniformly it is known as reciprocating follower. Flate faced cam with offset & spherical are the example.

② Oscillating or Rotating follower :-

When the uniform rotary motion of the cam is converted into predetermined oscillatory motion the follower is called oscillating follower. spherical faced follower.

③ Acc. to the path of motion of follower :-

(i) Radial follower :- when the motion of the follower is along an axis passing through the centre of the cam it is known as radial follower.

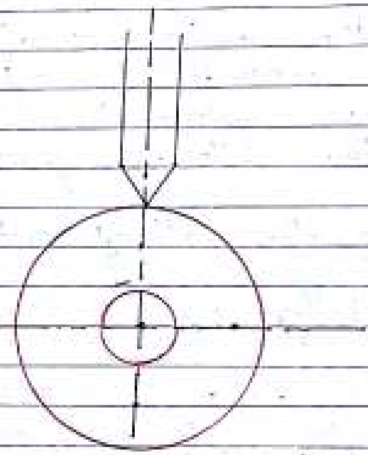
(ii) Offset follower :- when the motion of the follower is along axis away from the axis of the cam centre it is called offset follower.

◎ Classification of Cam ◎

(i) Radial Cam :- In radial cam the follower reciprocate in a direction ⊥ to the cam axis

(3) Cylindrical cam :–
In cyl. cam the follower reciprocat or oscillate in the direction parallel to the cam

• Terms Used In Radial Cam :–



* Base circle :– It is the smallest circle that can be drawn by the help of radius of cam.

* Trace point :– It is the reference pt on the follower and used to generate the pitch curve. In the roller follower the centre of roller represents the trace point.

• Pressure angle :– it is the angle b/w direction of the follower motion and a normal to pitch curve. This angle is very imp. to design a cam profile. If the pressure angle is too large in a reciprocating follower will jam in its bearing.

• Pitch point :– It is pt on the pitch curve having the max. pressure angle.

• Pitch circle :– it is circle drawn from the centre of the cam through the pitch point.

• Prime circle :– It is the smallest circle that ~~cam~~ can be drawn from the centre of the cam & centre to the pitch curve.

For knife edge follower the prime & base circle are identical.

For roller follower prime circle is large than base circle by the radius of the follower.

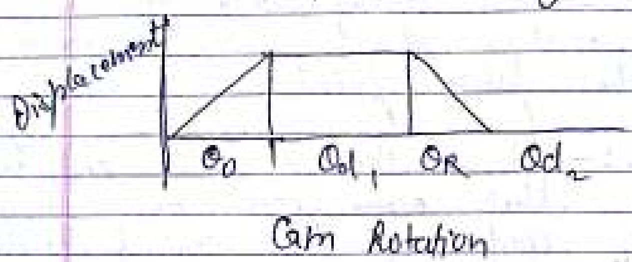
Lift or stroke :– it is the max. travel of the follower its lowest to the highest position.

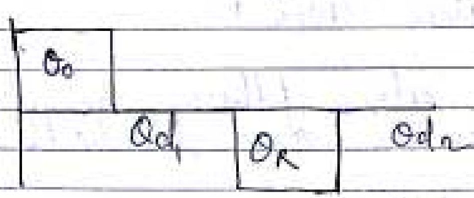
Motion of the Follower

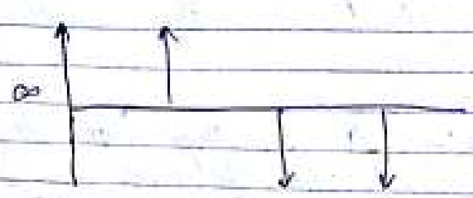
The follower during its travel may have one of the following motion.

(i) Uniform velocity
(ii) Simple harmonic motion
(iii) Uniform acc^n & retardation.
(iv) Cycloidal motion

Displacement, Velocity & acceleration diagram when the follower moves in uniform velocity



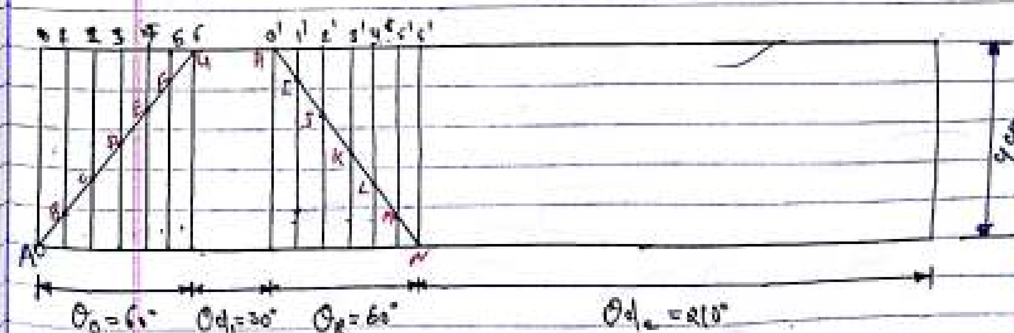


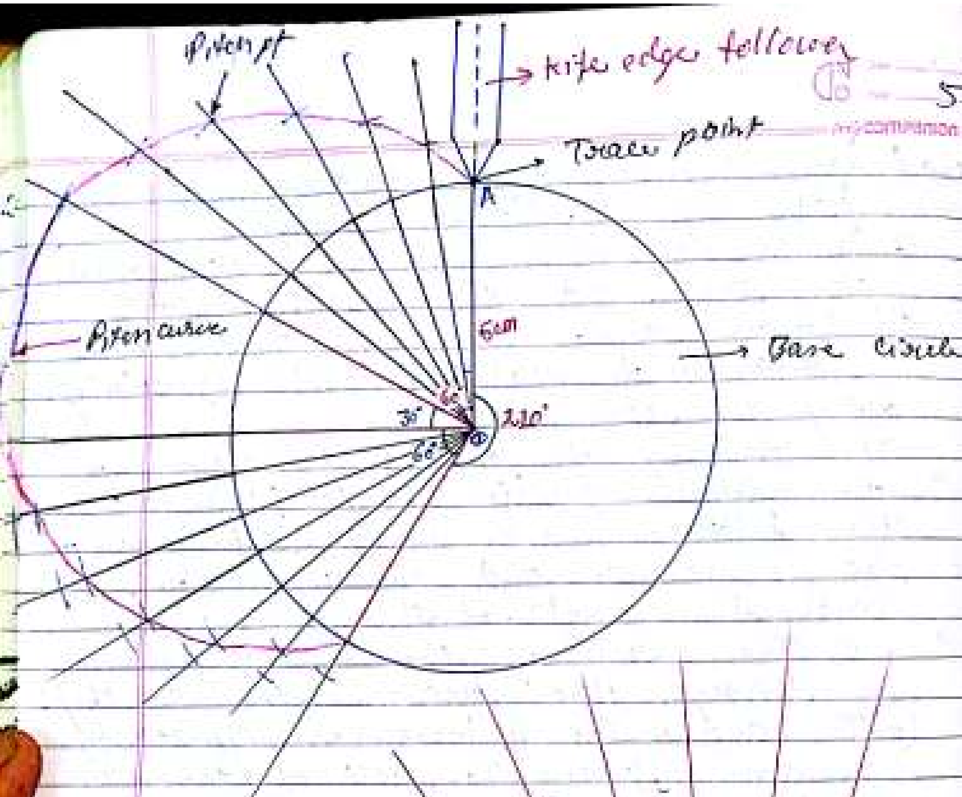


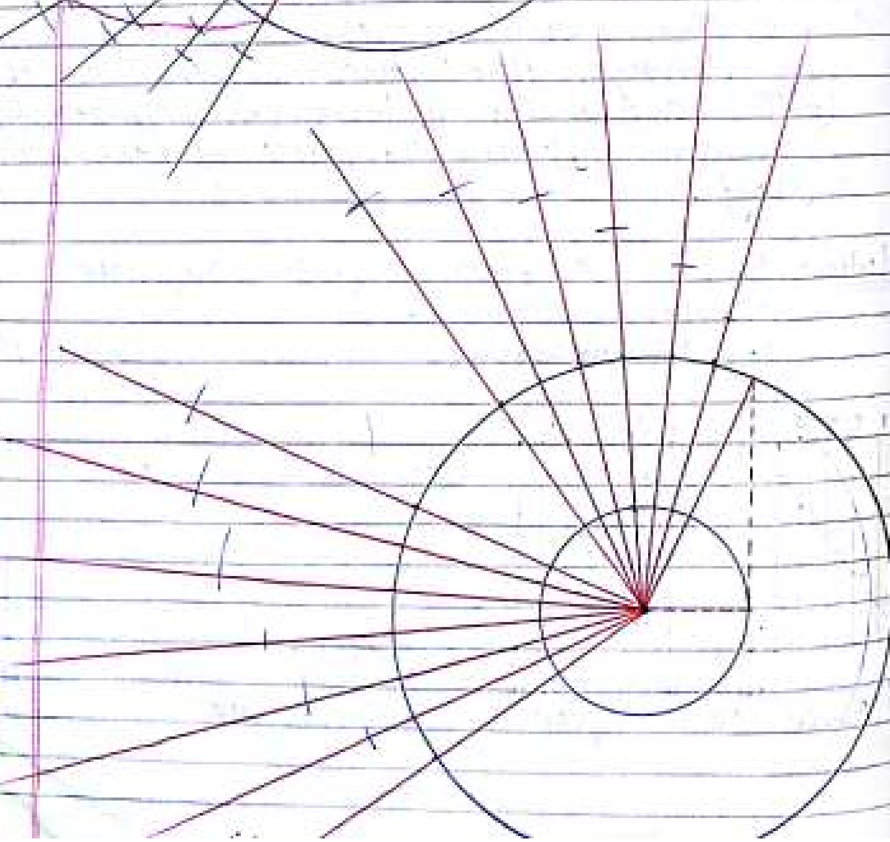
Ques> A cam is to give the following motion to a knife edge follower.
(i) Out stroke during 60° of cam rotation (ii) dwell for the next 30° of cam rotation.
(iii) return stroke during 60° of the cam rotation (iv) to dwell for the remaining 210° of the cam rotation.
The stroke of the follower is 40mm & the minimum radius of the cam is 50mm.
The follower moves with uniform velo during out stroke and return stroke.
Draw the profile of the cam.
(i) The axis of the follower passes through the axis of the cam shaft.
(ii) The axis of the follower is offset by 20 mm from the axis of the cam shaft.

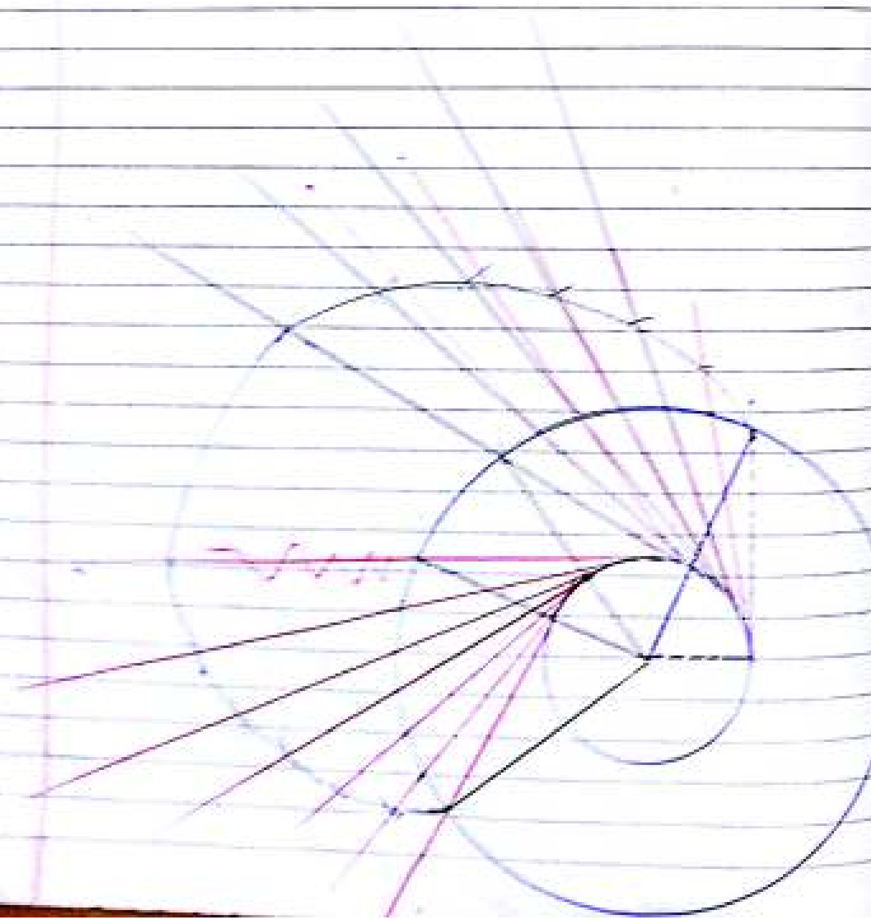
Solution> $\theta_O = 60°$, $\theta d_1 = 30°$, $\theta_R = 60°$, $\theta d_2 = 210$
$S = 40mm = 4cm$
$r = 50 mm = 5cm$

Scale -> 20° - 1cm



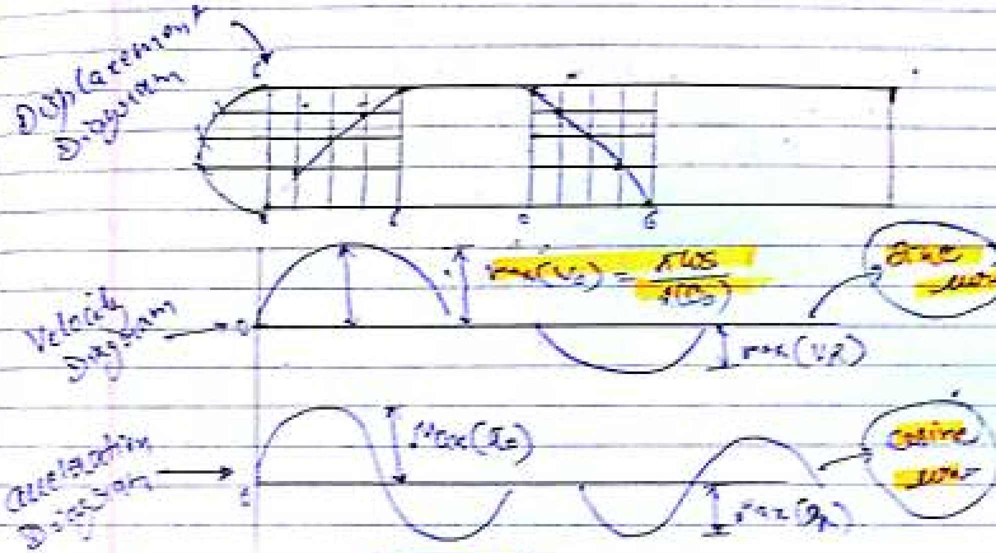

Pitch pt
Knife edge follower
Trace point
A
6cm
Pitch curve
Base circle
30°
60°
60°
210°





Date 2/2/2017

Displacement, Velocity and Acc'n Diagram for Simple Harmonic Motion

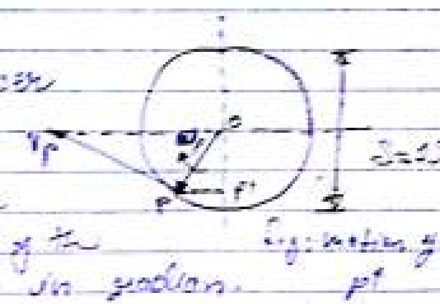


$\boxed{\theta = \omega t}$

Let,

S = Stroke of the follower

θ_o & θ_R = angular displacement during out stroke & return stroke of the follower respectively in radian.



ω → angular vel. of the cam in radian/sec.

Time required for the out stroke of the follower

$t_o = \frac{\theta_o}{\omega}$

consider a pt (P) moving at uniform const. speed ω radian/sec around circumference of circle with the stroke S as a dia. as shown in figure. The pt P' which is the projection of pt P on the diameter, executes a simple harmonic motion as the pt P rotates. The motion of the follower is similar to that of the pt P'

Periferal speed of the pt P'

$$V_P = \frac{\pi S}{2} \times \frac{1}{t_0} \quad \text{---} \quad \boxed{S = 2r}$$

$$V_P = \frac{\pi S}{2} \times \frac{1}{\frac{\theta_0}{\omega}}$$

$$V_P = \frac{\pi \omega S}{2(\theta_0)}$$

$$\boxed{(V_P)_{max.} = \frac{\pi \omega S}{2(\theta_0)_{max}}}$$

Periferal velocity for return stroke

$$V_R = \frac{\pi \omega S}{2(\theta_R)}$$

Accⁿ of the pt P $\quad$ --- $F_C = \frac{mV^2}{r}$

$$P = \frac{V^2}{r}$$

$$(a_0) = \frac{(V_P)_0^2}{OP}$$

$$(a_0) = \frac{\left(\frac{\pi \omega S}{2(\theta_0)}\right)^2}{S/2}$$

$$\boxed{a_0 = \frac{\pi^2 \omega^2 S}{2(\theta_0)^2}}$$

for return

$$a_R = \frac{\pi^2 \omega^2 S}{2(\theta_R)^2}$$

Ques 3) A cam is derived for a knife edge follower with the following data.
— cam lift 40mm during 90° cam rotation with SHM. Dwell for next 30°, during the next 60 cam rotation. The follower returns its original position with SHM. Dwell during remaining 180°. Draw the cam profile (i) the line of stroke passes through the cam shaft & offset 20mm from the axis of ~~centre~~ cam shaft. The ~~static~~ radius of circle of the cam is 40 mm. Determine the max vel. & accⁿ. of the follower for the out stroke & return stroke if the cam ~~st~~ rotates 240 rpm

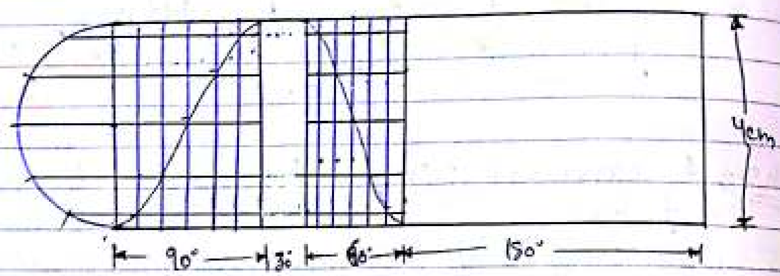
$$V_P = \frac{\pi \omega S}{2(\theta_0)} \quad , \quad S = 40 \times 10^{-3}\,m$$

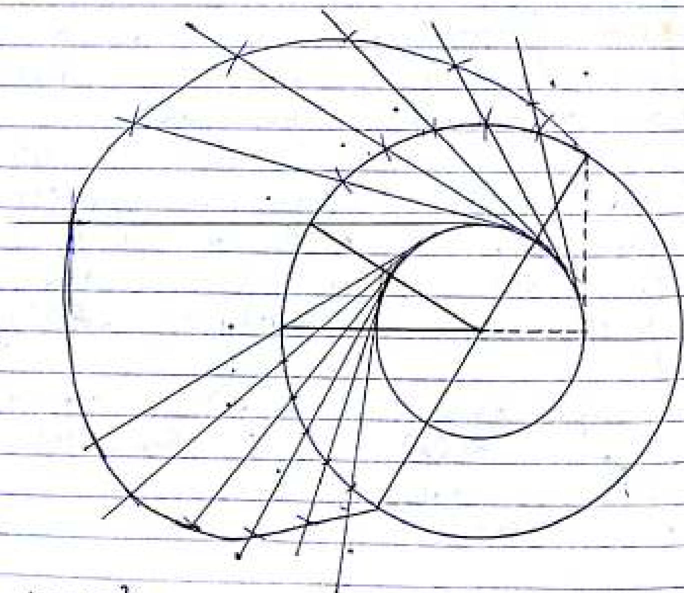
$$\theta_0 = 90° = \frac{\pi \times 90}{180} = \frac{\pi}{2}\ \text{radian}$$

$$\omega = \frac{2\pi N}{60}$$

$S = 40mm = 4cm$
$\theta_0 = 90°$
$\theta_{d_1} = 30°$
$\theta_R = 60°$
$\theta_{d_2} = 180°$

Scale → $30^\circ = 1\,cm$





Given –

$s = 40 \times 10^{-3}\ m$

$\theta_0 = 90^\circ = \frac{\pi \times 90}{180} = \frac{\pi}{2}$ radian

$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 240}{60} = 25.132\ m/s$

$V_P = \frac{\pi \omega s}{2(\theta_0)}$

$= \frac{\pi \times 25.132 \times 40 \times 10^{-3}}{2 \times (\pi/2)}$

$= 1.00528\ m/s$

$V_R = \frac{\pi \times 25.132 \times 40 \times 10^{-3}}{2 \times \pi/3} = 1.507\ m/s$

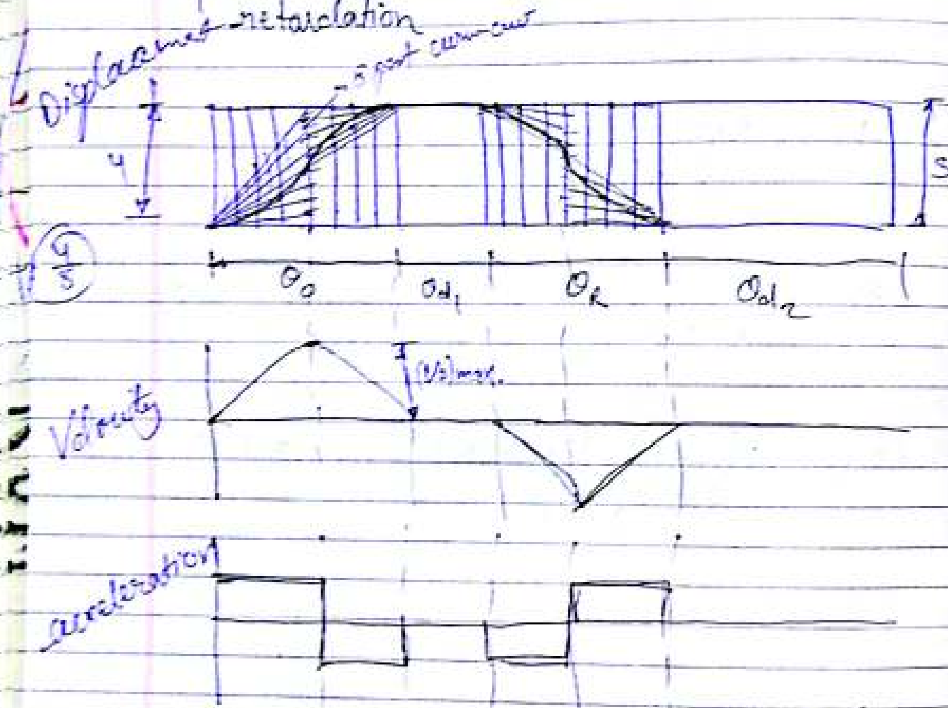
$a_0 = \frac{\pi^2 \omega^2 s}{2(\theta_0)^2} = \frac{\pi^2 \times (25.132)^2 \times 40 \times 10^{-3}}{2 \times (\pi/2)^2}$

$= \cancel{16.094}\ 50.53$

$a_R = \frac{\pi^2 \times (25.132)^2 \times 40 \times 10^{-3}}{2 \times (\pi/3)^2} = 113.7$

1/2/2017

Displacement, Velocity and Acc^n diagram when follower moves with uniform acc^n & retardation



$a = C$

$\frac{v}{t} = C$

$\boxed{v \propto t}$

$V_{mean} = \frac{V_{max}}{2}$

$(V_o)_{max} = \frac{2\omega S}{\theta_o}$

$V_R = \frac{2\omega S}{\theta_R}$

$(a_o)_{max} = \frac{4\omega^2 S}{(\theta_o)^2}$

$(a_R)_{max} = \frac{4\omega^2 S}{(\theta_R)^2}$

Displacement, Velocity and Acc^n. diagram when follower moves with cycloidal Motion

$S = 2\pi r$

$r = \frac{S}{2\pi}$

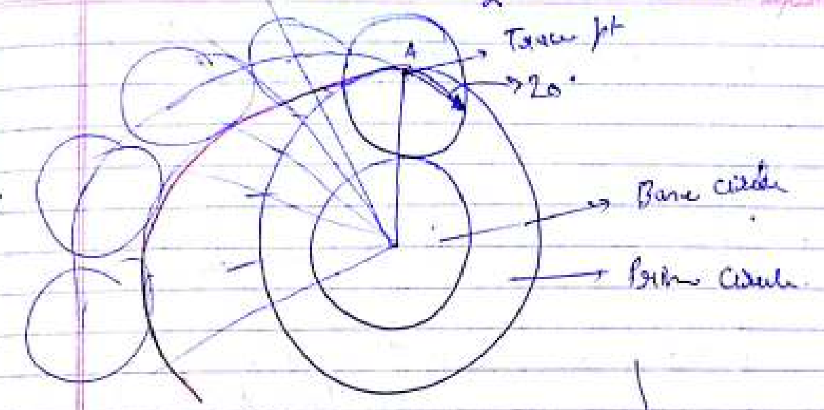
Ques A cam ~~to~~ with min. radius of rotation 25 mm rotating clock wise at a uniform speed is to be design to give a roller follower at the end of a valve rod motion described below $\theta_O = 120°$, $\theta_{d_1} = 30°$, $\theta_R = 60°$, $\theta_{d_2} = 150°$, $S = 50$ mm, dia of roller 20 mm & the dia of cam shaft is 25 mm. Draw the cam profile when it passes through the axis of the cam to shaft. The line of stroke is offset 15 mm from the axis of the cam shaft. Determine the max. acc^n & velocity when the cam shaft rotates 100 rpm also draw the vel. & acc^n diagram for one complete revolution of the cam assum the follower rotate with simple harmonic motion.

Scale → 20° → 1 cm

Base circle = radius of cam = 25

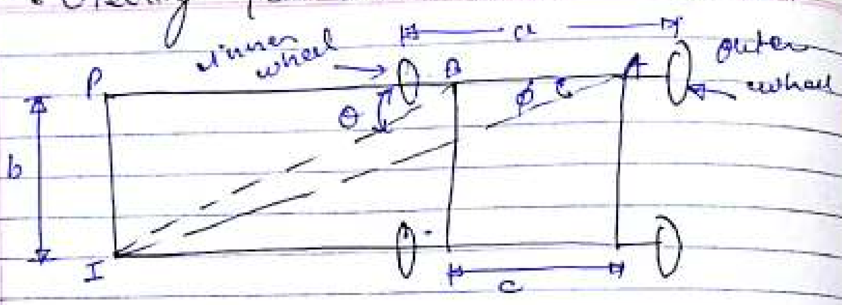
Prime circle = Base circle radius + roller radius

$= 25 + \frac{1}{2} \times 20 = 35$ mm



Date 7/2/2017

Steering Gear Mechanism :-



$$\boxed{\cot\phi - \cot\theta = \frac{c}{b}}$$ This eqn. is to be satisfied for steering gear mechanisms.

The steering gear mechanisms is used for changing the direction of two or more ~~each~~ of the wheel axel with the reference chesis. So as to move the automobile in desired path. Usually the two back wheel have a common axis which is fixed in the direction with reference to the chesis in the ~~in~~ steering is done by means of the front wheel.

In automobiles the front wheel are placed on the front axel which are pivoted at the pt A & B. The pts are fixed to the chesis which is shown in figure. The back wheels are place over the back axel at the two end of the the differential ~~tool~~ tube. When the vihicles take a turn the front wheel along with respective axle turn

about respective pivoited point. The back wheel remain straight and do not turn. There for the steering is done by means of front wheels only.

In order to avoid skidding (ie. slipping of the wheel) the two front wheel must turn about the same instanteneous centre (I). which lies on the ~~axle~~ axis of the back wheel. If the intesteneous centre of the two front wheel donot ~~co~~ coincide with the I.c of the back wheels the skidding on the front wheels ~~of~~ or back wheel will deffinetly takes place which will causes more wear & tear of the ~~tiers~~ tyres. Thus the condition for correct steering all the four wheel must be turn about the same instentoneous centre (Ic). The axis of the in

then the angle ϕ ~~sugton~~ subtended the axis of the outer wheel.

Let ~~a small a wheel track~~.

a – wheel track
b → wheel base
c → distance b/w pivoited pt ~~A~~ A B
~~from the Δ IBP~~

From the triangle IBP

$$\cot\theta = \frac{BP}{IP}$$

From the Δ IAP

$$\cot\phi = \frac{AP}{IP}$$

Done

$$\cot\phi = \frac{AP}{IP} = \frac{AB+BP}{IP}$$

$$\cot\phi = \frac{AB}{IP} + \frac{BP}{IP}$$

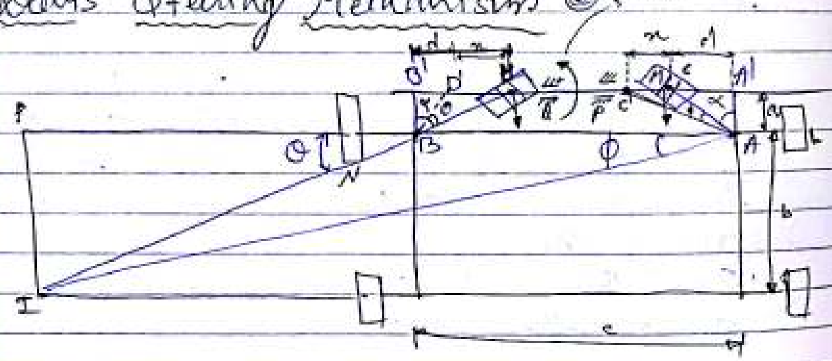
$$\cot\phi = \frac{c}{b} + \cot\theta$$

$$\boxed{\cot\phi - \cot\theta = \frac{c}{b}}$$

This is the fundamental eqn. for correct steering. If this condn. is satisfied there will be no skidding of the wheel when the vehicles take turn.

Davis Steering Mechanism

left turn



It is an exact steering gear mechanism. The slotted link AM & BH are attached to the front wheel axel which turn on pivotes A & B respectively. The rod CD constrainted move in direction of its length by sliding member at P & Q. Then constrained are connected to the slotted link AM & BH by a sliding and turning pair. The steering is effected by moving CD to right or left.

of its normal position C', D' shows the the position of CD for turing to the left.

let,
- $a \to$ vertical distance AB & CD
- $b \to$ wheel base
- $d \to$ horizontal distance b/w A'C & B'D
- $c \to$ distance b/w pivotes A & B
- $\alpha \to$ angle of inclination of the link AC & BD to the vertical.

From the $\Delta AA'C'$

$$\tan(\alpha+\phi) = \frac{A'C'}{AA'} = \frac{d+x}{a} \quad \text{---- (I)}$$

From the $\Delta AA'C$

$$\tan\alpha = \frac{d}{a} \quad \text{---- (II)}$$

From the $\Delta BB'D'$

$$\tan(\alpha-\theta) = \frac{d-x}{a} \quad \text{---- (III)}$$

we know the $\tan(\alpha+\phi)$

$$\tan(\alpha+\phi) = \frac{\tan\alpha + \tan\phi}{1-\tan\alpha \cdot \tan\phi}$$

$$\frac{d+x}{a} = \frac{d/a + \tan\phi}{1 - \frac{d}{a}\times\tan\phi}$$

$$\frac{d+x}{a}\left(1-\frac{d}{a}\tan\phi\right) = \frac{d}{a} + \tan\phi$$

$$\tan\phi = \frac{d+x}{a}\left(1-\frac{d}{a}\tan\phi\right) - \frac{d}{a}$$

$$= \frac{d+x}{a} - \frac{(d+x)d\tan\phi}{a^2} - \frac{d}{a}$$

$$\tan\phi = \frac{ax}{a^2+d^2+dx} \quad \text{---- (IV)}$$

$$\tan(\alpha-\theta) = \frac{\tan\alpha - \tan\theta}{1+\tan\alpha.\tan\theta}$$

$\tan(\alpha-\theta) =$

$$\tan\theta = \frac{ax}{a^2+d^2-dx} \quad --- (5)$$

We know,

$$\boxed{\cot\phi - \cot\theta = c/b}$$

$$\frac{1}{\tan\phi} - \frac{1}{\tan\theta} = \frac{c}{b}$$

$$\frac{a^2+d^2+dx}{dx} - \frac{a^2+d^2-dx}{ax} = \frac{c}{b}$$

$$\Rightarrow \frac{\cancel{a^2}+\cancel{d^2}+dx-\cancel{a^2}-\cancel{d^2}+dx}{ax} = \frac{c}{b}$$

$$\frac{2dx}{ax} = \frac{c}{b}$$

$$\frac{2d}{a} = \frac{c}{b}$$

$$\boxed{\frac{d}{a} = \tan\alpha}$$

$$\boxed{2\tan\alpha = \frac{c}{b}}$$

$$\therefore \boxed{\tan\alpha = \frac{c}{2b}}$$

20/2/2013

Ques ⇒ In a davis steering gear mechanism the distance b/w pivots of the front axle is 1.2m and the wheel base 2.7 m. Find the inclination of the track to the longitudinal axis of the car, when it is moving along straight path.

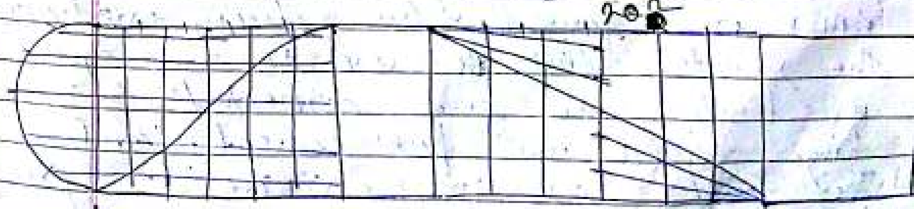
$$c = 1.2$$
$$b = 2.7\,m$$
$$\tan\alpha = \frac{c}{2B}$$
$$\alpha = \tan^{-1}\left(\frac{c}{2B}\right)$$
$$\alpha = 12.5°$$

Ques ⇒ Construct cam profile for following data-

(1) Cam radius = 25mm , $N = 200$ rpm (knife edge follower)
(2) lift = 40mm
(3) $\theta_o = 120°$ (with SHM)
(4) dwell b next dwell = 45°
(5) $\theta_R = 150°$ (with uniform acc^n & Retardation)

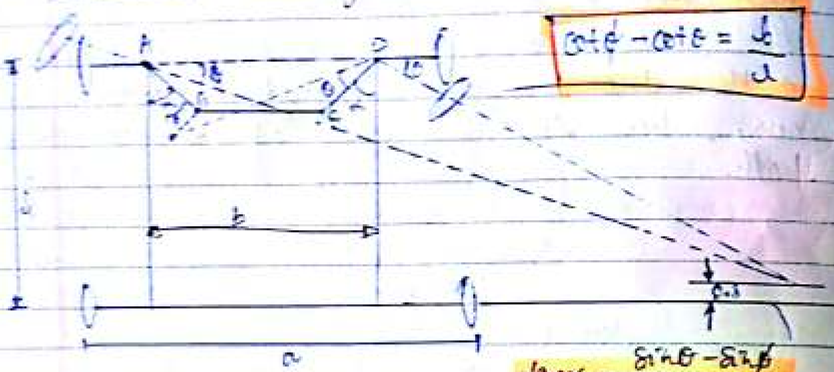
Also draw velocity, acc^n, displacement diagram.
Calculate (i) Max^m velocity at return stroke
(ii) Max acc^n at rise stroke.

$$\hookrightarrow a = \frac{\pi^2\omega^2 s}{2\theta_o^2} \qquad V = \frac{2\omega s}{\theta}$$



120 — 45 — 150 — 45

Ackerman Steering Mechanism



$$\boxed{\cot\phi - \cot\theta = \frac{b}{a}}$$

$$\tan\alpha = \frac{\sin\theta - \sin\phi}{2 - \cos\phi - \cos\theta}$$

⇒ Difference b/w Davis & Ackerman steering mechanism.

(1) The whole mechanism of the ackerman steering gear is a back of the front wheel where as in Davis steering gear it in front of the wheels.

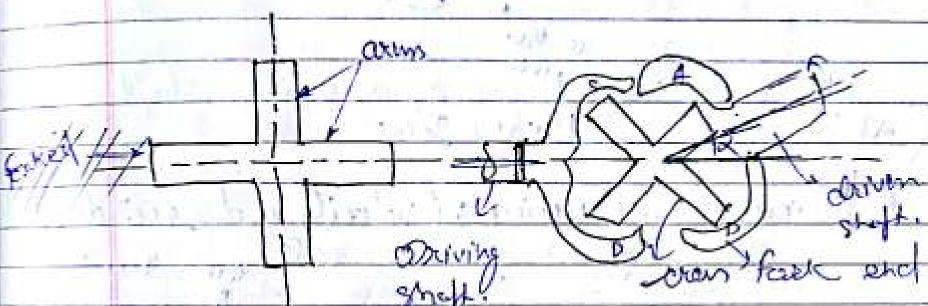
(2) The Ackerman gear consist of turning pairs (AB, BC, CD & AD) where as davis steering gear consist of sliding member.

(3) In ackerman steering gear the mechanism ABCD is a four bar chain mechanism the shorter link BC and AB are equal in length and are connected by hinge joint with front wheel axel. The longer link AB & CD are in equal length.

BC → track rod

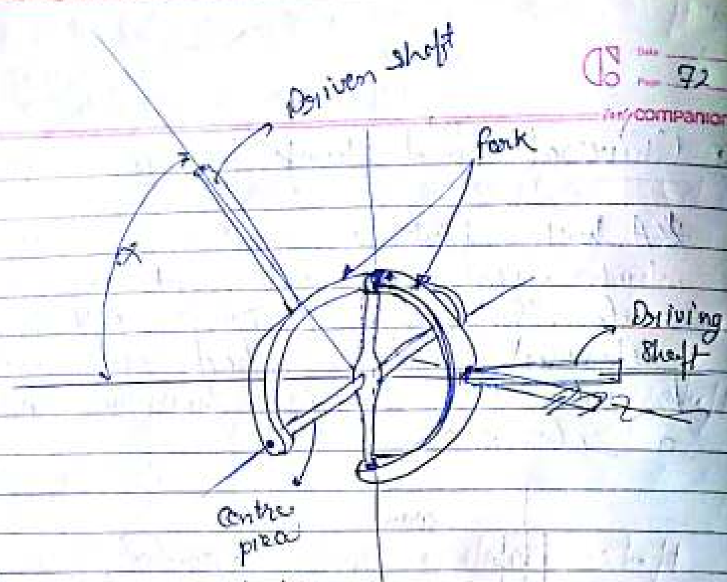
Universal or hook's joint

A hook joint is used to connect two shaft which are intersecting at an small angle. The end of each shaft is fork shaped and U-shaped and each fork provide two bearings for the arms of a cross.



The arm of the cross are ⊥ to each other the motion is transmitted from the driving shaft to the driven shaft. The inclination of the two shaft may be constant. But in actual practice it varies when the motion is transmitted.

The main application of hook joint is found in the transmission from the gear box to the differential. It is also used as a knee joint in milling M/c

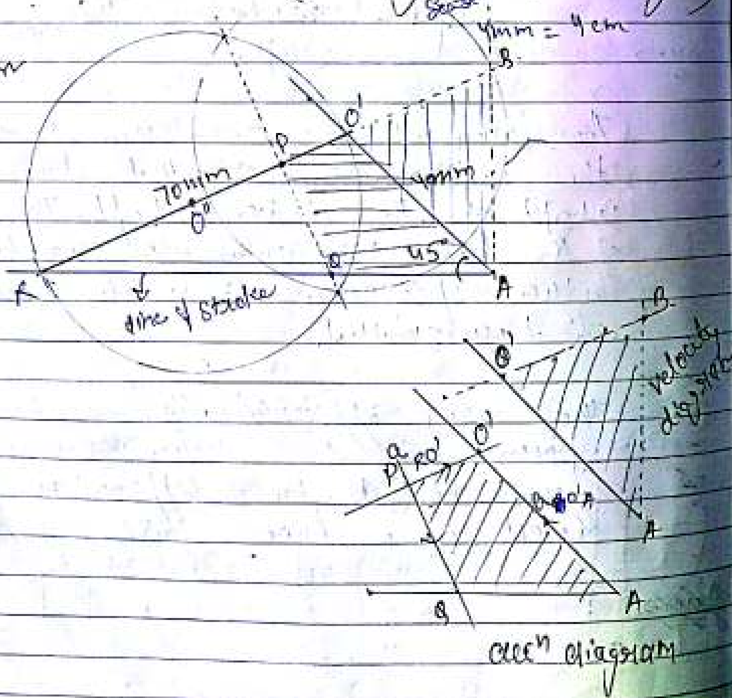


Hooke's Joint

21/2/2017

Klein's Construction : (Velocity and Acc^n diagram)

Crank = 40mm
Connecting rod = 70mm
$\theta = 45^\circ$



~~Max. & Mini. Speed of Driven Shaft~~

Ratio of the Shafts Velocities

The top and front view connecting the two shaft by the universal. It are shown in fig. Let the initial position of the cross be such that both axes lies in the plane of paper in front view while the arm AB attached to the driving shaft lies in the plane containing the axes of the two shaft. Let the driving shaft rotate through an angle θ so that the arm AB move in a circle to new position A_1B_1. A ~~litte~~ little consideration will shows that the arm CD will like an ellipse. The new position CD on the ellipse at an angle θ but the true angle must be on the circular path. To find the true angle project the pt C circular path. To find the true angle project the pt horizontally, intersecting the circle at C_2. Therefore the angle $COC_2 = \phi$ is the true angle turned by driven shaft.

Thus, when the driving shaft turns through an angle θ. The driven shaft turns through an angle φ.

In ΔOC_1M

$$\tan\theta = \frac{OM}{MC_1} \quad --(1)$$

In ΔOC_2M

$$\tan\phi = \frac{OM}{C_2N} = \frac{OM}{MC_1} \quad --(2) \qquad \boxed{C_2N = C_1M}$$

Divide (1) by (2)

$$\frac{\tan\theta}{\tan\phi} = \frac{OM}{MC_1} \times \frac{MC_1}{ON} = \frac{OM}{ON}$$

α = Angle of inclination of the driving and driven shaft
N = Speed of driving shaft in r.p.m.
N_1 = Speed of the driven shaft in r.p.m.

In Δ

$$\cos\alpha = \frac{OM}{OM_1} \Rightarrow OM = OM_1 \cos\alpha$$

$$\frac{\tan\theta}{\tan\phi} \Rightarrow \cos\alpha \Rightarrow \tan\theta = \cos\alpha \cdot \tan\phi \quad \text{--- (3)}$$

Let ω = angular velocity of the driving shaft

$$\omega = \frac{d\theta}{dt}$$

Let ω_1 = angular velocity of driven shaft

$$\omega_1 = \frac{d\phi}{dt}$$

Now differentiating w.r.t t

$$\sec^2\theta \frac{d\theta}{dt} = \sec^2\phi \cdot \cos\alpha \frac{d\phi}{dt}$$

$$\sec^2\theta \cdot \omega = \sec^2\phi \cdot \cos\alpha \cdot \omega_1$$

$$\frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\sec^2\phi \cdot \cos\alpha}$$

$$\frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\sec^2\phi \cdot \cos\alpha}$$

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta \cdot \sec^2\phi \cdot \cos\alpha} \quad \text{--- (4)}$$

$$\sec^2\phi = 1 + \tan^2\phi = 1 + \left(\frac{\tan\theta}{\cos\alpha}\right)^2$$

$$1 + \frac{\sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha}$$

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta \left(1 + \frac{\tan^2\theta}{\tan^2\alpha}\right) \cdot \cos\alpha}$$

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta \left(\frac{\cos^2\alpha + \frac{\sin^2\theta}{\cos^2\theta}}{\cos^2\alpha}\right) \cos\alpha}$$

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta \left(\frac{\cos^2\theta \cdot \cos^2\alpha + \sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha}\right) \cos\alpha}$$

$$\frac{\omega_1}{\omega} = \frac{\cos^2\theta (1 - \sin^2\alpha) + \sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha}$$

$$= \frac{1 - \sin^2\alpha \cdot \cos\theta}{\cos^2\theta \cdot \cos^2\alpha}$$

putting in equation (4)

$$\boxed{\frac{\omega_1}{\omega} = \frac{\cos\alpha}{(1 - \cos^2\theta \cdot \sin^2\alpha)}} = \frac{N_1}{N} \quad \text{--- (1)}$$

~~Max. & Min. Speed of driven shaft~~
~~we know the from the velocity~~

Max. and Mini. speeds of Driven shaft :-
The value of ω_1 will be max. for the given values of α of the denominator of the eqn. (1) is mini. This will happen $\cos^2\theta = 1$, $\theta = 0$, 180

$$(\omega_1)\,max = \frac{\omega \cos\alpha}{1 - \sin^2\alpha} = \frac{\omega \cos\alpha}{\cos^2\alpha}$$

$$\left[N_1(max) = \frac{N}{\cos\alpha}\right]$$

$$\boxed{(\omega_1)\,max = \frac{\omega}{\cos\alpha}} \quad \text{--- (11)}$$

Similarly the value of ω_1 is mini.

$$\left[N_1(mini) = N\cos\alpha\right] \quad \cos^2\theta = 0, \quad \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\boxed{(\omega_1)\,min. = \omega \cos\alpha}$$

~~Max^m or Min^m Speed of driven shaft~~

⇒ Condition for equal speed of driving + driven shaft

We know that $\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1-\cos^2\theta . \sin^2\alpha}$

$$\omega = \frac{\omega_1 (1-\cos^2\theta \sin^2\alpha)}{\cos\alpha}$$

for the equal speed :- $\boxed{\omega_1 = \omega}$

$$\cos\alpha = 1-\cos^2\theta . \sin^2\alpha$$
$$1-\cos\alpha = \cos^2\theta . \sin^2\alpha$$

$$\boxed{\cos^2\theta = \frac{1-\cos\alpha}{\sin^2\alpha}} \quad ----(3)$$

$$\sin^2\theta = 1-\cos^2\theta$$
$$= 1-\frac{1-\cos\alpha}{1-\cos^2\alpha}$$
$$= 1-\frac{(1-\cos\alpha)}{(1-\cos\alpha)(1+\cos\alpha)}$$

$$\sin^2\theta = \frac{\cos\alpha}{1+\cos\alpha} \quad ----(4)$$

3 ÷ 4

$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\cos\alpha}{\sin^2\alpha} \times \frac{1+\cos\alpha}{\cos\alpha}$$

$$\tan^2\theta = \frac{1-\cos^2\alpha}{\sin^2\alpha}$$

$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{\sin^2\alpha}{1+\cos\alpha} \times \frac{\cos\alpha}{(1+\cos\alpha)}$$

$$\boxed{\tan^2\theta = \cos\alpha}$$

$$\boxed{\tan\theta = \pm\sqrt{\cos\alpha}}$$

⇒ Max^m Fluctuation of Speed.

we know that max. speed of driven shaft

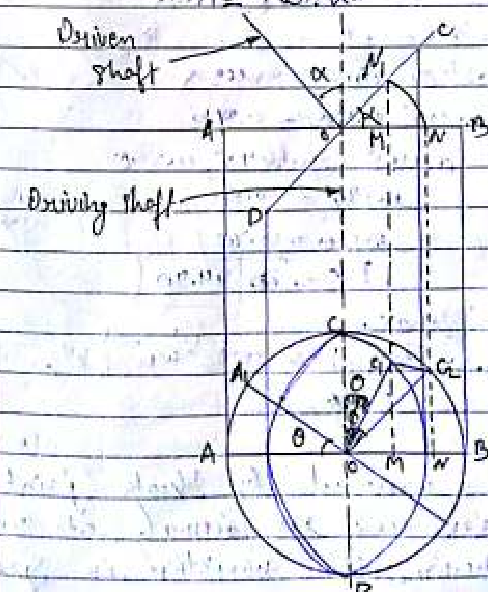
$$= (\omega_1)_{max} - (\omega_1)_{min}$$

$$= \frac{\omega}{\cos\alpha} - \omega\cos\alpha$$

$$= \omega\tan\alpha . \sin\alpha$$

⇒ Since, α is small angle therefore putting $\cos\alpha = 1$ & $\sin\alpha = \alpha$

$$= \omega . \alpha^2$$



$\phi \rightarrow$ driven
$\theta \rightarrow$ driving

⊛ Double hook Joint ⊛

We know that the velocity of driven shaft is not constant, but various from max. to minimum values in order to have a const. velocity ratio of the driving & driven shaft & intermediat shaft with a hook joint at each end is used. This type of joint known as double hook joint.

Ques→ The angle b/w the axis of the two shaft connected by hook joint is 18°. determine the angle turned by driving shaft when the vel. ratio is max. & unity.

$$\frac{\omega_1}{\omega} = 1 = \frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

$$1-\cos^2\theta \cdot \sin^2\alpha = \cos\alpha$$

$$1-0.095\cos^2\theta = 0.95$$

$$0.048 = 0.095\cos^2\theta$$

$$\cos^2\theta = 0.50$$

$$\cos\theta = 0.70$$

$$\boxed{\theta = 44.90}$$

when velocity ratio is max.

$$\cos^2\theta = 1 \quad \frac{\omega}{\omega_1} = 1$$

$$\theta = 0, \pi$$

Ques⇒ Two shafts are connected by hook joint the driving shaft revolve uniformaly at 500 rpm if the total permissible variation in speed of the driven shaft is not to exceed ±ve 6% of the mean speed. Find the greatest permissible angle b/w the central line of the shaft.

$$a x^2 + b x + c = 0$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 500}{60} = 52.36$$

Max. fluctuation of speed $= (\omega_1)_{max} - (\omega_1)_{min}$

$$= (6\% \cdot \omega) - (-6\% \cdot \omega)$$

$$= 12\% \cdot \omega$$

$$12\% \cdot \omega = \frac{\omega}{\cos\alpha} - \omega\cos\alpha$$

$$\cancel{\frac{12}{100} \times 52.36} \quad \frac{\omega - \omega\cos^2\alpha}{\cos\alpha} = 12\% \omega$$

$$\cancel{6.2832} =$$

$$\frac{\omega(1-\cos^2\alpha)}{\cos\alpha} = 12\% \omega$$

$$0.12\cos\alpha = 1-\cos^2\alpha$$

$$\cos^2\alpha + 0.12\cos\alpha - 1 = 0$$

$$\cancel{\alpha = 0.94}$$

$$\cos\alpha = 94.64$$

$$\alpha = 19.94$$

Dat 14/3/2017

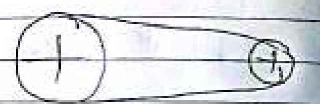
—: UNIT-IIII :—

* Belt Drives
- → low drive (< 8 m/s) todi m/s
- → Medium drive (8–22 m/s)
- → High drive (more than 22 m/s) compressor & generator (workshop)

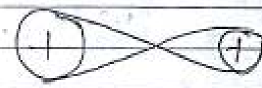
→ Flat belt

→ V-belt

→ Circular



open belt



Cross belt

* Velocity/Speed ratio *

It may be expressed mathematically –

d_1 = dia of a driver
d_2 = dia of the follower
N_1 = Speed of the driver in rpm
N_2 = Speed of the follower in rpm

length of the belt that passes over the driver in one minute $= \pi d_1 N_1$

Similarly:–
length of the belt that passes over the follower in one minute $= \pi d_2 N_2$

$\frac{T_1}{T_2} =$

Since, The length of the belt passes over the driver in one minute = length of the belt that passes over the follower in 1 minute.

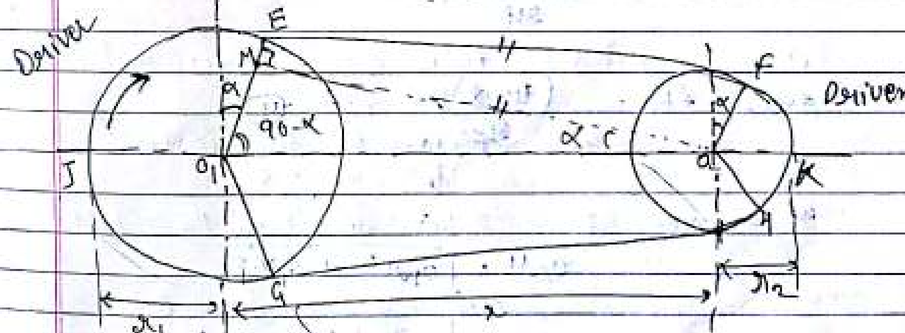
$$\boxed{\pi d_1 N_1 = \pi d_2 N_2}$$

$$\frac{N_1}{N_2} = \frac{d_2}{d_1}$$

If we consider the thickness of the belt

$$\frac{N_1}{N_2} = \frac{d_2 + t}{d_1 + t}$$

* Length of the open belt drive *



Let r_1 & r_2 radius of the larger and smaller pulley

x → distance b/w the centre of the pully
L → total length of the belt.

We know the length of belt (L)

$$L = \widehat{GJE} + EF + \widehat{FK} + HG$$

Angle = $\frac{\text{Arc}}{\text{radius}}$

$L = 2(JE + EF + FK)$ ---- (1)

From geometry of the fig.

$\sin\alpha = \frac{O_1M}{O_1O_2}$

$\sin\alpha = \frac{r_1 - r_2}{x}$

Since α is very small angle

$\sin\alpha \approx \alpha$

$\therefore \alpha = \frac{r_1 - r_2}{x}$ ---- (2)

Arc $JE = \frac{(90+\alpha)}{}r_1$ ---- (3)

Arc $FK = \frac{(90-\alpha)}{}r_2$ ---- (4)

$EF = MO_2$

$O_2M = \sqrt{O_1O_2^2 - O_1M^2}$

$= \sqrt{x^2 - (r_1 - r_2)^2}$

$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$

$= x\left(1 - \left(\frac{r_1 - r_2}{x}\right)^2\right)^{1/2}$

Apply binomial theorem

$\therefore O_2M = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \cdots\right]$

$O_2M = x - \frac{(r_1 - r_2)^2}{2x}$ ---- (5)

Putting in eq^n (1)

$L = 2(JE + EF + FK)$

$L = 2\left[(90+\alpha)r_1 + x - \frac{(r_1 - r_2)^2}{2x} + (90-\alpha)r_2\right]$

$= 2\left[r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2\frac{\pi}{2} - r_2\alpha\right]$

$L = 2\left[\frac{\pi}{2}(r_1 + r_2) + 2\alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$

$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$

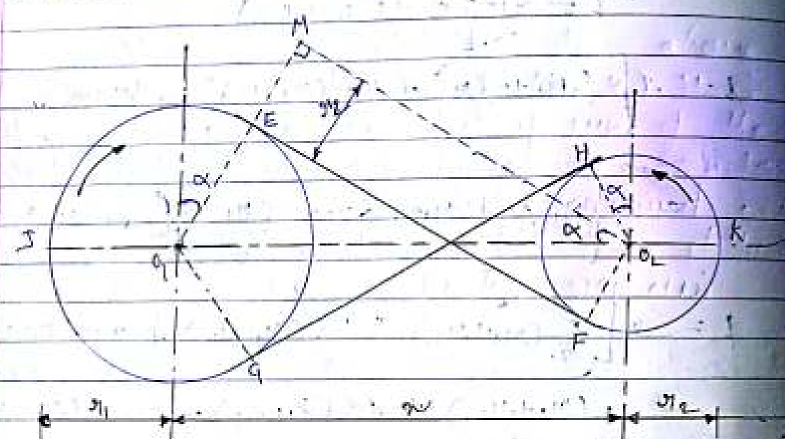
putting the value of α

$L = \pi(r_1 + r_2) + \frac{2(r_1 - r_2)}{x}(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$

$$\boxed{L = \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{2x}}$$

* Length of a Cross Belt drive :-

This case both the pulleys rotate in opposite direction.



Let, r_1 and r_2 = Radii of the larger and smaller belt drive.

x = distance b/w the centres of two pulleys (O_1, O_2)

$\boxed{L = \text{Total length of the belt}}$

From geometry O_2M will be $\perp$ to O_1E

Let the angle $MO_2O_1 = \alpha$ radians.

Total length of the belt (L)

$$L = \text{arc } GJE + EF + \text{arc } HKF + GH$$
$$= 2(\text{arc } JE + EF + \text{arc } FK) \quad \text{--- (1)}$$

From the fig.

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1 + r_2}{O_1O_2}$$

Since $\alpha <<<<$ [very small]

$\sin\alpha = \alpha$

$$\therefore \alpha = \frac{r_1 + r_2}{x} \quad \text{--- (A)}$$

$$\text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \quad \text{--- (2)}$$

Similarly

$$\text{arc } FK = r_2\left(\frac{\pi}{2} + \alpha\right) \quad \text{--- (3)}$$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

Applying binomial theorem

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + \cdots\right] = x - \frac{(r_1 + r_2)^2}{2x} \quad \text{--- (4)}$$

From eqn (1), (2), (3), & (4)

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2\frac{\pi}{2} + r_2\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

From (A)

$$L = \pi(r_1 + r_2) + 2\left(\frac{r_1 + r_2}{x}\right)(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$\boxed{L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}}$$ (In terms of pulley radii)

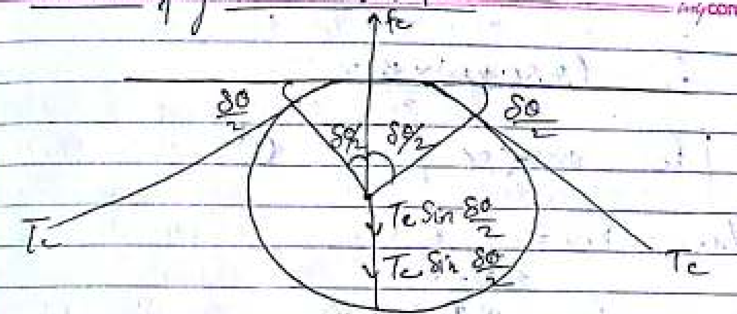
$$\boxed{L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}}$$

(In terms of pulley diameter)

Note :-

It may be noted that the above expression is a function of $(r_1 + r_2)$. It is thus ~~obvious~~ obvious that if sum of the ~~radii~~ radii of the two pulley be constant, then the length of the belt required will also remain constant, provided the distance b/w centres of the pulleys remain unchanged.

: Centrifugal Tension :-



While in motion as a belt passes on a pulley the centrifugal effect due to its own weight tends to lift the belt from the pulley. The centrifugal force produces equal tension on the two side of the belt i.e. on the tight side as well as slack side.

Consider a short element of belt,

Let, m mass per unit length of belt
T_c – centrifugal tension for tight side as well as slack side.
F_c → centrifugal force on the element.
r → radius of the pulley.
v → vel. of the belt
$\delta\theta$ → angle of lope of the element over the pulley.

We know that, F_c = mass of element × accn.
= mass per unit length × length of element × accn.

angle $= \frac{arc}{r}$

$\delta\theta \times r = l$

$f_c = (r\delta\theta \times m) \times \frac{v^2}{r}$

$\boxed{f_c = mv^2 \cdot \delta\theta}$ - - - - ①

also, $f_c =$

$\Sigma V = 0$

$f_c = 2 T_c \sin \frac{\delta\theta}{2}$

Since, $\delta\theta$ is very small

$\sin \frac{\delta\theta}{2} = \frac{\delta\theta}{2}$

$\boxed{f_c = T_c \, \delta\theta}$ - - - ②

② = ①

$mv^2 \cdot \cancel{\delta\theta} = T_c \cdot \cancel{\delta\theta}$

$\boxed{T_c = mv^2}$

Thus, centrifugal tension is independent of the tight & slack side tension

It depends only the vel. of the belt over the pulley.

Also, centrifugal stress $= \frac{\text{centrifugal tension}}{\text{crossectional area of belt}}$

$= \frac{T_c}{a}$

Total tension on the tight side

$T = T_1 + T_c$

S tight side

Total tension on the slack side =

$T = T_2 + T_c$

⇒ **Power Transmitted By The Pulley** *

Power $= \frac{w.D}{t} = \frac{f \times d}{t}$

$= f \times d/t = f \times v$

$\boxed{f = T_1 - T_2}$ $v = \frac{\pi D N}{60}$

Power $= (T_1 - T_2) \times v$ - - - - ③

⇒ Condition for the transmission of max. power :- we know that power transmitted by the belt

$P = (T_1 - T_2) \times v$

Where,

$T_1 \rightarrow$ tension in the tight of the belt in newton

$T_2 \rightarrow$ tension in the slack of the belt in N

$v \rightarrow$ vel. of the belt in m/s.

We also known that from tension ratio

$\frac{T_1}{T_2} = e^{\mu\theta} \Rightarrow T_2 = \frac{T_1}{e^{\mu\theta}}$ - - - ④

from ③ & ④

$P = T_1 \left(1 - \frac{1}{e^{\mu\theta}}\right) v$

$= T_1 v . C$ - - - - ⑤

where, $C = \left(1 - \frac{1}{e^{\mu\theta}}\right)$

Total tension –

$$T = T_1 + T_c$$
$$T_1 = T - T_c$$

T_1, max tension through which the belt can be subjected

T_c, centrifugal tension.

$\therefore \; P = (T - T_c)\, v_c$

$P = (T - mv^2)\, v_c$

for max. power, differentiate above eqn w.r.t v_c & equal to zero.

$P = Tv_c - mv^3{}_c$

$\frac{dP}{dV} = \cancel{\quad} \; T_c - 3mv^2{}_c = 0$

$T_c = 3mv^2{}_c$

$\boxed{T = 3mv^2}$

$T = 3mv^2$

$T = 3T_c$

$\boxed{T_c = \frac{T}{3}}$ $\quad T_1 = T - T_c$

$= 3T_c - T_c$

$\boxed{T_1 = 2T_c}$

for power transmission $\boxed{v = \sqrt{\frac{T}{3m}}}$

Que-7 A shaft rotating at 200 rpm drives another shaft 300 rpm & transmit 6 kW through a belt. The belt is 100mm wide & 10mm thick. The distance b/w the shaft is 4m. The small pulley is 0.5mm in dia. Calculate stress in the belt if
① An open belt drive
② Cross belt drive
Take coeff. friction 0.3

$P = 6\,kW$

$\mu = 0.3$

$x = 4m$

$\frac{N_1}{N_2} = \frac{d_2}{d_1}$

$\frac{200}{300} = \frac{0.5}{d_1}$

$d_1 = 0.75$

$v_1 = \frac{\pi D N_1}{60}$

$= \frac{\pi \times 0.75 \times 200}{60}$

$v_1 = 7.85\ m/s$

for open belt –
$\theta = \pi - 2\alpha$
cross belt
$\theta = \pi + 2\alpha$

$\sin\alpha = \frac{r_1 - r_2}{x}$

$\alpha = \cancel{\tan}\sin^{-1}\left(\frac{r_1 - r_2}{x}\right)$

$= \sin^{-1}\left(\frac{0.375 - 0.25}{4}\right)$

$\boxed{\alpha = 1.8°}$

$\theta = 180 - 2 \times 1.8$
$= 176.4°$
$= \frac{\pi}{180} \times 176.4$
$= 3.08$ rad.

$\frac{T_1}{T_2} = e^{\mu\theta} = e^{(0.3 \times 3.08)}$

$\frac{T_1}{T_2} = 2.52$

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$P = (T_1 - T_2)\, v$

$6 \times 10^3 = (T_1 - T_2) \times 7.85$

$6 \times 10^3 = (8.52\, T_2 - T_2) \times 7.85$

$6 \times 10^3 = 11.932\, T_2$

$$\boxed{T_2 = 503\ N}$$

$$\boxed{T_1 = 1267.18\ N}$$

We know that from the max tension in the belt

$T_1 = \sigma b t$

$$\sigma = \frac{T_1}{bt}$$

$$\sigma = \frac{1267.18}{100 \times 10}$$

$$\boxed{\sigma = 1.267\ N/mm^2}$$

(ii) $\theta = \pi + \alpha 2$

$$\sin\alpha = \frac{r_1 + r_2}{x}$$

$$\boxed{\sigma = 1.18\ N/mm^2}$$

Ans. (ii)

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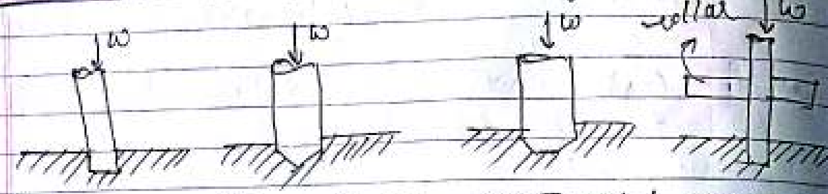
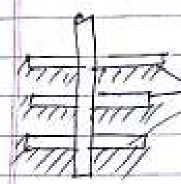
Date 7/8/2017

Friction in Pivots and Collar bearing :



(a) Flat pivot (b) Conical pivot (c) Truncated pivoted (d) Single collar



When a rotating shaft is subjected to an axial load (w). The thrust (axial force) is taken either by pivots or collar. example are the shafts of steam turbine and propeller shaft of a ship.

* Collar Bearing :-

A collar bearing or simply a collar is provided at any position along the shaft and bears the axial load on ~~meeting~~ mating surfaces.

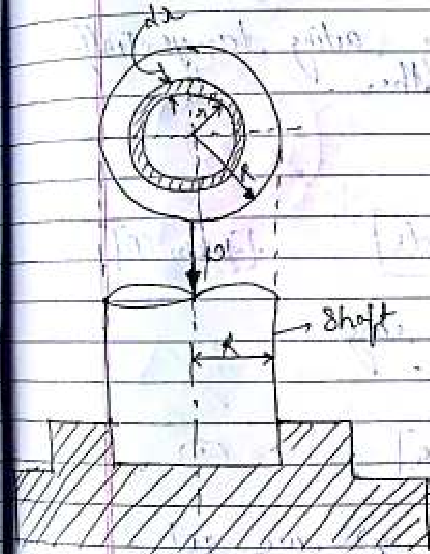
The surface of a collar may be plane (flat) or conical shaped.

Pivot bearing :-
When the axial load is taken by the end of shaft which is inserted

in the bearing is called pivot bearing.

(1) Flat pivot bearing :-

Assumption :- 1) Uniform pressure theory
2) Uniform wear theory
$p.r = c$
r_1, r_2 (inner & outer) radius



Let, W load transmitted over the bearing surface
R → radius of bearing surface
p → intensity of pressure per unit area of bearing surface b/w the rubbing surface
μ → coeff. friction b/w shaft & bearing.

fig: Flat pivot or foot step bearing.

Consider :- i) UPT
ii) UWT

(1) Uniform pressure theory :-

When the pressure uniformly distributed over the bearing area

Consider a ring of radius r & thickness dr of the bearing area. Area of the bearing surface = $2\pi r\,dr$

Load transmitted to the ring $(\delta w) = p \cdot 2\pi r\,dr$ ---- ①

⇒ Frictional resistance –

Fr sliding on the ring acting tangentially at a radius r then –

$$F_r = \mu \cdot \delta w$$

$$\boxed{F_r = \mu \cdot 2\pi r\,dr} \quad \text{---- [from ①]}$$

⇒ Frictional torque on the ring

$$T_r = F_r \times r$$

$$\boxed{T_r = 2\pi\mu p r^2\,dr} \quad \text{---- ②}$$

By integrating the above eqn we get

$$\int_0^R T_r = \int_0^R 2\pi\mu p r^2\,dr$$

$$\boxed{T = \frac{2}{3}\mu w R}$$

(ii) When we consider uniform wear theory

$p \propto \frac{1}{r}$, $pr = c$

$$p = \frac{c}{r}$$

$$p = \frac{w}{\pi R^2}$$

Load transmitted on the disc –

$$\delta w = p \times 2\pi r\,dr$$

$$= \frac{c}{r} \cdot 2\pi r\,dr$$

$$\delta w = 2\pi c\,dr$$

⇒ The total load transmitted to the ring

$$w = \int_0^R \delta w = \int_0^R 2\pi c\,dr = 2\pi R c$$

$$\boxed{c = \frac{w}{2\pi R}}$$

Frictional torque –

$$T_r = 2\pi\mu p r^2\,dr$$

$$T_r = 2\pi\mu \times \frac{c}{r} \cdot r^2\,dr$$

$$T_r = 2\pi\mu c\,r\,dr$$

For large part

$$\int_0^R T_r = \int_0^R 2\pi\mu c\,r\,dr$$

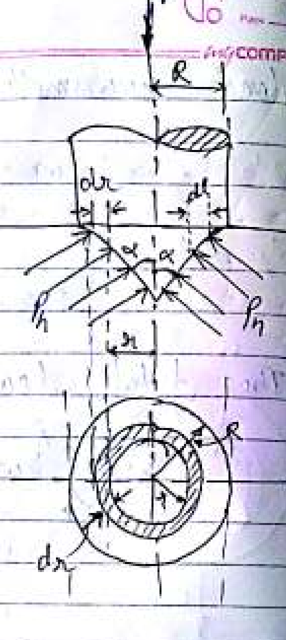
$$T_R = 2\pi\mu c \frac{R^2}{2}$$

$$T = \pi\mu \times \frac{w}{2\pi R} \times R^2$$

$$\boxed{T = \frac{1}{2}\mu w R}$$

20/3/2017

(2) Conical Pivot Bearing



Let P_n intensity of pressure normal to the cone.

$\alpha \rightarrow$ Semi angle of the cone

$\mu \rightarrow$ coeff. of friction b/w shaft & bearing

$R \rightarrow$ radius of shaft.

Let us consider a small ring of radius (r) and thickness 'dr'. Let dl is the length of the ring along the cone.

Here, $dl = dr \operatorname{cosec} \alpha$

Area of the ring $= 2\pi r \, dl$
$= 2\pi r \, dr \operatorname{cosec}\alpha$

Consider first case :-

(1) Uniform pressure theory :-

We know the normal load acting on the ring

$\delta w_n = P_n \times \text{Area}$
$= P_n \times 2\pi r \, dr \operatorname{cosec}\alpha$
~~$\delta w = \delta w_n \sin\alpha$~~

Vertical load acting on the ring (δw_n) = vertical component of δw_n.

$\delta w = \delta w_n \sin\alpha$
$= P_n \times 2\pi r \, dr \operatorname{cosec}\alpha \sin\alpha$
$= P_n \times 2\pi r \, dr$

Total vertical load transmitted to the bearing.

$$W = \int_0^R \delta w = \int_0^R P_n 2\pi r \, dr$$

$$\boxed{W = \pi R^2 P_n}$$

$$\boxed{P_n = \frac{W}{\pi R^2}}$$

We also know the frictional force on the ring acting tangentially

$F_r = \mu \delta w_n = \mu P_n 2\pi r \, dr \operatorname{cosec}\alpha$

and frictional torque acting on the ring

$T_r = F_r \times r$
$= 2\pi \mu P_n \operatorname{cosec}\alpha \times r^2 dr$

Total torque

$$T = \int_0^R 2\pi \mu P_n \operatorname{cosec}\alpha \, r^2 dr$$

$$T = 2\pi \mu \frac{W}{\pi R^2} \cdot \frac{R^3}{3} \operatorname{cosec}\alpha \quad \left(P_n = \frac{W}{\pi R^2}\right)$$

$$T = \frac{2}{3} \mu W R \operatorname{cosec}\alpha$$

$$\boxed{T = \frac{2}{3} \mu W L}$$

(ii) When we consider uniform wear theory :-

Let 'P_r' be the normal intensity of pressure at a distance 'r' from the central axis.

In case of uniform wear the intensity of pressure varries –

$$P_r \times r = C$$
$$P_r = \frac{C}{r}$$

Load transmitted to the ring (δw)

$$\delta w = P_r . 2\pi r . dr.$$
$$= \frac{C}{\cancel{r}} . 2\pi \cancel{r} . dr$$
$$\int_0^W \delta w = \int_0^R C . 2\pi\, dr.$$
$$W = 2\pi C R$$
$$C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring.

$$T_r = 2\pi \mu P_r . \operatorname{cosec}\alpha . r^2 dr$$
$$T_r = 2\pi\mu . \frac{C}{\cancel{r}} . \operatorname{cosec}\alpha . r^2 dr \text{ ——— } [P_r = \tfrac{C}{r}]$$
$$\int_0^T T_r = \int_0^R 2\pi\mu C . \operatorname{cosec}\alpha . r . dr$$

$$T = \cancel{2\pi}\mu \times \frac{W}{\cancel{2\pi} R} . \operatorname{cosec}\alpha . \frac{R^2}{2}$$

~~T = 2πμ~~

$$\boxed{T = \frac{1}{2}\mu W R \operatorname{cosec}\alpha}$$

Ques-> A conical pivot bearing support a vertical shaft of 200 mm dia. It is subjected to a load of 30 kN. The angle of the cone 120° and coeff. of friction 0.25. Find the power loss in friction when the speed is 140 rpm. Assuming uniform pressure, uniform wear.

Solution->
$d = 200\,\text{mm} \Rightarrow R = 100\,\text{mm}$

$W = 30 \times 10^3\, N.$

$2\alpha = 120° \Rightarrow \alpha = 60°$ $\qquad P = T \times \omega$

$\mu = 0.25$

$N = 140\, rpm$ $\qquad \omega = \frac{2\pi N}{60} = \frac{2\pi \times 140}{60}$

$\omega = 14.661$

(i) If ($T = \frac{2}{3} \mu . W R \operatorname{cosec}\alpha$

$= \frac{2}{3} \times 0.25 \times 30 \times 10^3 \times 100 \times \operatorname{cosec} 60°$

$= 57.735\ N\text{-}m$

$P = T \times \omega$

$= 57.735 \times 14.661$

$= 846\, W$

(ii) $T = \frac{1}{2} \mu W R \operatorname{cosec}\alpha = \frac{1}{2} \times 0.25 \times 30 \times 10^{-3} \times 100 \times \operatorname{cosec} 60°$

$= 43.3\ N\text{-}m$

$P = T \times \omega = 43.3 \times \frac{2\pi 140}{60}$

$= 634.8$ watt

conical

Ques → A ~~truncated~~ pivot support a load of 20 kN $2\alpha = 120°$ and the intensity of normal pressure is not to exceed $0.3\ N/mm^2$. The do twice the di. Find the outer and inner radius of the bearing surface. If the shaft rotates at 200 rpm. $\mu = 0.1$. Find the power absorbed in friction. Assume uniform pressure.

$2\alpha = 120° \Rightarrow \alpha = 60°$ $\quad r_1 \rightarrow$ external

$W = 20\,kN$

$= 20 \times 10^3\,N$

$p = 0.3\ N/mm^2$ $\quad p_n = \frac{W}{\pi(r_1^2 - r_2^2)}$

$r_1 = 2r_2$

$$T = \frac{2}{3}\mu\, W \operatorname{cosec}\alpha \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)$$

$p_n = \frac{W}{\pi(4r_2^2 - r_2^2)}$

$0.3 = \frac{20 \times 10^3}{\pi\, 3r_2^2}$

$T = \frac{2}{3} \times 0.1 \times 20 \times 10^3 \operatorname{cose}(60)\left(\frac{168.2^3 - 84.1^3}{168.2^2 - 84.10^2}\right)$

$T = 302174.8 = 302.2$ N-m

$3r_2^2 = \frac{20 \times 10^3}{\pi (0.3)}$

$r_2^2 = \sqrt{\frac{20 \times 10^3}{\pi (0.3)(3)}}$

$\omega = \frac{2\pi N}{60} \Rightarrow \frac{2 \times \pi \times 200}{60}$

$P = \frac{2 \times \pi \times 200}{60} \times 302.2$

$= 6322$ watt $= 6.32$ kW

$\boxed{r_2 = 84.10}$

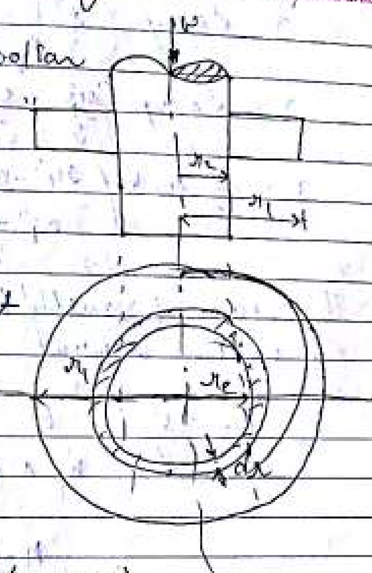
$\boxed{r_1 = 168.2}$

Foot Step (Pivots) Collar (Thrust)

* Flat Collar Bearing *

r_1 - External radius of collar

r_2 - Internal radius



Consider a single flat collar bearing supporting a shaft.

Consider a small element (ring) radius r and thickness dr. Area of the bearing surface

$A = \pi(r_1^2 - r_2^2)$

(1) Case - I (uniform pressure)

When the pressure is uniformly distributed over the bearing surface then intensity of pressure

$$\boxed{p = \frac{W}{A} = \frac{W}{\pi(r_1^2 - r_2^2)}}$$

We also know that frictional torque ~~taken~~ transmitted on the ring.

$T_r = 2\pi\mu p r^2 dr$

Integrating the above equation within the limit for total torque

$T = \int T_r = \int_{r_2}^{r_1} 2\pi\mu p r^2 dr$

$$T = 2\pi\mu p\left(\frac{r^3}{3}\right)_{r_2}^{r_1}$$

$$T = 2\pi\mu p\left(\frac{r_1^3}{3} - \frac{r_2^3}{3}\right)$$

$$T = \frac{2}{3}\mu W\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)$$

In case of multiple collar, intensity of pressure equal load/area.

$$\boxed{p = \frac{W}{A} = \frac{W}{n\left(\pi(r_1^2 - r_2^2)\right)}}$$

$n \to$ no. of collar.

→ Case-II uniform wear theory

$$p_r r = C$$

$$p_r = \frac{C}{r}$$

$$\delta W = p_r \times 2\pi r\,dr$$

$$\delta W = \frac{C}{\cancel{r}} \times 2\pi \cancel{r}\,dr = 2\pi C\,dr$$

$$\boxed{\delta W = 2\pi C\,dr}$$

For total load

$$W = \int_{r_2}^{r_1} \delta W = \int_{r_2}^{r_1} 2\pi C\,dr$$

$$W = 2\pi C(r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

Frictional torque

$$T_r = f_r \times r$$

$$T_r = (\mu\,\delta W)\,r$$

$$\int_{r_2}^{r_1} T_r = \int_{r_2}^{r_1} 2\pi\mu C r\,dr$$

$$T = 2\pi\mu C\left(\frac{r_1^2}{2} - \frac{r_2^2}{2}\right) = \cancel{2\pi}\mu \times \frac{W}{\cancel{2\pi}(r_1 - r_2)}\left(\frac{r_1^2}{2} - \frac{r_2^2}{2}\right)$$

$$T = \mu W\left(\frac{r_1 + r_2}{2}\right) \to \text{mean radius}$$

$$T = \mu W R$$

$$T = \int_{r_2}^{r_1} 2\pi\mu\left(\frac{W}{2\pi(r_1 - r_2)}\right) r\,dr = \int_{r_2}^{r_1}\left(\frac{\mu W}{r_1 - r_2}\right) r\,dr$$

$$= \frac{\mu W}{r_1 - r_2}\left(\frac{r_1^2}{2} - \frac{r_2^2}{2}\right)$$

23/5/2017 clutch - 297

Ques) An shaft has a no. of collar integral with it. The external dia of collar is 400mm and the shaft dia 250 mm if the intensity of pressure $0.35\ N/mm^2$ (uniform pressure) and the coef. of friction 0.05. Estimate (i) power absorbed when the shaft run 105 rpm carrying a load of 180 kN. (ii) The no. of collar required

$d_1 = 400\ mm \Rightarrow r_1 = 200\ mm$

$d_2 = 250\ mm \Rightarrow r_2 = \frac{250}{2} = 125\ mm$

$p = 0.35\ N/mm^2$

$n = ?,\ P = ?$

$T = \frac{2}{3}\mu W\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right) = \frac{2}{3}(0.05) \times 180\left(\frac{200^3 - 125^3}{200^2 - 125^2}\right)$

$= 1240\ Nm$

$p = \frac{W}{n\pi(r_1^2 - r_2^2)}$

$0.035 = \frac{150 \times 10^3}{n\pi(200^2 - 120^2)}$

$\boxed{n = 5.6 \simeq 6}$

$P = T \times \omega$

$\omega = \frac{2\pi \times 105}{60} = 10.995$

$P = 1240 \times 10.995$

$= 13.64\ kW$

Ques) A thrust of a propeller shaft in marine engine is taken up by a no. collar integral with the which drum dia

HR: mary

Ques) The thrust on the shaft is 200 kN and the speed is 75 rpm. Take the coeff. of friction 0.05. Assume intensity of pressure is uniform & equal to $0.3\ N/mm^2$. Find the external dia of the collar & the no. of collar required if the power loss in friction is not to exceed 16 kW

$d_2 = 300\ mm$	$p = 0.3\ N/mm^2$
$W = 200\ kN$	$r_1 = ?$
$N = 75\ rpm$	$n = ?$
$\mu = 0.05$	

$p = \frac{W}{n(\pi)(r_1^2 - r_2^2)} \Rightarrow \frac{1200 \times 10^3}{0.3 \times \pi \times (}$

$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 75}{60} = 7.85$

$\boxed{P = T \times \omega}$

$\frac{P}{\omega} = T \Rightarrow T = \frac{0.3\ N/mm^2}{7.85} = 0.0382$

$\boxed{T = 0.0382}$

$T = \frac{2}{3} \times 0.05 \times 200 \times 10^3\left(\frac{r_1^3 - 150^3}{r_1^2 - 150^2}\right)$

$0.0382 = 6666.66\left(\frac{r_1^3 - 150^3}{r_1^2 - 150^2}\right)$

$0.03 = \frac{200 \times 10^3}{}$

$$\omega = \frac{2\times\pi\times75}{60} = 7.854$$

$\boxed{d_1 = 447\ mm}$

Power loss ⇒ $P = T\times\omega$

$$\frac{P}{\omega} = T$$

$$\frac{16\times10^3}{7.854} = T$$

$$\boxed{T = 2037.2}$$

$$P = \frac{W}{\pi(1-)(r_1^2 - r_2^2)}$$

$$\omega \times T = \frac{2}{3}\mu W\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)$$

$$16\times10^3 = \frac{2}{3}\mu\,\omega\left(\frac{r_1^3 - r_2^3}{(r_1^2 - r_2^2)}\right)$$

$$16\times10^3 = \frac{2}{3}\times0.05\times200\times10^3\left(\frac{r_1^3 - 150^3}{r_1^2 - 150^2}\right)$$

$$2.4(r_1^2 - 150^2) = r_1^3 - 150^3$$

$$2.4r_1^2 - 54000 = r_1^3 - 150^3$$

$$2.4r_1^2 + r_1^3 = -150^3 - 54000$$

$$= 3424000$$

friction clutchs

UPT ⇒ $T = \frac{2}{3}\mu W\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)$

UWT ⇒ $T = \mu\,\omega\left(\frac{r_1 + r_2}{2}\right)$

$= \mu WR$

where

$$R = \frac{r_1 + r_2}{2}$$

$$T = n\mu WR$$

where
$n \Rightarrow$ no. of friction plate

Ques) Determine the max, mini and average pressure in a plate clutch when the axial force is 4kN. the d_2 = 50 mm and d_1 = 100 mm. Assume Uniform wear.

Solutions: $r_2 d_2 = 50$
$r_1 d_1 = 100$

$$W = 2\pi C(50 - 25)$$

$$\frac{4\times10^3}{2\pi(50-25)} = C$$

$\boxed{C = 25.465}$

$C = 12.73$

$P_{max}\times r_2 = C$

$P_{max} = \frac{C}{r_2}$

$\boxed{P_{max} = \cancel{1.0186}\ 0.2546}$

$P_{min}\times r_1 = C$

$P_{min} = \frac{C}{r_1}$

$\boxed{P_{min} = \cancel{0.5093}}$ 0.1273

$P_{avg} = \cancel{\frac{0.2546 + 0.1273}{2}}$ $\frac{W}{\pi(r_1^2 - r_2^2)}$

$\cancel{= 0.1910}$

$= 0.16976$

$T = n\mu W R$

$n \rightarrow$ no. of pairs of friction in contact surface

$R =$ mean radius $= \frac{r_1+r_2}{2}$

For multiplate clutch –

$n = n_1 + n_2 - 1$

$n_1 \rightarrow$ no. of pair in driving shaft

$n_2 \rightarrow$ no. of pair in driven shaft

Ques→ A single plate clutch both side effective (no. pair = 2) has outer & inner dia (300 & 200) mm. The max. intensity of pressure at any point is not to exceed 0.1 N/mm². If the eff. of friction = 0.3. Determine the power transmitted by a clutch at a speed 2500 rpm.

Solution→ no. of pair $= 2$

$d_1 = 300\,mm$

$d_2 = 200\,mm$

$p_{max} = 0.1\,N/mm^2$

$\mu = 0.3$

$\omega = \frac{2\pi\, 200}{60} = 20.94$

$T = \cancel{n}\mu \times W\left(\frac{r_1+r_2}{2}\right)$

$T = 0.3 \times 2094\left(\frac{150+100}{2}\right)$

$= 785.25$

$P = T \times \omega$

$= 16443.135$

$W = 2\pi C(r_1 - r_2)$

$= \cancel{2\pi A x}$

$P_{max} r_2 = C$

$0.1 \times 100 = C$

$\boxed{C = 10}$

$W = 2\pi \times 10\,(150-100)$

$= 3141.6$

$\boxed{W = 3.141\,k}$

$T = n \times \mu W R$

$T = n \times \mu W\left(\frac{r_1+r_2}{2}\right)$

$\boxed{T = 235.62}\ Nm$

$P = T \times \omega = 235.62 \times \cancel{20.94}\ \frac{2\times\pi\times 2500}{60}$

$= 61.68\,kW$

Q→ A single plate clutch is to be design for an automobile vehicle whose engine is rated to give 100 kW at 2400 rpm & the max torque 500 Nm the outer radius of friction plate is ~~25%~~ 25% more than in radius. The intensity of pressure b/w the plate is not to exceed 0.07 N/mm². ~~Is~~ $\mu = 0.3$. The helical spring require by this ~~the~~ clutch to provide axial force necessary to ~~engag~~ engage the clutch are 8. If the each spring has stiffness = 40 N/m. Determine the initial compression in the spring & the dimension of the friction plate.

$n = 2$

$P = 100\,kW \Rightarrow 100 \times 10^3\,W$

$N = 2400\,rpm$

$T = 500\,Nm \Rightarrow 500 \times 10^3\,Nmm$

$r_1 = 25\% \times r_2$

$= (100 + 25\%)\,r_2$

$p_{max} = 0.07\,N/mm^2$

$\mu = 0.3$

$n = 8$

$s = 40\,N/mm$

Scanned by CamScanner

$R = 140$
$S = 8770$

we know that stiffness of the spring
s = stiffness per spring × no. of spring

Axial force required to engage the clutch

Initial compression of the spring

Ques) A multidisk clutch has 5-plate having 4-pairs active friction surface. If the intensity of the pressure is not to exceed $0.127\ N/mm^2$. find the power transmitted at 500 rpm. $r_1 = 125$ mm, $r_2 = 75$ mm. Assume UWT & $\mu = 0.3$

Ques) A multidisk clutch has 3 disks on the driving shaft & 2 on the driven shaft $d_1 = 240$ mm, $d_2 = 140$ mm. Find the max. axial intensity of pressure b/w disk for transmitting 25 kW at 1575 rpm. $\mu = 0.3$

Soln.

$$\boxed{\begin{aligned} n &= n_1 + n_2 - 1 \\ &= 3 + 2 - 1 \\ &= 4 \end{aligned}}$$

* Dynamometer *

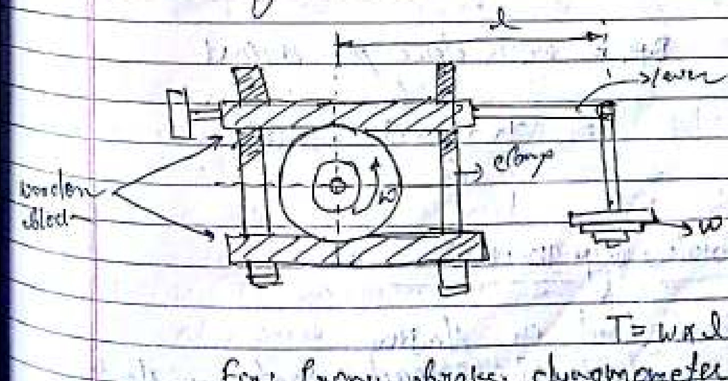
Dynamometer is a break work in addition it has a device to measure the frictional resistance, we may obtain torque transmitted ~~and ch~~ and hence the power of engine

Classification

① Absorption:- In this dynamometer the work done is converted in to heat by friction while we measured they can be used for the measurement of moderat powers only. exp:- ~~the~~ prony brake and rope brake.

② Transmission dynamometer:-

In this type the work is not adsorbe in the process but it utilize after the measurement. exampl:- belt transmission & torsion dynamometer.



$T = W \times l$

Fig: Prony brake dynamometer

Let W = weight at the outer end of the lever in newton

l = horizontal distance of the weight (W) from the centre of the pulley.

F = friction resistance b/w the block and pulley.

R = radius of the pulley.

N = speed of the shaft in rpm.

We know that moment of the frictional resistance or torque, $T = W \times l$
$= F \times R$

Work done in one revolution = T × angle turned in radian.
$= T \times 2\pi$

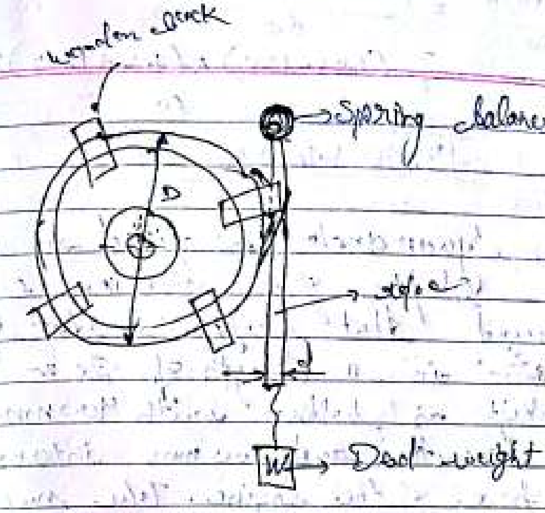
Work done per minute = $T \times 2\pi \times N$

We know that brake power the engine is

$$B.P. = \frac{\text{work done per minute}}{60}$$
$$= \frac{T \times 2\pi \times N}{60} \text{ watt.}$$

* Rope brake dynamometer *

W = Dead load in Newton
S = spring balance load (reading in Newton)
D = dia of the wheel in m.



d = dia of the rope in meter.
N = speed of the engine shaft in rpm

Net load on the brake = (W - S)
we know that distance moved in one revolution by $\pi(D+d) \times N$

Work done per revolution (work done per min.)
$= (W-S) \times \pi(D+d)N$

Brake power of the engine = $\frac{\text{w/d per minute}}{60}$
$= \frac{\pi(D+d)N}{60}$

In an lab exp. the following data where recorded with rope brake dia of the fly wheel 1 m, dia of the rope 12.5 mm. Speed of the engine = 200 rpm. Dead load on the brake 600 N.
Spring balance reading = 150 N.
Calculate the brake power of the engine

Ans: 5.7 KN

$$B.P = \frac{2\pi(D+d)N}{60} = \frac{(W-S)\,\pi(1.2+12.5)\,200}{60}$$

$$= \frac{(605-100)\,\pi(1.2+12.5)\,200}{60}$$

$$\boxed{B.P = 6.7\ kN}$$

Ques:- A torsion dynamometer is fitted to propeler shaft of a marien engine it is found that shaft ~~to stat~~ at 120 rpm twisted 2° in a length of 20 m and if the shaft is hollow with 400 mm external diameter and 200 mm internal dia. Find the power of the engine. Take modulus of rigidity for shaft material 80 G.Pa.

$$\frac{T}{I_p} = \frac{\tau}{r} = \frac{G\theta}{l} \quad (\text{Torsion equation})$$

$d = 400\,mm$, $d_i = 200\,mm$, $N = 120$ rpm
$\theta = 2°$

$$I_p = \frac{\pi}{64}(d_o^4 - d_i^4)$$

$$= \frac{\pi}{64}(400^4 - 200^4)$$

$$= \cancel{4.356\times10}\ 1.77\times10^{-3}\ m^4$$

$$\frac{T}{I_p} = \frac{G\theta}{l}$$

$$T = I_p \times \frac{G\theta}{l}$$

Belt - Self

$$P = \frac{2\pi NT}{60} = 2990\ KW$$

UNIT ⇒ IV

* Gear Train :

Some times two or more gears are made to mesh with each other to transmitt power from one shaft to another shaft such combination is called Gear train.

Gear train is necessary when it is required to obtain large speed reduction within small space.

The nature of gear train used depends on the required vel. ratio ~~the~~ and relative position of the axis of the shaft.

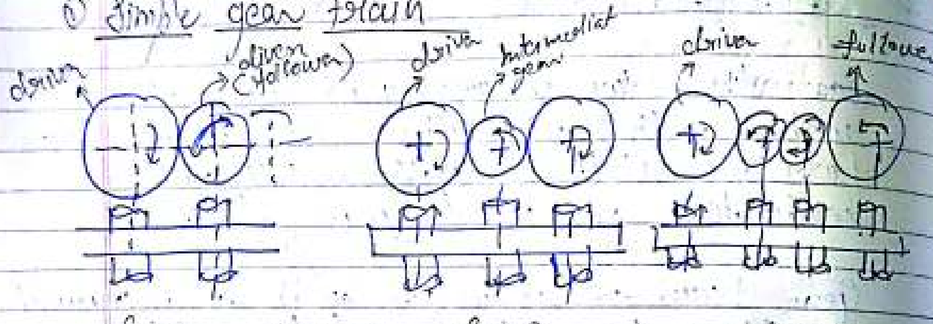
Gear train commonly used in various M/C (like lathe, drilling, milling etc.), automibiles vehicles. A series of gear capable of recieving and transmitting the motion from of one gear to another is called simple gear train.

* Types of Gear Train :-

The following are the main types of gear train

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train

① Simple gear train



fig(a) fig(b) fig(c)

When only one gear on each shaft mounted then series of gears capable of receiving and transmitting motion from one gear to another is called simple gear train.

Let N_1 = Speed of gear first.
T_1 = No. of teeth in first gear

Then, N_2, N_3, N_4 = speed of gear

T_2, T_3, T_4 = No. of teeth in gear second, third, fourth

From the figure (a). When take gear first and second then vel. ratio

$$\boxed{\frac{N_1}{N_2} = \frac{T_2}{T_1}} \text{ ------ } ①$$

When we consider gear 2, 3

$$\boxed{\frac{N_2}{N_3} = \frac{T_3}{T_2}} \text{ ---- } ②$$

When we considered gear no. 3, 4

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \text{ ------ } ③$$

Now, multiply eqn ①, ②, ③

$$\boxed{\frac{N_1}{N_4} = \frac{T_4}{T_1}}$$

There is no effect of intermediat gear therefore they known as idlers.

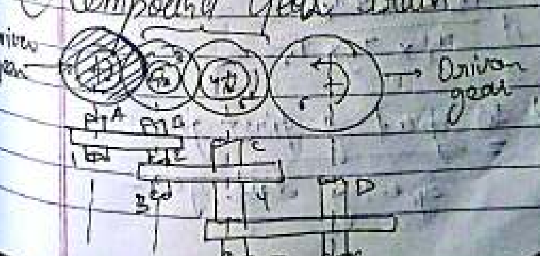
Train value = $\frac{1}{\text{speed ratio}} \Rightarrow \frac{N_4}{N_1} = \frac{T_1}{T_4}$

Train value = $\frac{\text{no. of teeth in driving gear}}{\text{no. of teeth in driven gear}}$

Note:-
When the intermediate gear is an odd no. then the direction of both gear will be same.

When the intermediat gear is an even no. then the direction both gear will be opposite to each other.

② Compound Gear train



When there are more than one gear on shaft as shown in figure are mounted it is called compound gear train.

Since G_1 is in mesh with G_2

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{--- (1)}$$

Similarly for G_3 & G_4

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \text{--- (2)}$$

For G_5 & G_6

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \text{--- (3)}$$

Now, multiplying equation (1), (2), (3) we get

$$\frac{N_1}{\cancel{N_2}} \times \frac{\cancel{N_3}}{\cancel{N_4}} \times \frac{\cancel{N_5}}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

Since $N_2 = N_3$, $N_5 = N_4$, because these are mounted on same shaft

$\therefore$ Speed ratio $\Rightarrow \dfrac{N_1}{N_6} = \dfrac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$

Speed ratio ~~equal to~~ $=$ $\dfrac{\text{speed of first gear}}{\text{Speed of last gear}}$

$=$ $\dfrac{\text{Product of no. of teeth on driven gear}}{\text{Product of no. of teeth on drivers gear}}$

Ques The gears of an m/c tool are given in the following table. The motor shaft is connected to gear (A) and rotates 975 rpm. The gear wheel B, C, D and E are fixed to parallel shaft rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F when the no. of teeth are given in table

	Gear	Teeth
1	A	20
2	B	50
3	C	25
4	D	75
5	E	26
6	F	65

~~Speed ratio~~ $\Rightarrow$

$$\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

$$\frac{975}{N_f} = \frac{50 \times 75 \times 65}{20 \times 25 \times 26}$$

$$\boxed{N_f = 52 \text{ rpm}}$$

③ Reverted Gear Train :-

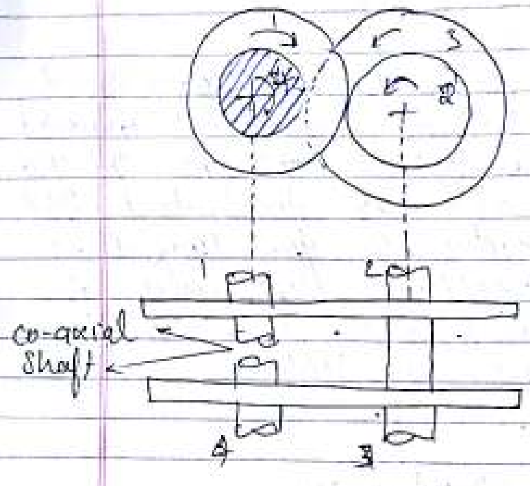


fig:(a)

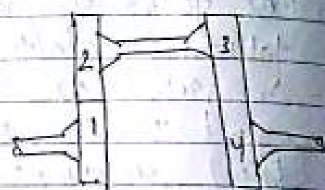


fig:(b)

When the axis of first gear and the last gear are coaxial then gear train known reverted gear trail.

If the axis of first and large last wheel of compound gear co-inside it is called reverted gear train.

Such an arrangement is used in clocks and in simple lathe where back gear is used to give the slow speed to the chuck from the fig (b).

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{—①}$$

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \text{—②}$$

① × ②

$$\frac{N_1}{\cancel{N_2}} \times \frac{N_2}{N_4} = \frac{T_2}{T_1} \times \frac{T_4}{T_3}$$

Since, $N_2 = N_3$

$$\boxed{\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}}$$

$x = r_1 + r_2 = r_3 + r_4$ [due to same central distance]

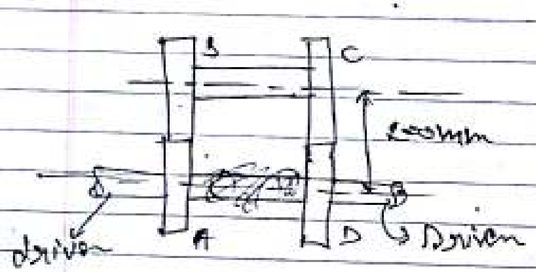
5/4/2017

Ques-> The speed ratio of the reverted gear train as shown in figure is to be 12. The module pitch of gear A & B is 3.125 mm and for gear C & D is 2.5 mm. Calculate the suitable no. of teeth for the gears, No gear is to have less than 24 teeth.

$m_A = m_B = 3.125$, $m_C = m_D = 2.5$

$\text{Module}(m) = \frac{D}{T} = \frac{2r}{T}$

$\therefore \boxed{r = \cancel{\frac{m}{2T}}}$ $\boxed{r = \frac{mT}{2}}$



$12 = \frac{N_A}{N_D} = \frac{N_A}{N_B} \times \frac{N_C}{N_D} = \sqrt{12} \times \sqrt{12}$

$\frac{N_A}{N_B} = \sqrt{12}$, $\frac{N_C}{N_D} = \sqrt{12}$

$r_A + r_B = r_C + r_D$

$\frac{m_A T_A}{2} + \frac{m_B T_B}{2} = \frac{m_C T_C}{2} + \frac{m_D T_D}{2}$

$m_A(T_A + T_B) = m_C(T_C + T_D)$

$\frac{N_A}{N_B} = \frac{T_B}{T_A} = \sqrt{12}$

$\frac{N_C}{N_D} = \frac{T_D}{T_C} = \sqrt{12}$

$x = r_1 + r_2 = r_3 + r_4$

$3.125(T_A + T_B) = 2.5(T_C + T_D)$

$3.125(T_A + \sqrt{12}\,T_A) = 2.5(T_C + \sqrt{12}\,T_C)$

$(13.93)(T_A) = 1.116\, T_C$

$\boxed{13.93\, T_A = 1.116\, T_C}$

$T_A = 28$

$T_B = 100$

$T_C = 36$

$T_D = 124$

~~$\frac{3.125\, T_A}{2} + 3...$~~

$3.125\left(\frac{T_A + T_B}{2}\right) = 2.5\left(\frac{T_C + T_D}{2}\right)$

$3.125\left(\frac{T_A + \sqrt{12}\cdot T_A}{2}\right) = 2.5\left(\frac{T_C + \sqrt{12}\cdot T_C}{2}\right)$

$\therefore \boxed{6.975\, T_A = 5.5858\, T_C}$

$\therefore \boxed{T_A = 0.8\, T_C}$

~~$13.93(0.8\, T_C) = 1.116\, T_C$~~

Que. A reverted gear train is used to provide a speed ratio of 10. The module of gear 1 & 2 is 3.2 mm and for gear 3 & 4 is 2 mm. Determine the suitable no. of teeth for each gear. No gear is to have 20 teeth. The distance b/w the shaft is 160 mm.

$r_1 + r_2 = r_3 + r_4 = x$

$r_1 + r_2 = 160$

$m_1 = m_2 = 3.2, \quad m_3 = m_4 = 2mm$

$$\frac{N_1}{N_4} = \frac{N_1}{N_2} \times \frac{N_3}{N_4} = \sqrt{10} \times \sqrt{10}$$

$$\therefore \frac{N_1}{N_2} = \sqrt{10}, \quad \frac{N_3}{N_4} = \sqrt{10}$$

$$\frac{m_1 T_1}{2} + \frac{m_2 T_2}{2} = \frac{m_3 T_3}{2} + \frac{m_4 T_4}{2}$$

$m_1 = m_2 \quad \therefore \quad m_3 = m_4$

$$m_1\left(\frac{T_1 + T_2}{2}\right) = m_3\left(\frac{T_3 + T_4}{2}\right)$$

$$3.2\left(\frac{T_1 + T_2}{2}\right) = 2\left(\frac{T_3 + T_4}{2}\right)$$

$$\frac{T_2}{T_1} = \sqrt{10}, \quad \frac{T_4}{T_3} = \sqrt{10}$$

$T_2 = \sqrt{10}.T_1, \quad T_4 = \sqrt{10}.T_3$

$m_1 \times (T_A + T_D) = m_3(T_C + T_D)$

$3.2(T_A + T_D) = 2(T_C + T_D)$

$r_1 + r_2 = 160$

$$\frac{m_1 T_1}{2} + \frac{m_2 T_2}{2} = 160$$

$$3.2\left(\frac{T_1 + T_2}{2}\right) = 160$$

$$3.2\left(\frac{T_1 + \sqrt{10}T_1}{2}\right) = 160$$

$$T_1 + \sqrt{10}T_1 = \frac{160 \times 2}{3.2}$$

$4.16\, T_1 =$

$\boxed{T_1 = 24.03}$

$T_1 = 24$

$T_2 = 75$

$T_3 = 38$

$T_4 = 121$

* Planetry or Epicyclic Gear Train *

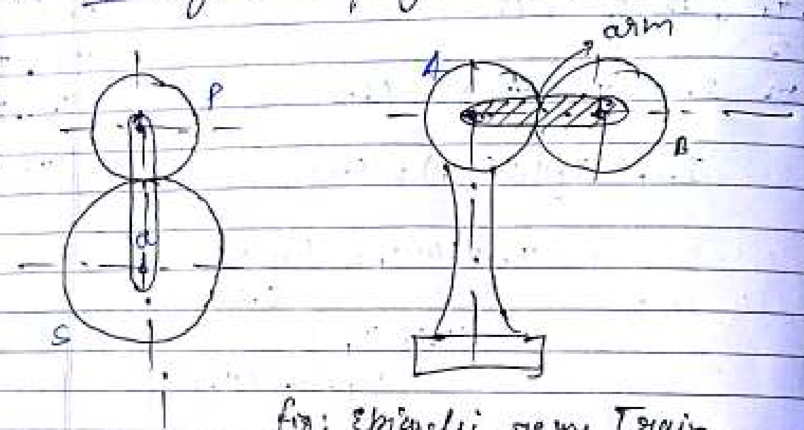


fig: Epicyclic gear Train.

A gear train having it relative motion of axis is called planetry or epicyclic gear train

In this the axis of at least one of the gears also move relative to the frame.

Consider two gear wheels S & P. The axis of which are connected by the arm (a) is fixed the wheel S and P constitute a simple gear train.

However, if the wheel S is fixed so that the arm 'a' can rotate about the axis of 'S' the, wheel 'P' would also move around 'S'. Therefore, it is an epicyclic gear train,

⇒ For the calculation for rpm and no. of teeth in gears we used tabular method of epicyclic gear train which are follows.

Step	Action	Rev. of arm	Rev. of S	Rev. of P
1	a is fixed, S takes +1 rev.	0	+1	$-\frac{T_S}{T_P}$
2	a is fixed, S takes +x rev.	0	+x	$-x\frac{T_S}{T_P}$
3	Add +y	+y	+y	+y
Total		+y	x+y	$y-x\frac{T_S}{T_P}$

Date 4/4/2017 $\frac{N_A}{N_B} = \frac{T_B}{T_A}$ $N_B = \frac{T_A}{T_B} \times N_A$ c.c.w +ve, c.w -ve

169

Ques, An epicyclic gear train consist of an arm and two gears A & B having 30 & 40 teeth respectively, the arm rotates about the centre of gear A at a speed of 80 rpm. c.c.w. Determine the speed of gear B if -

① The gear A is fixed
② The gear A revolves at 240 rpm c.w.

Action	revo. of arm C	revo. of gear A	revo. of gear B
arm fixed, A, +1	0	+1	$-\frac{T_A}{T_B}$
arm fixed, +x(A)	0	+x	$-x\frac{T_A}{T_B}$
add (+y)	+y	+y	+y
Total	y	x+y	$y - x\frac{T_A}{T_B}$

(i) $x+y=0$ [∴ fixed gear A]

$x = -y = -80$

$N_B = N_A \cdot \frac{T_A}{T_B}$ ⇒

170

$N_B = y - x\frac{T_A}{T_B}$

$= -80 - (-80) \times \frac{30}{40}$

$\boxed{N_B = 140 \text{ rpm}}$

(ii) ~~$x+y = -240$~~

~~$\boxed{x = -240 - y}$~~

~~$N_B = y - x\frac{T_A}{T_B}$~~

$= -240$

$x+y = -240$

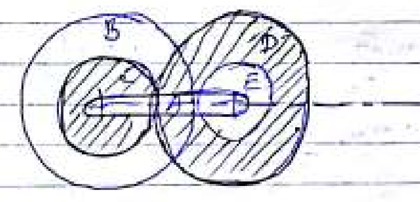
$x+80 = -240$, -320

$\boxed{x = -320}$

$N_B = 80 + 320 \times \frac{30}{40}$

$\boxed{N_B = 320}$

Ques In a reverted epicyclic gear train the arm A carries two gears B & C & the compound gear D, E. The gear B meshes with gear E & the gear C meshes with gear D. No. of teeth on gears B, C, D are 75, 30, 90 respectively. Find the speed and direction of gear C when the gear B is fixed and arm A makes 100 rpm in c.w direction.



$r_B + r_E = r_C + r_D = x$

$\frac{m_B T_B}{2} + \frac{m_E T_E}{2} = \frac{m_C T_C}{2} + \frac{m_D T_D}{2}$

$\frac{m}{2}(T_B + T_E) = \frac{m}{2}(T_C + T_D)$ .... [m(BECD) = 1]

$75 + T_E = 30 + 40$

$\boxed{T_E = 45}$

Action	revo. of arm (A)	revol. compound D-E	Revol. Gear B	Revol. G_C
arm A fixed D/E, +1	~~±~~0	+1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
arm A fixed +x, (A)	0	x	$-\frac{x T_E}{T_B}$	$-\frac{x T_D}{T_C}$
add	y	y	y	y
	y	x+y	$y - \frac{x T_E}{T_B}$	$y - \frac{x T_D}{T_C}$

$y = -100$

$x + y =$

$y - \frac{x T_E}{T_B} = \cancel{0}_B$ [fixed]

$-100 - x \times \frac{45}{75} = 0$

$\boxed{x = -166.66}$

$N_C = y - \frac{x T_D}{T_C} \Rightarrow -100 + 166.66 \times \frac{90}{30}$

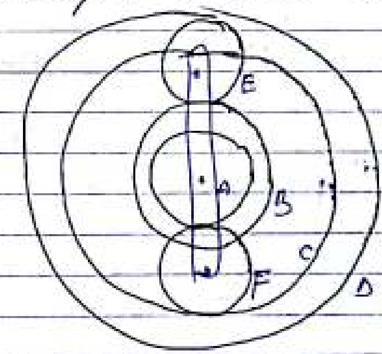
$\boxed{N_C = 399.98}$

6/04/2017

Example : 13-8 (446)

Ques→ An epicyclic gear train shown in fig. the compound wheels A, B as well as internal wheel C, D rotates about the axis O the wheels E & F rotates on the pins fixed to the arm (a) all the wheels are of the ~~sa~~ same modul. the no. of teeth on the wheel are $T_A = 52$, $T_B = 56$, $T_E = T_F = 86$ Determine the $\boxed{\text{speed of C}}$ (i) if the wheel D is fixed & arm rotate at 200 rpm cw direction

(ii) The wheel D rotates at 200 rpm ccw & arm 'a' rotates 20 rpm ~~clockwise~~ ~~ccc~~ ccw.



Action	arm	A/B	E	F	C	D
arm fixed +1, (A/B)	0	+1	$-\frac{T_B}{T_E}$	$-\frac{T_A}{T_F}$	$+\frac{T_A}{T_C}$	$+\frac{T_B}{T_D}$
arm fixed +x (A/B)	0	x	$-\frac{x T_B}{T_E}$	$-\frac{x T_A}{T_F}$	$\frac{x T_A}{T_C}$	$x \frac{T_B}{T_D}$
+y	y	y	y	y	y	y
Total	y	x+y	$y - x \frac{T_B}{T_E}$	$y - \frac{x T_A}{T_F}$	$y + \frac{x T_A}{T_C}$	$y + \frac{x T_B}{T_D}$

$y = 100$

$(N)_{A/B} = \frac{n - y}{}$
$= n + 100$

$D_D = D_B + 2D_E$

$m = \frac{D}{T}$

$D = mT$

$mT_D = mT_B + 2mD_E$

$\cancel{m}(T_D) = \cancel{m}(T_B + 2D_E)$

$T_D = T_B + 2D_E$

$= 56 + 2(36)$

$\boxed{T_D = 128}$

$\cancel{m}(T_C) = \cancel{m}(T_A + 2T_F)$

$T_C = T_A + 2T_F$

$= 52 + 2(36)$

$\boxed{T_C = 124}$

$y + x\frac{T_B}{T_D} = 0$

$100 + x\left(\frac{56}{128}\right) = 0$

$\frac{56x}{128} = -200$

$\boxed{x = \cancel{228.57}}$ 457.14

GEARS

Gears are used to transmit motion from one shaft to another shaft. This is accomplished by successively engaging the teeth of the gears.

In the transmitting of the motion or power b/w the two shaft the effect of slipping is to reduce the vel. ratio (belt & pulley)

In the precision required m/c in which definite vel. ratio is very important their only positive is used which is known as gears, teeth wheel or friction wheels. A gear drive also used when the distance b/w driver & follower is very small.

Advantages of Gear drive :-

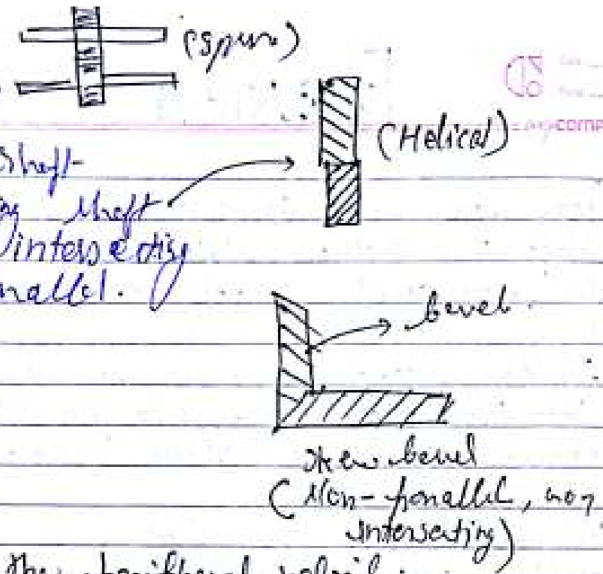
1. It transmitt exact vel. ratio.
2. It may be use to transmitt large power.
3. It has high efficiency.

Limitation :-

1. The manufacturing cost of gear is very high due to requirement of special tools and equipment.

* Classification *

1. According to the position of axis of shaft :-



=) Parallel shaft
=) Intersecting shaft
=) Non-intersecting
=) Non-parallel.

(2) Acc. to the peripheral velocity :-

1) Low (below 3m/s)
2) Medium (above 15 m/s)
3) High (

(3) Acc. to the type of gear :-

(1) External (2) Internal (3) Rack & pinion

$\boxed{\text{Gear Terminology}}$

10/4/2017

* Gear Terminology *

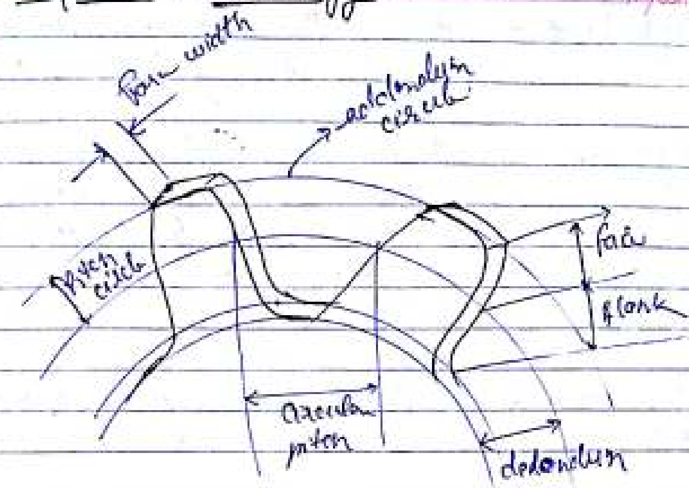


Fig: (a)

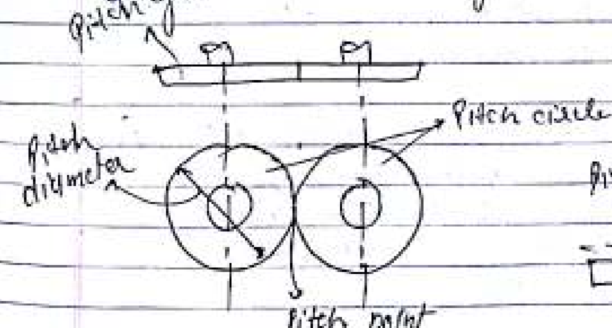


Fig (b)

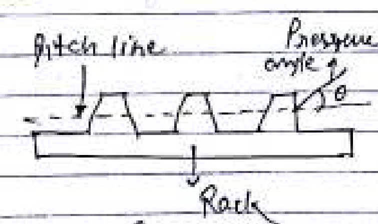


Fig: (c)

Pitch circle :- It is an imaginary circle by which pure rolling action occurs it would give the same motion as the actual gear.

Pitch circle dia :- It is the dia of the pitch circle. The size of gear specified by ~~pict~~ pitch circle dia.

Pitch point :- It is the common pt of contact b/w the two pitch circle.

Pressure :- It is the angle b/w common normal to two gear teeth at the pt of contact and the common tangent at the pitch pt. it is usually denoted by ϕ.

The standard pressure angle are 14.5° & 20°

Addendum :- It is the radial distance of a tooth from the pitch circle to the tip of the tooth

Dedendum :- It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

Circular pitch (P_c) :-

It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding on the next tooth.

It is usually denoted by P_c & mathematical

$$\text{Circular pitch} = \frac{2\pi R}{T} = \frac{\pi D}{T}$$

$$\boxed{P_c = \pi m}$$

For same circular pitch

$$\frac{D_1}{T_1} = \frac{D_2}{T_2}$$

Diametric Pitch :-

It is the ratio of no. of teeth to the pitch circle diameter.

$$\boxed{P_d = \frac{T}{D}}$$

$$P_c \times P_d = \frac{\pi D}{T} \times \frac{T}{D} = \pi$$

$$\boxed{P_c \times P_d = \pi}$$

Module :- It is the ratio of pitch circle dia to the no. teeth.

$$\boxed{m = \frac{D}{T}}$$

Path of contact :- It is the path traced by the point of contact of the two teeth from beginning to the end of engagement.

Length of path of contact :- It is the length of common normal cut off by addendum circle of the wheel & pinion.

Arc of contact :- it is the path traced by point on the pitch circle from the beginning to the end engagement of a given pair of teeth the arc of contact consist of two part i.e.

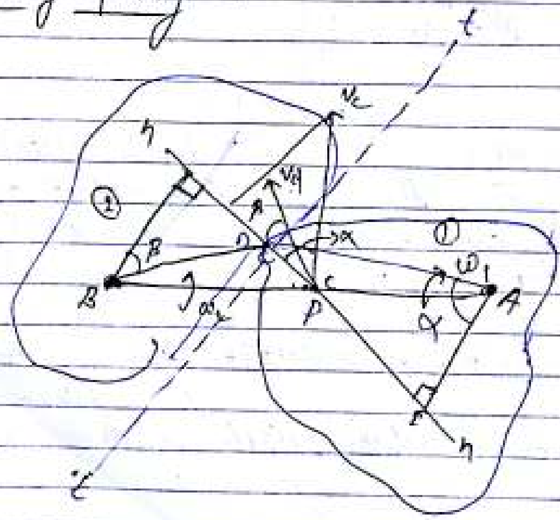
a) Arc of approach :-

It is the portion of the path of contact from the beginning of the engagement to the pitch point.

b) Arc of recess :-

It is the portion of the path of contact from the pitch to the end engagement of a pair of a teeth.

* **Law of Gearing** :-



Acc. to law of gearing

$$\omega_2 \times AP = \omega_1 \times BP$$

The law of gearing ~~stayed~~ states that the condition which must be fulfil by the gear tooth profile to maintain a constant angular velocity b/w the two gears as shown in fig.

The two bodies 1 & 2 represented a position of gear two gears in mesh.

A point 'C' on the tooth profile of the gear ① is in contact with a pt D. on the tooth profile of the gear ②.

The two curve in contact at point C & D must have common normal at the point.

Let it be n-n

Let ω_1 instantaneous angular vel. of gear ① in c.w direction, ω_2 instantaneous angular vel. of G_2 in c.c.w direction.

V_{BC} = linear vel. of C.

V_{BD} = linear vel. of D.

$B_C = \omega_1 AC$ in a direction ⊥ to AC at angle α to ~~n~~ n-n

$BD = \omega_2 \times BD$ in a direction ⊥ BD at angle β to n-n.

Now, if the curved surfaces of the teeth of two gears remains in contact, one surface may slide relative other along the common tangent (t-t)

The relative motion b/w the surfaces along the common normal (n-n) must be zero.

Component of V_c along n-n

$= V_c \cos\alpha$
$= V_d \cos\beta$

The relative motion along n-n.

$= V_c \cos\alpha - V_d \cos\beta$

Draw the $\perp$ AE & BF on n-n from pt A & B.

Then angle, $\angle CAE = \alpha$
$\angle DBF = \beta$

For the proper contact

$$V_c \cos\alpha - V_d \cos\beta = 0$$

$$\boxed{V_c \cos\alpha = V_d \cos\beta}$$

$$\cos\alpha\,(\omega_1 \times AC) = (\omega_2 \times BD)\cos\beta$$

$$\omega_1 \times \cancel{AC} \times \frac{AE}{\cancel{AC}} = \omega_2 \times \cancel{BD} \times \frac{BF}{\cancel{BD}}$$

$$\omega_1 \cdot AE = \omega_2 \cdot BF$$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{BF}{AE}}$$

From the similar Δ,
ΔAEP & ΔBFP,
we will.

$$\frac{\omega_1}{\omega_2} = \frac{BF}{AE} = \frac{BP}{AP}$$

$$\boxed{\omega_1 \times AP = \omega_2 \times BP} \ \star\star$$

⇒ Velocity of sliding:-

If the curved surface of the two teeth of the gears one & two remain in contact. One can have sliding motion relative to other along the common tangent (t-t) at C or D.

Vel. of sliding ~~equat~~ $= BC\sin\alpha - BD\sin\beta$

$= \omega_1 AC\sin\alpha - \omega_2 BD\sin\beta$

$= \omega_1 \times \cancel{AC} \times \frac{EC}{\cancel{AC}} - \omega_2 \times \cancel{BD} \times \frac{FD}{\cancel{BD}}$

$= \omega_1 EC - \omega_2 FD$

$= \omega_1 (EP + PC) - \omega_2 (FP - DP)$

$= \cancel{\omega_1 EP + \omega_1 PC - \omega_2 FP} + \omega_2 DP$

$= \cancel{\omega_1}$ $\quad \boxed{PC = PD}$

$= (\omega_1 + \omega_2) PC - \omega_2 FP + \omega_1 EP$

from the law of gearing

$\omega_1 \times EP = \omega_2 . FP$

$= (\omega_1 + \omega_2) Pc - \cancel{\omega_2 FP} + \cancel{\omega_2 FP}$

$\boxed{\text{Vel. of sliding} = (\omega_1 + \omega_2) Pc}$

⇒ Length of path of contact :-