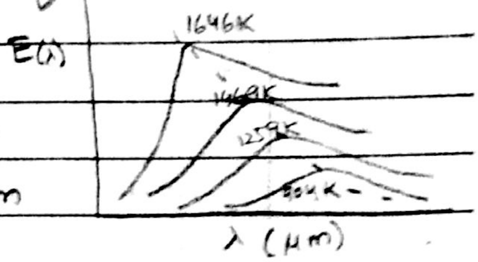
STATISTICAL PHYSICS
(UNIT II)
LECTURE I (a)
BLACK BODY RADIATION

BLACK BODY : Perfect black body is one that absorbs radiations of all wavelengths incident on it. when heated, it emits radiations of all wavelengths. Radiations are independent of nature of material
example : lamp-black.

Spectrum of Radiations emitted by a black body : A curve between intensity of radiation and wavelength for different temperatures is shown in the figure of a black body radiation



Features :-

① For a given temperature, the energy is not uniformly distributed in the radiation spectrum of a hot body.

② At a given temperature, intensity of radiation increases with wavelength & reaches a maximum value λ_m. Further increase in wavelength results in decrease in intensity

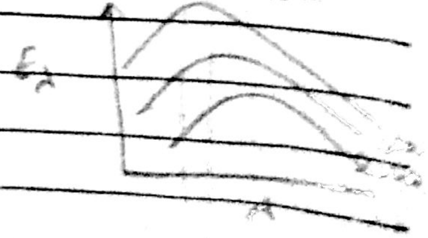
③ with increase in temperature, λ_m shifts to shorter wavelengths such that $\lambda_m \cdot T$ = constant = 0.2896 cm K
This is known as Wein's displacement law.

④ For all wavelengths, increase in T results in increase in Energy.

⑤ Area under each curve represents total energy emitted by a body for the range of wavelengths considered. This area increases with increase in temperature such that

$$E \propto T^4$$

This is known as Stefan's law.



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Wien's law

① In 1893, wien show that the wavelength corresponding to the maximum emission is inversely proportional to the absolute temp. of the black-body.

$$\lambda_m \propto 1/T$$

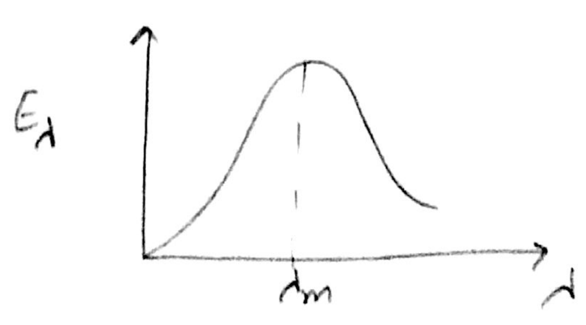
$$\lambda_m T = \text{Constant} = 2.893 \times 10^{-3}\ mK$$

This shows that with ↑ of temp. of the blackbody maximum intensity of radiation shifts towards shorter wavelength

② He assumed that in addition to thermodynamical principles, each oscillator emits radiation of only single wavelength.

③ Energy density (E_λ) of black-body radiation of wavelengths b/w λ and $\lambda + d\lambda$ is given by

$$E_\lambda\, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT}}\, d\lambda$$



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Rayleigh - Jeans law

① Wien's law agrees with experimental Results only at low wavelength but fails at large wavelength.

Lord Rayleigh and James Jeans used thermodynamical principles to explain black-body radiations.

② They assumed that emitted radiations have continously variable wavelength from 0 to ∞.

③ They used classical law of equipartion of energy i.e. average value $\bar{E} = \frac{1}{2}kT$

④ Energy density in the Range λ and $\lambda + d\lambda$ is

$$E_\lambda \, d\lambda = \frac{8\pi kT}{\lambda^4} \, d\lambda$$

Failure of Classical Theory to Explain Spectrum of radiation emitted by black-body

OR

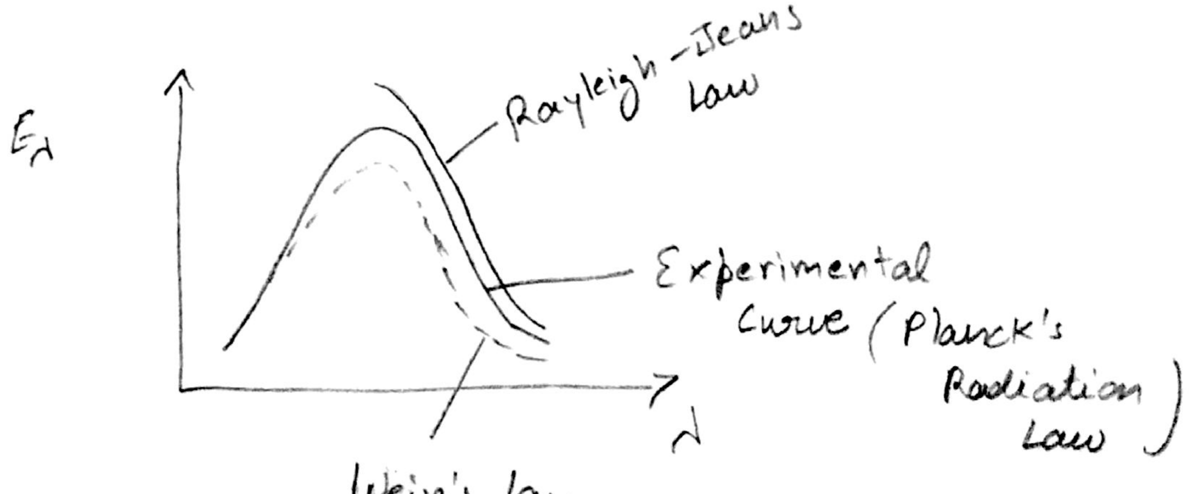
Ultraviolet Catastrophe

① Rayleigh - Jeans law agrees with experimental Results only for long wavelength and fails completely at Short wavelength. This result is known as Ultraviolet Catastrophe.

i.e $E_\lambda \to \infty$ when $\lambda \to 0$ in Rayleigh Jeans law whereas Experimental Result show that $E_\lambda \to 0$ when $\lambda \to 0$.

② Wien's law agree with experimental Results only in the low wavelength Region but fails in the large wavelength Region.

Hence it is concluded that there is discrepancy b/w experimental and theortical Results. So Classical physics fails to explain the Spectrum of Radiation emitted by a black-body.



Q-6.) (b) A Black body is one which absorbs heat radiations of all wavelength incident upon them When Such body is heated, it emits radiations of all types of wavelength. These radiations are independent of nature of the substance. Such heat radiations in a uniform temperature known as <u>Black body Radiations</u>

c) Planck derive theortical expression for energy distribution of black body on the basis of quantum theory of heat Radiations.

He made following assumptions

(i) A black-body chamber is filled up not only with radiation but also with simple harmonic oscillator which can vibrate with all possible frequencies. The vibration is one degree of freedom only.

(ii) Oscillator of black-body has a discrete energy instead of continous and is equal to integral Multiple of some minimum energy $E = nh\nu$

(iii) Oscillator can not radiate or absorb energy continously but can exchange energy with its surroundings in discrete values i.e. $0, h\nu,$

$2h\nu, 3h\nu, \ldots$ called Quantum.

Suppose N is the total No. of Planck oscillator and E their total Energy. Then energy per oscillator is given as

$$\bar{\varepsilon} = \frac{E}{N}$$

If $N_0, N_1, N_2, \ldots N_r$ are no. of oscillator having energy $0, \varepsilon, 2\varepsilon, \ldots r\varepsilon$ then

$$N = N_0 + N_1 + N_2 + \ldots N_r \quad \text{---} \; ①$$

$$E = 0 + \varepsilon N_1 + 2\varepsilon N_2 + 3\varepsilon N_3 + \ldots \quad ②$$

from Maxwell distribution formula

$$N_r = N_0 \, e^{-r\varepsilon/kT}$$

$$\therefore \quad N_1 = N_0 \, e^{-\varepsilon/kT}$$

$$N_2 = N_0 \, e^{-2\varepsilon/kT}$$

$\therefore$ ① equa becomes

$$N = N_0 \left[1 + e^{-\varepsilon/kT} + e^{-2\varepsilon/kT} + \ldots \right]$$

$$\left| N = \frac{N_0}{1 - e^{-\varepsilon/kT}} \right| \; \text{--} \; ③ \text{ equ.} \qquad \left(\because 1 + x + x^2 + \ldots = \frac{1}{1-x} \right)$$

and total Energy of Planck's oscillator

$$E = 0 + \varepsilon N_0 e^{-\varepsilon/kT} + 2\varepsilon N_0 e^{-2\varepsilon/kT} + \dots$$

$$E = N_0 \varepsilon e^{-\varepsilon/kT}\left(1 + 2e^{-\varepsilon/kT} + \dots\right)$$

$$\boxed{E = \frac{N_0 \varepsilon e^{-\varepsilon/kT}}{\left(1 - e^{-\varepsilon/kT}\right)^2}} \quad \text{--- ④ eq.} \qquad \left(\because 1 + 2x + 3x^2 + \dots = \frac{1}{(1-x)^2}\right)$$

Average energy of oscillator is

$$\bar{\varepsilon} = \frac{E}{N} = \frac{\varepsilon e^{-\varepsilon/kT}}{\left(1 - e^{-\varepsilon/kT}\right)}$$

using ③ & ④ eq.

$$\bar{\varepsilon} = \frac{\varepsilon}{\left(e^{\varepsilon/kT} - 1\right)}$$

$$\boxed{\bar{\varepsilon} = \frac{h\nu}{\left(e^{h\nu/kT} - 1\right)}} \quad \text{--- ⑤ equ.}$$

here $\varepsilon = h\nu$
acc. to planck
assumption

No. of Oscillator in frequency range ν and $\nu+d\nu$
is given as wavelength λ $\lambda+d\lambda$

$$N = \frac{8\pi\nu^2}{c^3}d\nu = \frac{8\pi c}{\lambda^5}d\lambda$$

∴ Energy density is given as

$$E\,d\nu = N\bar{E}$$

$$E\,d\nu = \frac{8\pi\nu^3}{c^3} \times \frac{h}{e^{h\nu/kT}-1}d\nu$$

in terms of wavelength

$$E\,d\lambda = \frac{8\pi}{c^3}\left(\frac{c}{\lambda}\right)^3 \frac{h}{e^{h\nu/kT}-1} \times \left(-\frac{c}{\lambda^2}d\lambda\right)$$

$$\boxed{E\,d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}d\lambda}$$

a) For small temp. λT is small

$$\therefore \frac{hc}{\lambda kT} >> 1$$

$$\therefore e^{hc/\lambda kT}-1 \simeq e^{hc/\lambda kT}$$

Q.

$$E\,d\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda kT} d\lambda$$

This is Wien's law

(b) For large temp. λT is large

$$\frac{hc}{\lambda kT} << 1$$

$$e^{hc/\lambda kT} = 1 + \frac{hc}{\lambda kT} + \left(\frac{hc}{\lambda kT}\right)^2 \frac{1}{2!} + \cdots$$

$$\simeq 1 + \frac{hc}{\lambda kT}$$

$$\therefore E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{\left(1 + \frac{hc}{\lambda kT} - 1\right)}$$

$$\boxed{E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda}$$

This is Rayleigh - Jean's law

Operator

① $\hat{H}\psi = E\psi$ where $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V$

operator   Eigen value

$= E$

② $p\psi = -i\hbar\frac{\partial\psi}{\partial x} = \underbrace{-i\hbar\nabla}\psi$

$\boxed{\hat{p} = -i\hbar\nabla}$

③ $\hat{E} = -\frac{\hbar^2}{2m}\nabla^2 + V$

④ K.E $= \hat{T} = -\frac{\hbar^2}{2m}\nabla^2$

Probability of wave function

$$P = \int_{-\infty}^{\infty}\psi_{(x)}\psi^*_{(x)}\,dx$$

Expectation value / Average value of p quantity

$$\langle p\rangle = \frac{\int_{-\infty}^{\infty}\psi^*_{(x)}\,p\,\psi_{(x)}\,dx}{\int_{-\infty}^{\infty}\psi_{(x)}\psi^*_{(x)}\,dx}$$

→ normalized function

Q① "We cannot know future because we cannot know present". Justify this statement with uncertainity principle. [2014] Re-appe

Sol. In classical physics, future motion of particle could be exactly predicted / determined from the knowledge of its position and momentum and all the forces acting upon it.

But Heisenberg denies this concept ∵ one cannot measure the exact position and momentum of particle simultaneously and accurately so future cannot be predicted. but only a Range of possibilities for the future motion of a partich.

Q-① Find smallest possible uncertainity in position of e^- moving with velocity 3×10^7 m/s. (given $m_0 = 9.1 \times 10^{-31}$ kg)

[2007]

Sol. $$\boxed{\Delta x_{min} \, \Delta p_{max.} = \frac{h}{2\pi}}$$

here $v \simeq c$

$$p = m v$$

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \approx \Delta p_{max}$$

$$\therefore \Delta x_{min} = \frac{h}{2\pi} \times \frac{\sqrt{1 - \frac{v^2}{c^2}}}{m_0 v} = 3.8 \times 10^{-12} \, m$$

Q-② Show that the uncertainity in the location of the particle is equal to de-broglie wavelength the uncertainity in its velocity is equal to the its velocity.

Sol. Given $\Delta x = \lambda$ Proof :- $\Delta v_x \simeq v_x$.

Sol. $$\Delta x \, \Delta p \simeq \textcircled{h}$$

$$\lambda \, \Delta p = h$$

$$\frac{h}{m v_x} \Delta p = h$$

$$\Delta p = m v_x \quad ; \quad m \Delta v_x = m v_x$$

$$\boxed{\Delta v_x = v_x}$$

Q③ Calculate de-broglies wavelength of

(i) a 46g golf ball moving with velocity 30 m/s

(ii) an e^- moving with velocity 10^7 m/s. which one is measurable?

[2010]

Sol. (i) for half ball $\lambda = 4.8 \times 10^{-34} = \frac{h}{mv}$

(not measurable)

(ii) $\lambda = 72$ nm (measurable) comes in Gamma Range

Q(1) Determine de-broglie wavelength of an e^- having K.E. 2 eV

Sol.

$$\lambda = \frac{hc}{\sqrt{K(K+2m_0c^2)}}$$

$$\lambda = \frac{h}{\sqrt{2m(E)}}$$

This is T.E. Not K.E.

$\lambda =$

Q-2.) Wave function of a certain particle is

$\psi = A \cos^2 x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

a) find the value of A

b) find the probability that particle is found b/w $x = 0$ and $x = \frac{\pi}{4}$

[2012]

Sol. a) Normalised wave-function

$$\int_{-\infty}^{\infty} \psi \psi^* dx = 1$$

$$\int_{-\pi/2}^{\pi/2} A^2 \cos^4 x \, dx = 1$$

$$2A^2 \int_0^{\pi/2} \cos^4 x \, dx = 1$$

$$2A^2 \times \frac{3\pi}{16} = 1$$

$$\boxed{A = \sqrt{\frac{8}{3\pi}}}$$

b) $$P = \int_0^{\pi/4} \psi \psi^* \, dx = A^2 \int_0^{\pi/4} \cos^4 x \, dx$$

$$P = \left(\sqrt{\frac{8}{3\pi}}\right)^2 \times \int_0^{\pi/4} \cos^4 x \, dx$$

$$\boxed{P = 0.462}$$

Q.1.) Write the physical significance of Zero point energy?

Sol. $E_1 = \frac{h^2}{8ma^2}$

This shows that particle in a box cannot be at Rest and have some minimum energy Known as Zero point Energy.

Q.2.) Acceptable function

$x \to 0 \;,\; \psi \to 0$

$x \to \pm\infty \;,\; \psi \to 0$

a.) $\frac{1}{x}$ ☒ b) $\frac{1}{1+x^2}$ ☑

c) e^{x} ☒ d) e^{-x^2} ☑

e) $\sin x$ ☑

Oscillate b/w −1 and +1

Q-3) a) $E_n = \frac{n^2 h^2}{8mL^2}$, $L = 0.1 \times 10^{-9} m$

$n=1$, $E_1 = \frac{(6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.1 \times 10^{-9})^2} = 59.83 \times 10^{-19}$

$n=2$, $E_2 = 4 \times 59.83 \times 10^{19} = 239.34 \times 10^{-19}$

$n=3$, $E_3 = 9 \times 59.83 \times 10^{19} = 538.51 \times 10^{-19}$

$\lambda_1 = \frac{hc}{E_1}$

$\lambda_1 = \frac{(6.6 \times 10^{-34}) \times 3 \times 10^{8}}{59.83 \times 10^{-19}}$

$\lambda_1 = 0.331 \times 10^{-7}$

$\lambda_2 = \frac{hc}{E_2}$

$\lambda_2 = \frac{19.8 \times 10^{-26}}{239.3 \times 10^{-19}}$

$\lambda_2 = 0.082 \times 10^{-7}$

$\lambda_3 = \frac{hc}{E_3}$

$\lambda_3 = \frac{19.8 \times 10^{-2}}{538.51 \times 10^{-}}$

$\lambda_3 = 0.037 \times 10^{-7}$

(b) $\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$

for First Excited state $n=2$

$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi}{L} x$

$$P = \int_{0.45L}^{0.55L} \psi(x)\, \psi^*(x)\, dx = \int_{0.45L}^{0.55L} |\psi(x)|^2 dx$$

$$= \left(\frac{2}{L}\right) \int_{0.45L}^{0.55L} \sin^2\left(\frac{2\pi}{L} x\right) dx$$

$$P = \frac{1}{L} \int_{0.45L}^{0.55L} \left\{ 1 - \cos\left(\frac{4\pi x}{L}\right) \right\} dx$$

$$= \frac{1}{L} \left[(L)_{0.45L}^{0.55L} + \left\{ \sin\left(\frac{4\pi x}{L}\right) \times \frac{L}{4\pi} \right\}_{0.45L}^{0.55L} \right]$$

$$= \frac{1}{L} \left[(0.1) L + \frac{L}{4\pi} (0.1201 - 0.098) \right]$$

$$= \frac{1}{L} \left[(0.1) L + 0.0215 \frac{L}{4\pi} \right]$$

$$= 0.1 + 1.710 \times 10^{-3}$$

$$P \approx 0.1$$

(c) (i) There is a wave-function associate with every physical state of the system contain entire description. Such function must be finite, single valued and continous and must be normalised.

(ii) Every physical observable is associate with an operator. Physical observable may be energy, p, position etc.

ie. $$E = -\frac{\hbar^2}{2m} \nabla^2 + V$$

$$p = \frac{h}{i} \nabla$$

(iii) The measurement of an physical observable can provide the values λ is given as

$$P\psi = \lambda\psi$$

λ is Eigen values and P associate with observable in state ψ leaving function unchanged

(iv) If a system is in a state describe by normalised wave function then average value of observable correspond to operator P is given as

$$<a> = \int_{-\infty}^{\infty} \psi^* P \psi \, dV$$

(v) The wave function of a system evolves in time according to time-dependent Schrodinger's equation

$$\hat{H}\psi(x,t) = i\hbar \frac{\partial\psi}{\partial t}$$