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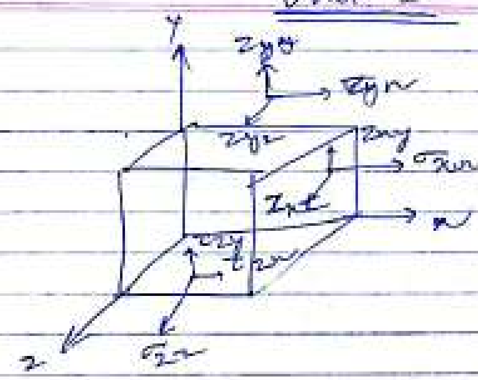
7/1/2017

Unit - I



$\mu = -\left[\frac{\text{lateral strain}}{\text{linear strain}}\right]$

⇒ Strain in x-direction —
Due to σ_{xx}

$$\epsilon_{xx}^{(1)} = \frac{\sigma_{xx}}{E}$$

Strain in y-direction due to σ_{xx}

$$\epsilon_{yy}^{(1)} = -\frac{\mu\sigma_{xx}}{E}$$

Strain in z-direction due to σ_{xx}

$$\epsilon_{zz}^{(1)} = -\frac{\mu\sigma_{xx}}{E}$$

⇒ Strain in x, y, z due to σ_{yy}

$$\epsilon_{xx}^{(11)} = -\frac{\mu\sigma_{yy}}{E}$$

$$\epsilon_{yy}^{(11)} = \frac{\sigma_{yy}}{E}$$

$$\epsilon_{zz}^{(11)} = -\frac{\mu\sigma_{yy}}{E}$$

Strain in x, y, z due σ_{zz}

$$\epsilon_{xx}^{(1)} = \frac{-\mu\sigma_{zz}}{E}$$

$$\epsilon_{yy}^{(1)} = \frac{-\mu\sigma_{zz}}{E}$$

$$\epsilon_{zz}^{(1)} = \frac{\sigma_{zz}}{E}$$

⇒ Combined strain in x-direction

$$\epsilon_{xx} = \epsilon_{xx}^{(1)} + \epsilon_{xx}^{(2)} + \epsilon_{xx}^{(3)}$$

$$= \frac{\sigma_{xx}}{E} - \frac{\mu\sigma_{yy}}{E} - \frac{\mu\sigma_{zz}}{E}$$

$$= \frac{1}{E}\left(\sigma_{xx} - \mu\sigma_{yy} - \mu\sigma_{zz}\right)$$

→ Combined strain in y-direction

Shear strain (γ)

$$\gamma_{xy} = \gamma_{yx} = \frac{\tau_{xy}}{G} = \frac{\tau_{yx}}{G}$$

Similarly :-

$$\gamma_{yz} = \gamma_{zy} = \frac{\tau_{zy}}{G} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \gamma_{xz} = \frac{\tau_{zx}}{G} = \frac{\tau_{xz}}{G}$$

$$\begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \gamma_{xy} \\ \gamma_{zx} \\ \gamma_{xz} \end{bmatrix}_{6\times1} = \begin{bmatrix} \frac{\sigma_{xx}}{E} & -\frac{\mu\sigma_{yy}}{E} & -\frac{\mu\sigma_{zz}}{E} \\ & & \\ \end{bmatrix}_{6\times6} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{bmatrix}_{6\times1}$$

$$\begin{bmatrix} \frac{1}{E} & -\frac{\mu}{E} & -\frac{\mu}{E} & 0 & 0 & 0 \\ -\frac{\mu}{E} & \frac{1}{E} & -\frac{\mu}{E} & 0 & 0 & 0 \\ -\frac{\mu}{E} & -\frac{\mu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix}$$

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \mu\left(\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}\right)$$

$$\epsilon_{yy} = \frac{\sigma_{yy}}{E} - \mu\left(\frac{\sigma_{zz}}{E} + \frac{\sigma_{xx}}{E}\right)$$

$$\epsilon_{zz} = \frac{\sigma_{zz}}{E} - \mu\left(\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E}\right)$$

$$(1-\mu)\epsilon_{xx} + \mu\epsilon_{yy} + \mu\epsilon_{zz}$$

$$(1-\mu)\left[\frac{\sigma_{xx}}{E} - \mu\left(\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}\right)\right] + \mu\left(\frac{\sigma_{yy}}{E} - \mu\left(\frac{\sigma_{zz}}{E} + \frac{\sigma_{xx}}{E}\right)\right) + \mu\left(\frac{\sigma_{zz}}{E} - \mu\left(\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E}\right)\right)$$

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$\Rightarrow \frac{\sigma_x}{E} - \mu\left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right) - \left[\frac{\mu\sigma_x}{E} - \mu^2\left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)\right]$

$+\mu\frac{\sigma_{yy}}{E} - \mu^2\left(\frac{\sigma_z}{E} + \frac{\sigma_x}{E}\right) + \mu\frac{\sigma_z}{E} - \mu^2\left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E}\right)$

$\Rightarrow \frac{\sigma_x}{E} - \cancel{\frac{\mu\sigma_y}{E}} - \cancel{\frac{\mu\sigma_z}{E}} - \frac{\mu\sigma_x}{E} + \cancel{\frac{\mu^2\sigma_y}{E}} - \frac{\mu^2\sigma_z}{E}$

$+\cancel{\frac{\mu\sigma_y}{E}} - \frac{\mu^2\sigma_z}{E} - \frac{\mu^2\sigma_x}{E} + \cancel{\frac{\mu\sigma_z}{E}} - \frac{\mu^2\sigma_x}{E}$

$- \cancel{\frac{\mu^2\sigma_y}{E}}$

$\frac{\sigma_x}{E} - \mu\frac{\sigma_x}{E} - 2\mu^2\frac{\sigma_z}{E} - 2\mu^2\frac{\sigma_x}{E}$

$\frac{\sigma_{xx}}{E}\left[1-\mu-2\mu^2\right]$

$\frac{\sigma_x}{E}(\mu+1)(1-2\mu)$

$\Rightarrow \therefore \sigma_{xx} = \frac{E}{(1-2\mu)(1+\mu)}\left((1-\mu)\epsilon_{xx} + \mu\epsilon_{yy} + \mu\epsilon_{zz}\right)$

$\Rightarrow \sigma_{yy} = \frac{E}{(1-2\mu)(1+\mu)}\left(\mu\,\epsilon_{xx} + (1-\mu)\epsilon_{yy} + \mu\epsilon_{zz}\right)$

$\Rightarrow \sigma_{zz} = \frac{E}{(1-2\mu)(1+\mu)}\left(\mu\epsilon_{xx} + \mu\epsilon_{yy} + (1-\mu)\epsilon_{zz}\right)$

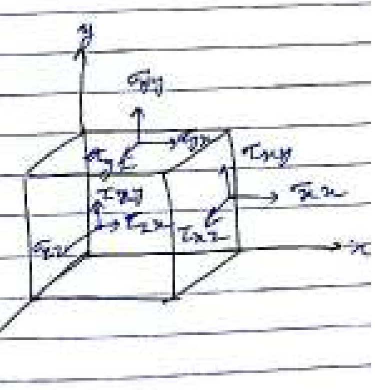
Let $\boxed{\frac{E}{(1-\mu^2)(1+\mu)} = K}$ where $\gamma_{xy} \to$ O.I.e

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{bmatrix} = \begin{bmatrix} K(1-\mu) & K(\mu) & K(\mu) & 0 & 0 & 0\\ K(\mu) & K(1-\mu) & K(\mu) & 0 & 0 & 0\\ K(\mu) & K(\mu) & K(1-\mu) & 0 & 0 & 0\\ 0 & 0 & 0 & G & 0 & 0\\ 0 & 0 & 0 & 0 & G & 0\\ 0 & 0 & 0 & 0 & 0 & G\end{bmatrix} \cdot \begin{bmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \epsilon_{zz}\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{zx}\end{bmatrix}$$

$\Rightarrow$ This is a generalized Hooke's law in three Dimension.

Stress Tensor

$$\begin{array}{c|ccc|l} \text{dir}^n \rightarrow & x & y & z & \\ \hline & \sigma_{xx} & \tau_{xy} & \tau_{xz} & x \text{ face} \\ \sigma_{3D} = & \tau_{yx} & \sigma_{yy} & \tau_{yz} & y \\ & \tau_{zx} & \tau_{zy} & \sigma_{zz} & z \end{array}$$



Que what will be the shear stress in y-dir due to z and z-dir due to y

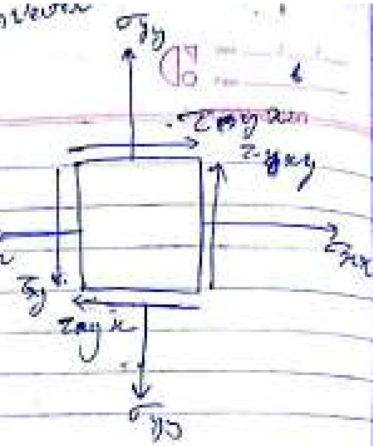
$$\begin{bmatrix} 5 & 20 & 10\\ 10 & 30 & 5\\ 15 & 20 & 0\end{bmatrix}$$

$\tau_{yz} = 5, \quad \tau_{zy} = 20$

$\boxed{0^{th} \text{ order tensor}}$, $\boxed{1^{st} \text{ order tensor}}$ → vector

↓
scalar $\boxed{2^{nd} \text{ order tensor}}$

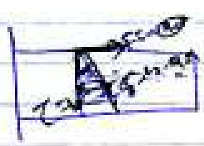
* Plane Stress Condition *



$$\sigma_{zz} = \tau_{zy} = \tau_{zx} = 0$$

$$\boxed{\epsilon_{zz} \neq 0}$$

$$\sigma_{2D} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$$



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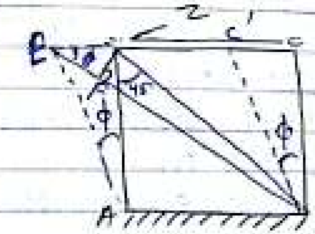
Shear Strain



Strain in BD

$$= \frac{DE - DF}{DF}$$

$$= \frac{DE - DF}{BD}$$

$$= \frac{DE - DF}{AB\sqrt{2}}$$

$\cos 45 = \frac{AB}{BD}$

$BD = \frac{AB}{\cos 45} = AB\sqrt{2}$

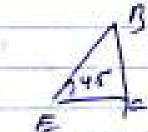
$$= \frac{EF}{AB\sqrt{2}}$$

$$= \frac{1}{2} \times \frac{BE}{AB}$$

$$= \frac{1}{2} \times \tan\phi$$

$\tan\phi <<<$

$$= \frac{1}{2} \times \phi = \frac{1}{2} \times \frac{\tau}{G} = \frac{1}{2}\gamma$$



$\cos 45 = \frac{EF}{BE}$

$$\epsilon_{3D} = \begin{bmatrix} \epsilon_{xy} & \gamma_{xy}/2 & \gamma_{xz}/2 \\ \gamma_{yx}/2 & \epsilon_{yy} & \gamma_{yz}/2 \\ \gamma_{zx}/2 & \gamma_{zy}/2 & \epsilon_{zz} \end{bmatrix}$$ Stress tensor

* Plane strain Condition :-

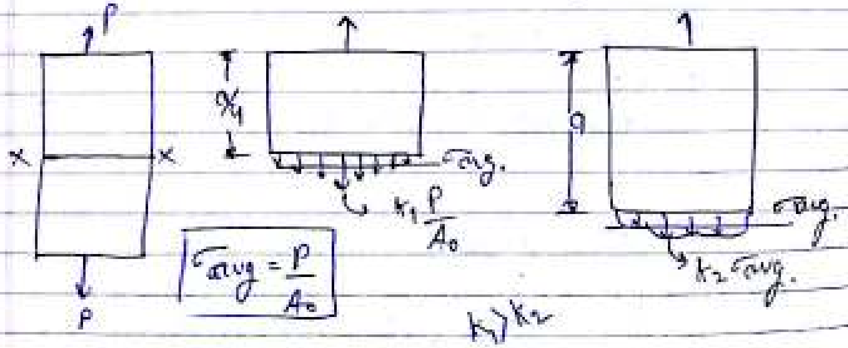
$$\gamma_{zz} = \gamma_{zy} = \gamma_{zx} = 0$$

but $\epsilon_{zz} = 0$, $\sigma_{zz} \neq 0$

$$\epsilon_{2D} = \begin{bmatrix} \epsilon_{xx} & \gamma_{xy}/2 \\ \gamma_{yx} & \epsilon_{yy} \end{bmatrix}$$

- ## Saint Venant's Principle

Point load on a surface give rise to stress concentration near the pt of application. Stress concentration is the increase in stress along the crossection that may be caused by a pt load or any other discontinuity such as holes which bring abrupt or sudden change in the crossection area as shown below in the rectangular bar –



Date 10/11/2017

Ductile → yield stress, Ultimate stress
Brittle → yield stress

Theory of Failures

Failure Stress :-

FOS :- It is used to determine safe dimension of an component under strength criteria.

$$N \text{ or fos} = \frac{\text{Failure of Stress}}{\text{Permissible Stress or allowable or working}}$$

Strength Criteria

$$\sigma_{\text{Induced}} \leq \sigma_{\text{permissible}} \text{ – (for safe design)}$$

$$\sigma_{\text{Induced}} \leq \frac{\text{failure stress}}{N}$$

$$\frac{P}{\pi/4\, d^2} \leq \frac{\text{failure stress}}{N}$$

$$\frac{4P}{\pi d^2} \leq \frac{\text{failure stress}}{N}$$

$$\sqrt{\frac{4PN}{\pi \sigma}} \leq d \Rightarrow d\uparrow \rightarrow N\uparrow$$

TOF :-

It is used to design the m/c component when it's subjected to both normal & shear stresses on the crossection of the component due to various loads acting on ~~load~~ component.

eg:- I.C. engine crank shaft, power transmission shaft etc.

* TOF are used in design of above given example due to unability of failure stress under similler loading condition. This is basically used to stablish a relationship b/w stresses developed under combine loading cond^n. & properties obtain from tension test. i.e. S_{yt} or S_{ut}

[I] Max. principle stress theory. or Max. normal stress theory or Rankine's theory:-

For safe design $\sigma_1 \leq \sigma_{yt}$ or σ_{ut}

$$\leq \frac{S_{yt}}{N} \text{ or } \frac{S_{ut}}{N}$$

This theory suitable for the safe design of brittle material components under any stat of stress condition because brittle material are weak in tension.

This theory is not suitable for the safe design of ductile material component under every state of stress condition because ductile material weaks in shear.

[II] Max. Shear stress theory :- or Guest & Tresca

$$\tau_{max} \leq \frac{S_{ys}}{N} \text{ or } \frac{S_{yt}}{2N}$$

It is suitable for ductile material component under any stat of stress condition because ductile material weeks in shear.

In plane $\tau_{max} = \pm\left(\frac{\sigma_1-\sigma_2}{2}\right)$

Absolute,

$$\tau_{max} = \left[\left|\frac{\sigma_1-\sigma_2}{2}\right|, \left|\frac{\sigma_2-\sigma_3}{2}\right|, \left|\frac{\sigma_3-\sigma_1}{2}\right|\right]$$

[III] Max. Principle strain theory :- or St. Venants

$$\epsilon_1 = [\epsilon_{y.p}]_T$$

(For safe design) $\frac{1}{E}[\sigma_1 - \mu\sigma_2] \leq [\epsilon_{y.p}]_T$ [in 2D]

$$\frac{\sigma_1}{E} \leq \frac{S_{yt}}{NE} \quad \text{[in 1D]}$$

$\sigma \leq \sigma_{permissible}$

$\leq \frac{S_{yt}}{N}$

$$\frac{1}{E}[\sigma_1 - \mu\sigma_2] \leq \frac{S_{yt}}{N.E}$$

$$\boxed{\sigma_1 - \mu\sigma_2 \leq \frac{S_{yt}}{N}}$$

[IV] Total strain Energy Theory :- or Haigh's theory.

$$\frac{\text{Total strain Energy}}{\text{volume}} = \frac{1}{2}\sigma_1\epsilon_1 + \frac{1}{2}\sigma_2\epsilon_2 + \frac{1}{2}\sigma_3\epsilon_3$$

$$\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2+\sigma_3)\right]$$

$$\epsilon_2 = \frac{1}{E}\left[\sigma_2 - \mu(\sigma_1+\sigma_3)\right]$$

$$\epsilon_3 = \frac{1}{E}\left[\sigma_3 - \mu(\sigma_2+\sigma_1)\right]$$

$$\frac{T.S.E.}{Vol.} = \frac{1}{2}\left\{\sigma_1 \times \frac{1}{E}\left(\sigma_1 - \mu(\sigma_2+\sigma_3)\right) + \sigma_2 \times \frac{1}{E}\left[\sigma_2 - \mu(\sigma_1+\sigma_3)\right] + \sigma_3 \times \frac{1}{E}\left[\sigma_3 - \mu(\sigma_2+\sigma_1)\right]\right\}$$

$$= \frac{1}{2}\left\{\frac{\sigma_1^2}{E} - \cancel{\frac{\mu\sigma_2\sigma_1}{E}} - \frac{\sigma_1\mu\sigma_3}{E} + \frac{\sigma_2^2}{E} - \cancel{\frac{\sigma_2\mu\sigma_1}{E}} - \frac{\sigma_2\sigma_3\mu}{E} + \frac{\sigma_3^2}{E} - \frac{\sigma_3\mu\sigma_2}{E} - \frac{\mu\sigma_1\sigma_3}{E}\right\}$$

$$= \frac{1}{2}\left\{\frac{\sigma_1^2}{E} + \frac{\sigma_2^2}{E} + \frac{\sigma_3^2}{E} - 2\left(\frac{\mu\sigma_1\sigma_2}{E}\right) - 2\left(\frac{\mu\sigma_1\sigma_3}{E}\right) - 2\left(\frac{\mu\sigma_2\sigma_3}{E}\right)\right\}$$

$$\boxed{\frac{T.S.E}{Vol.} \le \frac{S_{yt}^2}{N^2} \times \frac{1}{2E}}$$

$$= \frac{1}{2E}\left\{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)\right\}$$

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Ques) Principle stress in a cast iron body are 40 MPa tensile and 90 MPa compressive, 3rd principle stress being zero. Determine the FOS based on the elastic limit, if the criteria of failure in principle stress theory. The elastic limit in simple tension is 80 MPa & in simple compression 450 MPa for cast iron.

Solution: $S_{yt} = 40$ MPa , $S_{yc} = 90$ MPa ,

FOS =

$\sigma_1 = \sigma_{per.}$	$\sigma_1 = \sigma_{per}$
$\sigma_1 = \frac{\text{failure}}{N}$	$\sigma_1 = \frac{\text{failure}}{N}$
$\le \frac{S_{yt}}{N}$	$\le \frac{S_{uc}}{N}$
$40 \le \frac{80}{N}$	$90 \le \frac{450}{N}$
$\boxed{N \le 2}$	$N \le \frac{450}{90}$
	$\boxed{N \le 5}$

Ques) Principle stresses in a mild steel body are 50 MPa tensile and 40 MPa compressive, 3rd principle stress being zero. Determine FOS based on the elastic limit, if the criteria of failure in max. shear strain theory. The elastic limit in simple tension as well in compression 300 MPa. What will be the FOS of the stress with 75 MPa also tensile instead of compressive.

$\tau_{max} \le \tau_{per}$

Solution: $\frac{\sigma_1 - \sigma_2}{N} \le \frac{S_{yp}}{N}$ or $\frac{S_{yt}}{2N}$

$$\frac{\sigma_1 - \sigma_2}{\cancel{2}} \le \frac{S_{yt}}{\cancel{2}N}$$

$$N \le \frac{S_{yt}}{\sigma_1 + \sigma_2}$$

$$N \le \frac{210}{50+90}$$

$$\boxed{N \le 1.5}$$

If cable in nature

★★ Abs. $\tau_{max} \le \tau_{per}$

$$\text{larger of } \left[\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_3 - \sigma_1}{2}\right| \right]$$

$$\therefore \frac{\sigma_2}{\cancel{2}} \le \frac{S_{yt}}{\cancel{2}N}$$

$$\frac{90}{2} \le \frac{210}{N}$$

$$N \le \frac{210 \times 2}{90}$$

$$\boxed{N \le 4.6}$$

Ques: A steel tube of 40 mm mean dia. & 2 mm thick is under simple torsion. Determine the torque that can be transmitted by the tube if the criterion of failure is
(i) Max. shear stress theory.
~~(ii) Max. principle str.~~

take fos 3 ~~for~~ elastic limit of steel $250 N/mm^2$
& $\mu = 0.3$

Solution: $D = 40mm + 2mm = 42 mm$
$d = 40mm - 2mm = 38 mm$

$$\tau = \frac{16T}{\pi d^3}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{T}{J} R$$

$$\tau = \frac{T(D_o/2)}{\frac{\pi}{32}(D_o^4 - D_i^4)}$$

$\tau_{max} \le \tau_{per}$

$$\frac{T(D_o/2)}{\frac{\pi}{32}(D_o^4 - D_i^4)} = \frac{S_{yt}}{2N}$$

$$\frac{16\, T D_o}{\pi(D_o^4 - D_i^4)} = \frac{S_{yt}}{2N}$$

$$T = \frac{S_{yt}}{2N} \times \frac{\pi(D_o^4 - D_i^4)}{(16 D_o)}$$

$T = 192 Nm$

7/1/2017

[V] Max. Distortion Energy Theory or Max. shear strain energy theory or Von-Mises and Henkey's theory.

For safe design —

$$\frac{\text{Max. distortion energy}}{\text{Volume}} \le \left(\frac{\text{Max. dist energy}}{\text{~~yield point~~ vol.}}\right)_{y.p}$$

$$\frac{\text{Total strain Energy}}{\text{Volume}} = \frac{\text{Volumetric Strain Energy}}{\text{Volume}} + \frac{\text{Dist. strain Energy}}{\text{Volume}}$$

$$\frac{\text{Total strain Energy}}{\text{Volume}} - \frac{\text{Volumetric Strain Energy}}{\text{Volume}} = \frac{\text{Dist. strain Energy}}{\text{Volume}}$$

$$\frac{D.S.E}{\text{Volume}} = \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2 - 2\mu(\sigma_1\sigma_2+\sigma_3\sigma_1+\sigma_2\sigma_3)\right] - \frac{1}{2}\sigma_{avg}\,e_v$$

$$e_1 = \frac{1}{E}\left(\sigma_1 - \mu(\sigma_2+\sigma_3)\right]$$

$$e_2 = \frac{1}{E}\left(\sigma_2 - \mu(\sigma_1+\sigma_3)\right]$$

$$e_3 = \frac{1}{E}\left(\sigma_3 - \mu(\sigma_2+\sigma_1)\right]$$

$$e_v = \frac{1}{E}\left(\sigma_1 - \mu(\sigma_2+\sigma_3)\right] + \frac{1}{E}\left(\sigma_2 - \mu(\sigma_1+\sigma_3)\right] + \frac{1}{E}\left[\sigma_3 - \mu(\sigma_2+\sigma_1)\right]$$

$$= \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2+\sigma_3) + \sigma_2 - \mu(\sigma_1+\sigma_3) + \sigma_3 - \mu(\sigma_2+\sigma_1)\right]$$

$$= \frac{1}{E}\left[\sigma_1+\sigma_2+\sigma_3 - \mu\sigma_2 - \mu\sigma_3 - \mu\sigma_1 - \mu\sigma_3 - \mu\sigma_2 - \mu\sigma_1\right]$$

$$= \frac{1}{E}\left[\sigma_1+\sigma_2+\sigma_3 - 2\mu\sigma_2 - 2\mu\sigma_3 - 2\mu\sigma_1\right]$$

$$e_v = \frac{1}{E}\left[\sigma_1+\sigma_2+\sigma_3 - 2\mu(\sigma_1+\sigma_2+\sigma_3)\right]$$

$$= \frac{1}{E}\left[(\sigma_1+\sigma_2+\sigma_3)(1-2\mu)\right]$$

$$e_v = \frac{(1-2\mu)}{E}(\sigma_1+\sigma_2+\sigma_3)$$

$$\frac{D.S.E}{Vol.} = \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2 - 2\mu(\sigma_1\sigma_2+\sigma_3\sigma_1+\sigma_2\sigma_3)\right] - \frac{1}{2}\left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)\times\frac{(1-2\mu)}{E}(\sigma_1+\sigma_2+\sigma_3)$$

$$= \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2 - 2\mu(\sigma_1\sigma_2+\sigma_3\sigma_1+\sigma_2\sigma_3)\right] - \frac{1}{6E}(1-2\mu)(\sigma_1+\sigma_2+\sigma_3)^2$$

$$= \frac{1+\mu}{6E}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

In 1-D

$$= \frac{1+\mu}{6E}\left[(\sigma_1-\cancel{\sigma_2})^2+(\cancel{\sigma_2}-\cancel{\sigma_3})^2+(\cancel{\sigma_3}-\sigma_1)^2\right]$$

$$= \frac{1+\mu}{\cancel{6}_3 E}(\cancel{2}\sigma_1^2) = \frac{1+\mu}{3E}\times\sigma_1^2$$

$$\boxed{\frac{D.S.E}{Vol.} \le \left(\frac{S_{yt}}{N}\right)^2 \times \frac{1+\mu}{3E}}$$

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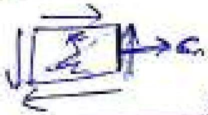


In 2-D

$$\frac{1+\mu}{6E}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\cancel{\sigma_3})^2+(\cancel{\sigma_3}-\sigma_1)^2\right]\le\left(\frac{1+\mu}{3E}\right)\left(\frac{S_{yt}}{N}\right)^2$$

$$\frac{1+\mu}{6E}\left[(\sigma_1-\sigma_2)^2+\sigma_2^2+\sigma_1^2\right]\le\left(\frac{1+\mu}{3E}\right)\left(\frac{S_{yt}}{N}\right)^2$$

$$\left(\frac{1+\mu}{6E}\right)6\tau^2\le\frac{1+\mu}{3E}\left(\frac{S_{yt}}{N}\right)^2$$

$$S_{ys}^2\le\left(\frac{S_{yt}}{N}\right)^2\times\frac{1}{3}$$

$$\boxed{S_{ys}\le\frac{(S_{yt})}{\sqrt{3}}}$$

* Max. shear stress theory

abs. $\tau_{max}\le S_{yt}/2N$

$$\frac{\sigma_1-\sigma_2}{2}\le\frac{S_{yt}}{2N}$$

$\sigma_1=\tau$, $\sigma_2=-\tau$

$$\frac{\tau+\tau}{2}\le\frac{S_{yt}}{2N}$$

$$\frac{2\tau}{2}\le\frac{S_{yt}}{2N}$$

$$\tau\le\frac{S_{yt}}{2N}$$

$$\boxed{\frac{S_{ys}}{N}\le\frac{S_{yt}}{2N}}$$

* Max. principle stress theory -

$$\sigma_1\le\frac{S_{yt}}{N}$$

$$\sigma_N\le\frac{S_{yt}}{N}$$

$$\tau\le\frac{S_{yt}}{N}$$

$$S_{ys}=S_{yt}$$

$\boxed{\mu=0.33}$

		S_{ys}/S_{yt}
MPST	$S_{ys}=S_{yt}$	1
MSST	$S_{ys}=S_{yt}/2$	0.5
MPSt.T	$S_{ys}=S_{yt}/(1+\mu)$	0.75
MPSET	$S_{ys}=S_{yt}/\sqrt{2(1+\mu)}$	0.37
MDEET	$S_{ys}=\frac{S_{yt}}{\sqrt{3}}$	0.577

* Max. principle strain theory.

$$\epsilon_1\le(\epsilon_{up})_T$$

$$\frac{1}{E}(\sigma_1-\mu\sigma_2)\le(\epsilon_{up})_T$$

$$\frac{1}{E}(\tau+\mu\tau)\le\frac{S_{yt}}{NE}$$

$$\tau(1+\mu)\le S_{yt}$$

$$\tau=\frac{S_{yt}}{(1+\mu)}$$

* Total strain energy theory

$\sigma_3=0$

$$\frac{1}{2E}\left[\tau^2+\tau^2+2\mu(\tau^2)\right]\le\left(\frac{S_{yt}}{N}\right)^2\times\frac{1}{2E}$$

$$2\tau^2 + 2\mu\tau^2 \le \left(\frac{S_{yt}}{N}\right)^2$$

$$2\tau^2(1+\mu) \le (S_{yt})^2 \qquad n=1$$

$$\tau^2 \le \frac{(S_{yt})^2}{2(1+\mu)}$$

$$\tau \le \frac{(S_{yt})^2}{\sqrt{2(1+\mu)}}$$

20/11/2017

TOF Summary Chart

$$\frac{M_e}{I_e} = \frac{\sigma_e}{y_e}$$

$\frac{M}{I} = \frac{\sigma}{y}$	$\frac{T}{J} = \frac{\tau}{R}$
$\frac{M}{\frac{\pi d^4}{64}} = \frac{\sigma}{d/2}$	$\frac{T}{\frac{\pi d^4}{32}} = \frac{\tau}{d/2}$
$\sigma = \frac{32M}{\pi d^3}$	$\tau = \frac{16T}{\pi d^3}$

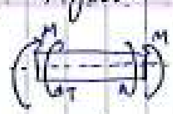
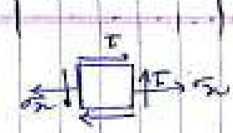
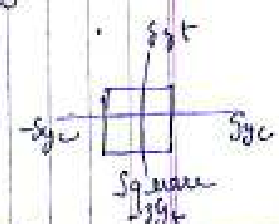
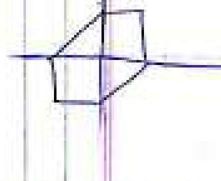
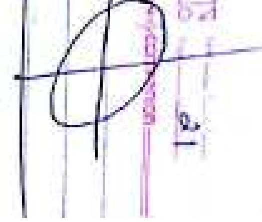
$$\cancel{M_e = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 +}}$$

$$M_e = \frac{1}{2}\left[\sqrt{\left(\frac{\sigma\pi d^3}{32}\right)^2 + \left(\frac{\tau\pi d^3}{16}\right)^2} + M\right]$$

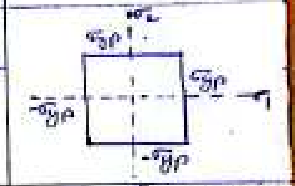
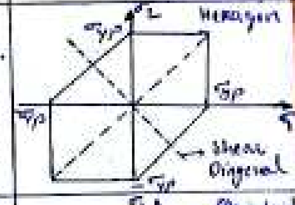
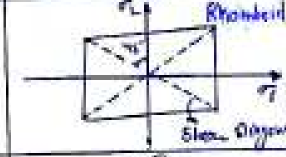
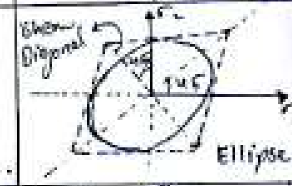
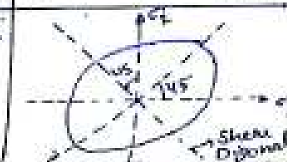
$$= \frac{1}{2}\left[\sqrt{\frac{\sigma^2\pi^2 d^6}{(32)^2} + \frac{\tau^2\pi^2 d^6}{16^2}} + M\right]$$

$$= \frac{1}{2}\left[M + \sqrt{\frac{\sigma^2\pi^2 d^6}{1024} + \frac{\tau^2\pi^2 d^6}{256}}\right] = \frac{\pi d^3}{64}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

TOF Summary Chart

TOF	3-D Equation	2-D equation	figure		Graphical Represent^n
MPST	$\sigma_1 \le \frac{S_{yt}}{N}$	$\sigma_1 \le \frac{S_{yt}}{N}$	 $M_e = \frac{1}{2}\left[M + \sqrt{M^2+T^2}\right]$ $T_e = \frac{1}{2}\sqrt{M^2+T^2}$	 $\frac{1}{2}\left[\sigma + \sqrt{\sigma^2+4\tau^2}\right] \le M_e\,per.$	
MSST	Larger of $\left[\left\lvert\frac{\sigma_1-\sigma_2}{2}\right\rvert, \left\lvert\frac{\sigma_2-\sigma_3}{2}\right\rvert, \left\lvert\frac{\sigma_3-\sigma_1}{2}\right\rvert\right]$	for uncertain nature: $\sigma_1 \le \frac{S_{yt}}{N}$; Also $\frac{\sigma_1}{2} \le \frac{S_{yt}}{2N}$; $\frac{\sigma_1-\sigma_2}{2} \le \frac{S_{yt}}{2N}$ unlike	$T_e = \frac{1}{2}\left[\sqrt{M^2+T^2}\right]$	$\frac{1}{2}\sqrt{\sigma^2+4\tau^2} \le \tau_{per.} \le \frac{S_{ys}}{N} \le \frac{S_{yt}}{2N}$	
MDET	$\left(\frac{1+\mu}{6E}\right)\left\{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right\}$	$\left(\frac{1+\mu}{6E}\right)\left\{(\sigma_1-\sigma_2)^2 + \sigma_1^2 + \sigma_2^2\right\}$	$M_e = \frac{1}{2}\left[\sqrt{M^2 + \frac{3T^2}{4}}\right]$ $T_e = \frac{1}{2}\sqrt{M^2+T^2}$	$M_e = \frac{\pi d^3}{64}\sqrt{\sigma^2+3\tau^2}$ $M_e \le M_{per.}$ $\sqrt{\sigma^2+3\tau^2} \le S_{yt}$	

TOF [SUMMARY CHART]

TOF	EQUATION (3-D)	EQUATION (2-D)	Shaft (M, T)	Element (σ, τ)	GRAPHICAL REPRESENTATION
Maximum Principal Stress Theory (Rankine's)	$\sigma_1 \le \frac{S_{yt}}{N}$	$\sigma_1 \le \frac{S_{yt}}{N}$	$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$	$\frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right] \le$ M_e. per.	
Max. Shear Stress Theory. Guest & Tresca or Coulomb's	Larger of - $\tau_{max} = \left[\left\|\frac{\sigma_1-\sigma_2}{2}\right\|, \left\|\frac{\sigma_2-\sigma_3}{2}\right\|, \left\|\frac{\sigma_3-\sigma_1}{2}\right\|\right]$	Like in nature - $\sigma_1 \le \frac{S_{yt}}{N}$, Also $\frac{\sigma_1}{2} \le \frac{S_{yt}}{2N}$ Unlike in nature - $\frac{\sigma_1 - \sigma_2}{2} \le \frac{S_{yt}}{2N}$	$T_e = \frac{1}{2}\left[\sqrt{M^2 + T^2}\right]$	$\frac{1}{4}\left[\sqrt{\sigma^2 + 4\tau^2}\right] \le \tau_{per.}$ $\le S_{ys}/N$ $\le S_{yt}/2N$	
Maximum Principal Strain Theory St. Venant's	—	$\sigma_1 - \mu\sigma_2 \le \frac{S_{yt}}{N}$	—	—	
Maximum/T. Strain Energy Theory. Haigh's and Beltrami	$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)\right] \le \frac{S_{yt}^2}{N^2} \times \frac{1}{2E}$	$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1\sigma_2) \le \frac{S_{yt}^2}{N^2}$	—	—	
Max. Distortion Energy Theory, Von-Mises, Henky's, Max. Shear Strain Energy	$\left(\frac{1+\mu}{6E}\right)\left\{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right\} \le \left(\frac{S_{yt}}{N}\right)^2 \times \frac{1+\mu}{3E}$	$\left(\frac{1+\mu}{6E}\right)\left\{(\sigma_1-\sigma_2)^2 + \sigma_2^2 + \sigma_1^2\right\}$	$M_e = \frac{1}{2}\left[\sqrt{M^2 + \frac{3T^2}{4}}\right]$	$M_e = \frac{\pi d^3}{32}\sqrt{\sigma^2 + 3\tau^2}$ $M_e \le M_{per.}$ $\sqrt{\sigma^2 + 3\tau^2} \le S_{yt}$	

23

Ques:) A mild steel shaft of 'd' 100mm is subjected to max. T of 12 kNm, max. bending moment 8 kNm at a particular section. Determine 'N'. Acc. to max. shear stress theory if the $S_{yt} = 240$ MPa.

Ques: A shaft is subjected to a max T of 10 kNm & M. bending moment of 12 kNm. At a particular section. Determine the dia of shaft acc. to M.S.S.T if elastic limit in simple tension is 180 MPa.

① $S_{yt} = 240$ MPa, $d = 100$ mm, $T_{max} = 12$ kNm
$M = 8$ kNm

$\frac{1}{2}\left[\sqrt{M^2 + T^2}\right] \le \tau_{per}$

$\tau_{per} = \frac{S_{ys}}{N} = \frac{S_{yt}}{2N}$

$T_e = ?$

$T_e = \frac{1}{2}\sqrt{M^2 + T^2}$

$= \frac{1}{2}\sqrt{8^2 + 12^2}$

$= 7.211$ kNm

$\frac{1}{2}\left(\sqrt{8^2 + (12)^2}\right) = \tau_{per}$

$\tau_{per.} = 7.211$

$\frac{S_{ys}}{N} = 7.211$

$\frac{S_{yt}}{2N} = 7.211$

$\frac{240}{2 \times 7.211} = N = 16.$

$7.211 \times 10^3 \le \frac{\pi}{16} d^3 \tau_{per}$

$T = \frac{\pi d^3}{16}\tau$

$7.211 \times 10^3 = \frac{\pi}{16} d^3 \frac{S_{yt}}{2N}$

$N = \frac{\pi d^3 S_{yt}}{2 \times 16 \times 7.211 \times 10^3}$

$\boxed{N = 3.26}$ Ans

1st $FS = 1$

(2) $T_e = \frac{1}{2}\sqrt{M^2 + T^2}$

$= \frac{1}{2}\sqrt{10^2 + 14^2} = 8.602$

$8.602 \le \frac{\pi d^3}{16} \tau_{per}$

$8.602 \le \frac{\pi d^3}{16} \frac{S_{yt}}{2N}$

$N \le \frac{\pi d^3}{16} \frac{S_{yt}}{2 \times 8.602}$

$N = 1$

$\frac{1 \times 16 \times 2 \times 8.602}{\pi \times S_{yt}} = d^3$

$\boxed{d = 0.78 \text{ m}}$

For Summary Chart

* Max. Principle stress Theory:-

$\frac{M_e}{I_e} = \frac{\sigma_e}{y_e}$

$\frac{M}{I} = \frac{\sigma}{y}$

$\frac{M}{\frac{\pi d^4}{64}} = \frac{\sigma}{d/2}$

$\frac{M \times 64}{\pi d^4} = \frac{2 \times \sigma}{d}$

$\boxed{\sigma = \frac{32M}{\pi d^3}}$

$\boxed{M = \frac{\sigma \pi d^3}{32}}$

$\frac{T}{J} = \frac{\tau}{R}$

$\frac{T}{\frac{\pi d^4}{32}} = \frac{\tau}{d/2}$

$\frac{T \times 32}{\pi d^4} = \frac{2 \times \tau}{d}$

$\boxed{\tau = \frac{T \times 16}{\pi d^3}}$, $\boxed{T = \frac{\tau \pi d^3}{16}}$

$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{1}{2}\left[\frac{\sigma \pi d^3}{32} + \sqrt{\left(\frac{\sigma \pi d^3}{32}\right)^2 + \left(\frac{\pi d^3 \tau}{16}\right)^2}\right]$

$M_e = \frac{1}{2}\left[\frac{\sigma \pi d^3}{32} + \frac{\pi d^3}{32}\sqrt{\sigma^2 + 4\tau^2}\right]$

$= \frac{\pi d^3}{64}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$

$M_e \le M_{per}$

$M_e \le \frac{\pi d^3}{32} \sigma_{per}$

$\frac{\pi d^3}{64}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right] \le \frac{\pi d^3}{32} \sigma_{per}$

$\boxed{\frac{\sigma + \sqrt{\sigma^2 + 4\tau^2}}{2} \le \sigma_{per}}$

* Maximum Shear Stress theory :-

$$T_e = \frac{1}{2}\sqrt{M^2+T^2}$$

$$T_e = \frac{1}{2}\sqrt{\left(\frac{\sigma\pi d^3}{32}\right)^2+\left(\frac{\tau\pi d^3}{16}\right)^2} = \frac{1}{2}\times\frac{\pi d^3}{32}\sqrt{\sigma^2+4\tau^2}$$

$$T_e = \frac{\pi d^3}{64}\sqrt{\sigma^2+4\tau^2}$$

$$T_e \le T_{per}.$$

$$\frac{\pi d^3}{64_4}\sqrt{\sigma^2+4\tau^2} \le \frac{\pi d^3}{16}\tau_{per}.$$

$$\boxed{\frac{\sqrt{\sigma^2+4\tau^2}}{4} \le \tau_{per}}$$

* Maximum Distortion Energy Theory :-

$$M_e = \frac{1}{2}\sqrt{M^2+\frac{3T^2}{4}}$$

$$M_e = \frac{1}{2}\sqrt{\left(\frac{\sigma\pi d^3}{32}\right)^2+\frac{3}{4}\left(\frac{\tau\pi d^3}{16}\right)^2}$$

$$= \frac{\pi d^3}{64}\sqrt{\sigma^2+\frac{3}{\cancel{4}}\times\cancel{4}\tau^2}$$

$$= \frac{\pi d^3}{64}\sqrt{\sigma^2+3\tau^2}$$

$$M_e \le M_{per}.$$

$$\frac{\pi d^3}{64_2}\sqrt{\sigma^2+3\tau^2} \le \frac{\pi d^3}{32}\sigma_{per}.$$

$$\frac{\sqrt{\sigma^2+3\tau^2}}{2} \le \sigma_{per}$$

$$\frac{\sqrt{\sigma^2+3\tau^2}}{\cancel{2}} \le \frac{S_{yt}}{\cancel{2}}$$

$$\boxed{\sqrt{\sigma^2+3\tau^2} \le S_{yt}}$$

23/1/17

Q :- Principle Stresses at a pt in a elastic material are 100 M.Pa. tensile, 50 MPa comp., 50 MPa tensile. Determine the FOS against all the five T.O.F. The elastic limit in simple tension 250 MPa $\mu = 0.3$

Q→ A bolt is acted upon by an axial pull of 18 KN along with transverse shear force of 10 KN. The determine the diameter of the bolt required acc. to the all five theories. The elastic limit of bolt material is 250 M.Pa, FOS = 2.5, $\mu = 0.3$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}$$

$$\tau = \frac{P_1}{A}$$

$$\sigma_x = \frac{P_2}{A}$$

$$\sigma_1 \le \frac{S_{yt}}{N}$$

$$\frac{\sigma_1-\sigma_2}{2} \le \frac{S_{yt}}{2N}$$

$\sigma_y = 0$

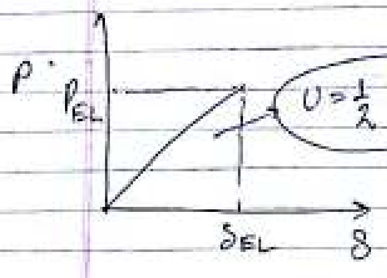
$\sigma_1 =$

Strain Energy (U)

It is defined as the energy by a member when work done by the load deforms that member.

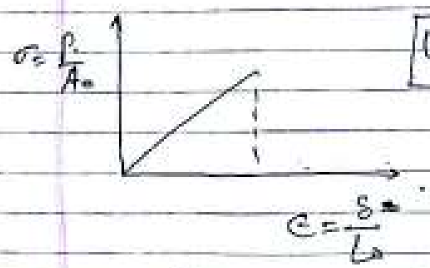
Resilience:— It is energy absorbed by the member within the elastic limit.

Toughness:— It is the energy absorbed the member before its fracture.



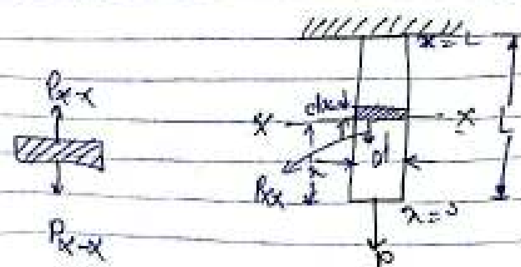
$U = \frac{1}{2} P_{EL} \times \delta_{EL}$

Proof of Resilience



$$\boxed{U = \frac{1}{2} \sigma_{EL} \times \epsilon_{EL}}$$

Modulus of resilience



$dU = \frac{1}{2} P_{x-x} \times \delta_{x-x}$

$= \frac{1}{2} P_{xx} \left(\frac{P_{xx} dx}{A_{x-x} E} \right)$

$= \frac{1}{2} \frac{(P_{x-x})^2 dx}{A_{xx} \cdot E}$ ---- (1)

$= \frac{1}{2} \left(\frac{P_{xx}}{A_{xx}} \right)^2 \cdot \frac{A_{xx} \cdot dx}{E}$

$= \frac{1}{2} \frac{\sigma_{xx}^2}{E} \cdot A_{xx} \cdot dx$

$= \frac{1}{2} \times \sigma_{xx} \times \frac{\sigma_{xx}}{E} \cdot A_{xx} \cdot dx$

$dU = \frac{1}{2} \times \sigma_{xx} \times \epsilon \cdot A_{xx} \cdot dx$ ---- (2)

When, $x = l$

$dU = \frac{1}{2} \sigma \times \epsilon \times A \times L$

$\frac{dU}{Vol.} = \frac{1}{2} \cdot \sigma \epsilon$

$\therefore \boxed{\frac{\text{Proof of Resilience}}{\text{Volume}} = \text{Modulus of Resilience.}}$

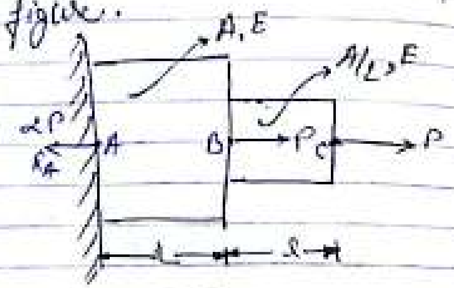
* From equation (1). If we have to calculate the strain energy in the above bar then—

$$U = \int_0^l \frac{(P_{xx})^2 dx}{2 \cdot A_{xx} \cdot E}$$

So, Generalised equation —

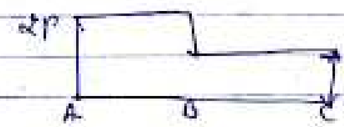
$$\boxed{U = \int_a^b \frac{(P_{xx})^2 dx}{2 \cdot A_{xx} \cdot E}}$$

Ques → Determine the strain energy in the given stepped bar due to the axial load as shown in the figure.



$$U_1 = \int_a^b \frac{(P_{x})^2\,dx}{2 \cdot A_x \cdot E}$$

⇒ $\boxed{R_A = 2P}$



$$U_1 = \int_0^l \frac{(2P)^2\,dx}{2x \cdot A \times E}$$

$$= \int_0^l \frac{4P^2 \times dx}{2AE} = \left(\frac{2 \times P^2 \times x}{AE}\right)_0^l = \frac{2 \times P^2 \times l}{AE} = \frac{2P^2 l}{AE}$$

$$U_2 = \int_0^l \frac{P^2 \times dx}{2 \cdot A/2 \cdot E} = \int_0^l \frac{P^2\,dx}{AE} = \left(\frac{P^2 \cdot x}{AE}\right)_0^l$$

$$\boxed{U_2 = \frac{P^2 \times l}{AE}}$$

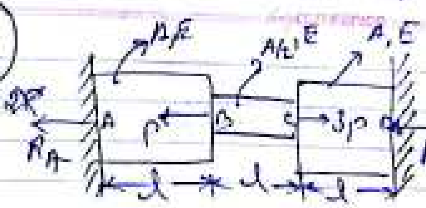
$$\boxed{U = U_1 + U_2 = \frac{3P^2 l}{AE}}$$

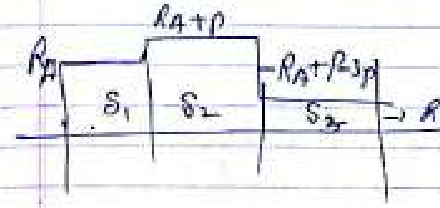
$U = \int_0^l$

when (A, E)

$$R_D = \frac{-P(l) + 3P(2l)}{3l}$$

$$R_D = \frac{5P}{3}$$





$$\boxed{R_A = \frac{-P \times l + 3Pl}{3l}}$$

$$R_A = P/3$$

$$\delta_1 + \delta_2 + \delta_3 = \frac{R_A \times l}{AE} + \frac{(R_A + P)(l)}{AE} + \frac{(R_A + P - 3P)l}{AE}$$

$$0 = R_A l + R_A l + Pl + R_A l + Pl - 3Pl$$

$$0 = 3R_A l + 2Pl - 3Pl$$

$$0 = 3R_A l - Pl$$

$$3R_A l = Pl$$

$$R_A = \frac{Pl}{3l}$$

$$\boxed{R_A = 0}$$

$$0 = \frac{R_A(l)}{AE} + \frac{(R_A + P)(l)}{AE} + \frac{(R_A + P - 3P)(l)}{AE}$$

$$0 = (R_A)(l) + (R_A)(l) + (P)(l) + (R_A)(l) + (P)(l) - (3P)(l)$$

$$0 = 3R_A l - Pl$$

$$3R_A l = Pl$$

$$R_A = \frac{Pl}{3l}$$

$$\boxed{R_A = P/3}$$

$$U_1 = \int_0^l \frac{(P/3)^2\,dx}{2 \cdot A \times E}$$

$$= \frac{(P/3)^2}{2AE} \int_0^l dx$$

$$= \left(\frac{P^2\,x}{9 \times 2 \times A \times E}\right)_0^l = \frac{P^2 l}{18AE}$$

$$U_2 = \int_0^l \frac{(P/3+P)^2 dx}{2A_{xx} \cdot E} = \int_0^l \frac{(4P/3)^2 dx}{2 \cdot A_{x-x} \cdot E}$$

$$= \frac{16P^2}{18 A/2 \cdot E}\int_0^l dx = \frac{16 \cdot 2P^2}{9 \cdot 18 A \cdot E}\left[x\right]_0^l$$

$$= \frac{16P^2 l}{9AE}$$

$$U_3 = \int_0^l \frac{(P/3+P-3P)^2 dx}{2 \cdot A \cdot E} = \int_0^l \frac{(-5P/3)^2 dx}{2 \cdot A \cdot E}$$

$$= \frac{25P^2}{18AE}\int_0^l dx = \frac{25P^2}{18AE}\left[x\right]_0^l$$

$$= \frac{25P^2 l}{18AE}$$

$$U_1 + U_2 + U_3 = \frac{P^2 l}{18AE} + \frac{16P^2 l}{9AE} + \frac{25P^2 l}{18AE}$$

$$= \frac{P^2 l}{AE}\left(\frac{1}{18} + \frac{16}{9} + \frac{25}{18}\right)$$

$$= \frac{29P^2 l}{9AE}$$

Ques → Principle stress at a pt in a elastic material are 100 M.Pa. tensile, 25 M.Pa compressive and 50 M.Pa tensile. ~~Detete~~ Determine the F.O.S (N) against all the five T.O.F.. The elastic limit in simple tension is 220 M.Pa. and $\mu = 0.3$

Solution → $\sigma_1 = 100$ M.Pa, $\sigma_2 = -25$ M.Pa, $\sigma_3 = 50$ M.Pa.

$(S_{yt})_t = 220$ M.Pa., $\mu = 0.3$

① Max. principle ~~shear~~ stress theory –

$$\sigma_1 = \frac{S_{yt}}{N} \Rightarrow 100\text{ M.Pa} = \frac{220\text{ M.Pa}}{N}$$

$$N = \frac{220\text{ M.Pa}}{100\text{ M.Pa}}$$

$$\boxed{N = 2.20}$$

② Max. Shear stress theory –

If like in nature –

$$\sigma_1 \le \frac{S_{yt}}{2N}$$

$$N \le \frac{S_{yt}}{2\sigma_1} = \frac{220}{2 \times 100}$$

$$\boxed{N \le 1.1}$$

If unlike in nature –

$$\frac{\sigma_1 - \sigma_2}{2} \le \frac{S_{yt}}{N}$$

$$\frac{100-(-25)}{2} \le \frac{220}{N}$$

$$62.5 \le \frac{220}{N}$$

$$\boxed{N = 3.52}$$

③ Max. principle strain energy theory –

$$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1\sigma_2) \le \frac{S_{yt}^2}{N^2}$$

$$(100)^2 + (-25) - 2(0.3)\left(100(-25)\right) \le \frac{220^2}{N^2}$$

$$N^2 = \frac{48400}{12\cancel{2}25}$$

$\boxed{N = 1.997} \simeq 2$ $\boxed{\cancel{N = 20.05}}$ $\boxed{\cancel{N = 3.99}}$

$$\frac{1}{2E}\left\{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right\} \le \frac{\sigma_{yt}^2}{N^2} \times \frac{1}{2E}$$

$$(100)^2 + (25)^2 + (50)^2 - 2(0.3)\left(100(-25) + (-25)(50) + (50)(100)\right) \le \frac{220^2}{N^2}$$

$$12375 \le \frac{48400}{N^2}$$

$\boxed{N = 1.97} \cong 2$

④ Max. principle strain theory :-

$$\sigma_1 - \mu(\sigma_2) \le \frac{\sigma_{yt}}{N}$$

$$100 - 0.3(-25) \le \frac{220}{N}$$

$\boxed{N = 2.04}$

⑤ Max. distortion Energy Theory :-

$$\left(\frac{1+\mu}{3E}\right)\left\{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right\} \le \left(\frac{\sigma_{yt}}{N}\right)^2 \times \frac{1+\mu}{3E}$$

$$\frac{1}{2}\left\{(100+25)^2 + (-25+50)^2 + (50-100)^2\right\} \le \left(\frac{220}{N}\right)^2$$

$$\frac{1}{2}\left\{15625 + 625 + 2500\right\} \le \frac{48400}{N^2}$$

$\boxed{N = 2.27}$

Solution → ~~Given — $P = 24$ kN $= 24\times10^3$ N, $S = 12$ kN $= 12\times10^3$ N, $\sigma_1 = 300$ MPa, $\mu = 0.25$, $N = 0.3$~~

Given –

$P = 16$ kN $= 16\times10^3$ N, $S = 10$ kN $= 10\times10^3$ N

$\sigma_2 = 250$ MPa

$\mu = 0.3$

FOS $= 2.5$

Sol. $P_1 = 16\times10^3$ N, $P_2 = 10\times10^3$ N

$$\tau = \frac{P_1}{A} = \frac{16\times10^3 N}{\frac{\pi}{4}d^2} = \frac{64\times10^3 N}{\pi d^2}$$

$$\sigma_x = \frac{P_2}{A} = \frac{10\times10^3}{\frac{\pi}{4}d^3} = \frac{40\times10^3 N}{\pi d^3}$$

$\boxed{\sigma_y = 0}$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{1}{2}\left(\frac{40\times10^3 N}{\pi d^3}\right) + \sqrt{\frac{1}{4}\left(\frac{40\times10^3 N}{\pi d^3}\right)^2 + \left(\frac{64\times10^3 N}{\pi d^3}\right)^2} \quad [\sigma_y = 0]$$

$$\sigma_1 = \frac{20\times10^3 N}{\pi d^3} + \sqrt{\frac{1}{4}(10\times$$

$$\sigma_1 = \frac{40\times10^3 N}{\pi d^3}\left\{\frac{1}{2} + \sqrt{\frac{1}{4}(1) + \left(\frac{64}{40}\right)^2}\right\}$$

$$\sigma_1 = \frac{40\times10^3}{\pi d^3}\left\{2.2\right\}$$

$$\boxed{\sigma_1 = \frac{87.1\times10^3}{\pi d^3} N/m}$$

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$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}$

$= \frac{1}{2}\left(\frac{40\times10^3 N}{\pi d^2}\right) - \sqrt{\frac{1}{4}\left(\frac{40\times10^3 N}{\pi d^2}\right)+\left(\frac{64\times10^3}{\pi d^2}\right)^2}$

$= \frac{40\times10^3 N}{\pi d^2}\left\{\frac{1}{2} - \sqrt{\frac{1}{4}+\left(\frac{64}{40}\right)^2}\right\}$

$\boxed{\sigma_2 = -\frac{47.1\times10^3 N}{\pi d^2}}$

(i) Max. principle stress theory —

$\sigma_1 \le \sigma_{yp}$ (277.24)

$\sigma_1 \le \frac{S_{yt}}{N}$

$\frac{87.1\times10^3 N}{\pi d^2} = \frac{250 \text{ M.Pa}}{2.5}$

$\frac{2.5\times87.1\times10^3}{\pi\times250} = d^2$

$\boxed{d = 16.51}$ mm

(ii) Max. shear stress theory :—

$\tau_{max.} = \frac{\sigma_1-\sigma_2}{2}$

$\tau_{max.} \le \sigma_{yp.}$

$\frac{\sigma_1-\sigma_2}{\cancel{2}} \le \frac{S_{yt}}{\cancel{2}N}$

$\frac{87.1\times10^3 N}{\pi d^2} - \left(-\frac{47.1\times10^3 N}{\pi d^2}\right) = \frac{250 \text{ MPa}}{2.5}$

$\frac{1}{d^2}(42717.2) = 100$

$d^2 = \frac{42717.2}{100}$

$\boxed{d = 20.67 \text{ mm}}$

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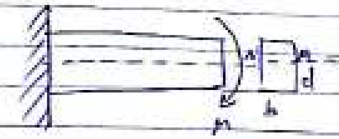
21/1/17

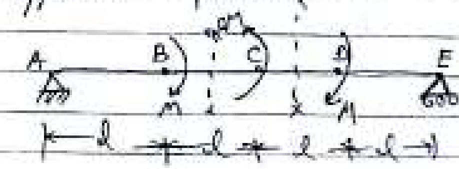
Strain Energy In Prismatic Beam Under pure bending

$U = \int_a^b \frac{(M_{x-x})^2\,dx}{2.E.I_{N-A}}$



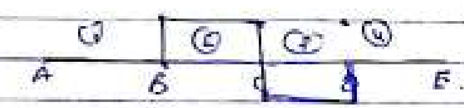
Ques→ Find the strain energy of a simply supported beam under the applied forces.

Ques→ $U_1 = \int_a^b \frac{(M_{xx})^2\,dx}{2.E.I_{N.A}}$



$\boxed{U_1 = 0}$

$U_2 = \int_l^{2l} \frac{M^2\,dx}{2.E.I_{NA}}$

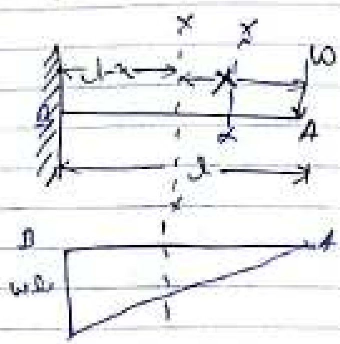


$= \frac{M^2}{2.E.I_{NA}}\int_l^{2l} dx = \frac{Ml^2}{2.E\,I_{NA}}$

$U_3 = \int_{2l}^{2l}$

Que → Determine the strain energy at a point A, x, B ? →

$$\cancel{U = \int_a^b \frac{(P_{x-x})^2 \, dy}{2 \cdot A_{x-x}/E}}$$

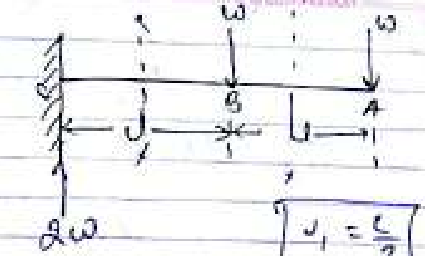


$$\cancel{U_A = \int_0^x \frac{(\omega)^2 \, dx}{2 \cdot A_{x-x} \cdot C}}$$

$$= \frac{\omega^2}{2A_{x-x}E} \left\{ x \right\}_0^x$$

$$U = \int_a^b \frac{(M_{x-x})^2 \, dx}{2 \cdot A_{x-x} \cdot I_{NA}}$$

Determine strain energy



$$U = \int_0^l \frac{(\omega x)^2 \, dx}{2 \cdot A_{x-x} I_{NA}}$$

$$= \frac{\omega^2}{2A_{x-x} \cdot I_{NA}} \int_0^l x^2 \, dx$$

$$U_1 = \frac{\omega^2}{2 \cdot A_{x-x} \cdot I_{NA}} \left[\frac{x^3}{3} \right]_0^l = \frac{\omega^2 l^3}{6 A_{x-x} \cdot I_{NA}}$$

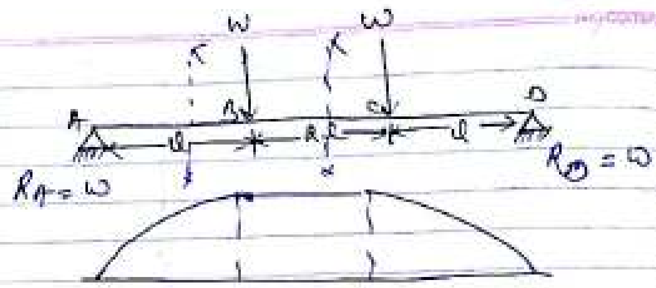
$$\boxed{U_1 = \frac{1}{3}}$$

$$\boxed{U_2 = \frac{1}{6}}$$

$$U_2 = \int_0^{2l} \frac{(\omega x + \omega(x-l))^2 \, dx}{2 \cdot A_{x-x} \cdot I_{NA}} =$$

$$\boxed{U_T = \frac{5}{6} \frac{\omega^2 l^3}{E_1}}$$

Ques →



$$U_1 = \int_0^l \frac{(Wx)^2\,dx}{2\,A\,x\,I_{NA}} = \frac{W^2}{2\,A\,x\,I_{NA}} \int_0^l x^2\,dx$$

$$= \frac{W^2}{2\,A\,x\,I_{NA}} \left[\frac{x^3}{3}\right]_0^l$$

$$\boxed{U_1 = \frac{W^2 l^3}{6\,A\,I}}$$

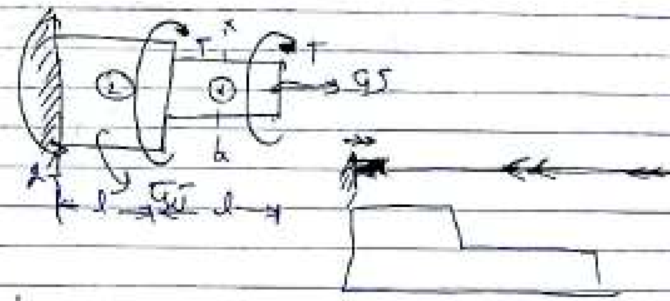
$$U_2 = \int_0^{2l} \frac{(-W(l+x) + Wx)^2\,dx}{2.A.I.} = \int_0^{2l} W^2(1+x+x)^2$$

$$\boxed{U_T = 2\int_0^l \frac{(-Wx)^2\,dx}{2EI} + \frac{M^2 L}{2EI}}$$

Strain Energy due to Torsion

$$U = \int_a^b \frac{(T_x)^2\,dx}{2.GJ}$$

Ques →



$$U_1 = \int_0^l \frac{(T_x)^2\,dx}{2\,G.J}$$

$$= \frac{T^2}{2GJ} \int_0^l dx = \frac{T^2}{2GJ} \left[x\right]_0^l = \frac{T^2 l}{2GJ}$$

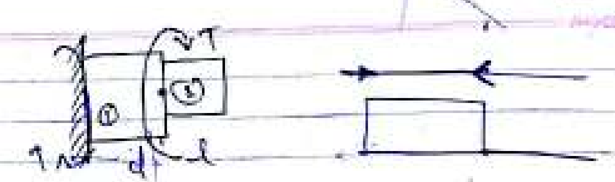
$$U_2 = \int_0^{2l} \frac{(2T)^2\,dx}{2\,GJ} = \frac{4T^2}{2GJ} \int_0^{2l} dx$$

$$= \frac{4T^2}{2GJ} \left(x\right)_0^{2l}$$

$$= \frac{4T^2 \times \cancel{2}l}{\cancel{2}\,GJ} = \frac{4T^2 l}{GJ}$$

$$\boxed{U_{Total} = \frac{5T^2 l}{2GJ}}$$

Ques.→



$$U = \int_0^l \frac{(T \cdot l)^2\,dx}{2GJ} = \frac{T^2}{2GJ}\int_0^l dx$$

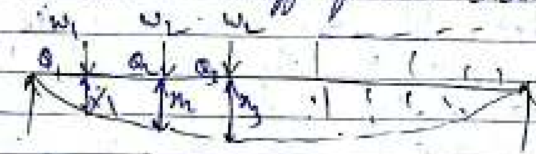
$$\boxed{U_T = \frac{T^2 l}{2GJ}}$$

27/11/2017 Castigliano's 1st Theorem (Deflection from strain energy)

If a structure is subjected to a no. of external loads (or couple) the partial derivative of the total strain energy with respect to any load or couple provides the deflection in the direction of that load (or couple)

$U_1 \neq \frac{1}{2}$

Let U = total strain energy of the structure



$U = \frac{1}{2} W\delta$ Let → W_1, W_2, W_3 ----

external load of the pt Q_1, Q_2, Q_3 ---

⇒ M_1, M_2, M_3 ---- external couples at the same pt.

⇒ Then acc. to this theorem, ~~deflection~~ Deflection at respective pt. and directions are.

$$\frac{\partial U}{\partial W_1}, \frac{\partial U}{\partial W_2}, \frac{\partial U}{\partial W_3}, \ldots$$

⇒ At angular rotation of the couple at the applied pts. are previous θ.

$$\frac{\partial U}{\partial M_1} \to \theta_1, \frac{\partial U}{\partial M_2} \to \theta_2, \frac{\partial U}{\partial M_3} \to \theta_3 - - -$$

★ (δ) sudden loading = 2(δ) static, gradual loading

★ (σ) sudden = 2(σ) gradual

Proof :-
Let x_1, x_2, x_3 ---- be the displacement in the direction of gradually applied load.

Then, $U_1 = \frac{1}{2}w_1x_1 + \frac{1}{2}w_2x_2 + \frac{1}{2}w_3x_3 + \dots$ ---- (A)

Let W_1 be increased to $W_1 + \delta w_1$ & due to the increased load δw_1, let $\delta x_1, \delta x_2, \delta x_3$ ---- be the simp. increments in x_1, x_2, x_3 ----

Then increased in external work done,

$$\delta U = \frac{1}{2}w_1\delta x_1 + \frac{1}{2}w_1\delta x_1$$

$$= \frac{1}{2}(w_1+\delta w_1)\delta x_1 + \frac{1}{2}w_2\delta x_2 + \frac{1}{2}w_3\delta x_3 \dots$$

All loads ~~ex~~ except δw_1 exist already and thus treated as suddenly applied load. Where as δw_1 is gradually applied.

$$\delta U = w_1\delta x_1 + \frac{1}{2}\delta w_1\delta x_1 + w_2\delta x_2 + w_3\delta x_3 \dots \text{ (1)}$$

Let the term $\frac{1}{2}\delta w_1\delta x_1$ be the very small so, we are going to neglect it.

$$\delta U = w_1\delta x_1 + w_2\delta x_2 + w_3\delta x_3 \dots \text{ (2)}$$

However, if loads $w_1+\delta w_1$, w_2, w_3 ---- ~~at~~ the had been applied gradually from zero,

Then, total Strain energy = $U + \delta U$

$$T.S.E = \frac{1}{2}(w_1+\delta w_1)(x_1+\delta x_1) + \frac{1}{2}w_2(x_2+\delta x_2) + \frac{1}{2}w_3(x_3+\delta x_3) + \dots \text{ (3)}$$

(3) − (A)

~~$T.S.E - U_1 = T.S.E = \frac{1}{2}w_1x_1 + \frac{1}{2}w$~~

$$T.S.E = \left(\frac{1}{2}w_1 + \frac{1}{2}\delta w_1\right)(x_1+\delta x_1) +$$

$$= \frac{1}{2}wx_1 + \frac{1}{2}w\delta x_1 + \frac{1}{2}\delta w_1 + \frac{1}{2}\delta w_1\delta x_1 + \frac{1}{2}w_2(x_2) + \frac{1}{2}w_2\delta x_2 + \frac{1}{2}w_3x_3 + \frac{1}{2}w_3\delta x_3$$

$$= \frac{1}{2}wx_1 + \frac{1}{2}w\delta x_1 + \frac{1}{2}$$

$\Rightarrow [(U+\delta U) - U]$

$$\delta U = \frac{1}{2}(w_1\delta x_1 + \delta w_1x_1) + \frac{1}{2}(w_2\delta x_2) + \frac{1}{2}(w_3\delta x_3) \dots \text{ (4)}$$

$$2\delta U = w_1\delta x_1 + \delta w_1x_1 + w_2\delta x_2 + w_3\delta x_3$$

Subtracting $\boxed{4-2}$

$$2\delta U - \delta U = \delta w_1$$

$$\boxed{\frac{\partial U}{\partial w_1} = x_1}$$

Similarly, we increase $w_2 \to w_2 + \delta w_2$

$$\boxed{\frac{\partial U}{\partial w_2} = x_2}$$

$w_3 \to w_3 + \delta w_2$

$$\boxed{\frac{\partial U}{\partial w_3} = x_3}$$

In case of bending beam $U = \int_0^l \frac{M^2 dx}{2EI}$

Then, deflection –

$$\delta_i = \frac{\partial U}{\partial w_i}$$

and

$$\theta_i = \frac{\partial U}{\partial M_i}$$

① Que → Write a short note on the following topic
a) ~~Sir~~ St. Venant's principle
b) Rebe Recealiers
c) Toughness

② Q → From the stress tensor at a pt shown below determine the shear stress acting on xz-plane

and yz-plane along z-direction. ⑧

$$\sigma = \begin{bmatrix} 5 & 10 & 5 \\ 10 & 20 & 15 \\ 5 & 15 & 0 \end{bmatrix}$$

③ Q → Calculate the strain energy stored in a bar 200 cm long, 5 cm wide, & 4 cm thick, when it subjected to a tensile load of 6 tonne, Take $E = 2.0 \times 10^6$ kg/cm² (6.5)

④ Q → Determine the thickness of a thin ~~strain~~ cylindrical pressure vessel of dia. 500 mm using Max. shear strain energy theory if permissible stress is 100 MPa. & $\mu = 0.3$ (4.5)

⑤ Q → Derive castigliano's first theorem for statistically determinate structure. Also find the displacement & slope ~~at~~ at tip of a cantilever beam loaded at the free end. Assume the flexural rigidity of the beam EI to be const. for the beam. ⑧

Solution: 4)

By Max. shear strain Energy –

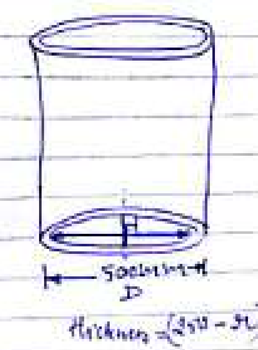
$\sigma_{per.} = 100$ MPa

$= 100 \times 10^6$ Pa

$\mu = 0.3$

diameter = 500 mm

$\tau =$



1 tonne = 1000 kg

σ_t = 6 tonnes = 6000 kg

E = 2×10^6 kg/cm^2

$$U=\int_a^b \frac{(P_x x)^2\,dx}{2\cdot A x x\cdot E} = \int_0^{200} \frac{(6000\,kg)^2\,dx}{2\cdot(200\times3640\times2\times10^6)} \frac{cm^2}{kg}$$

$$= \frac{600}{2\times3640\times2\times10^6}(200)\ kg\,cm^2$$

$$\boxed{U = 8.3\times10^{-6}\ kg\,cm^2}$$

(2) $\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{z} \end{bmatrix}$

Given ⇒ $\sigma \begin{bmatrix} 5 & 10 & 5 \\ 10 & 20 & 15 \\ 5 & 15 & 0 \end{bmatrix}$

Ques: find the deflection at the free end of a cantilever which carries a pt. load at free end.

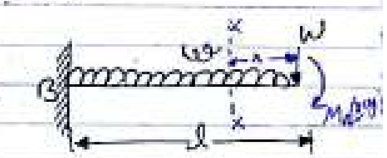
$$U=\int_0^l \frac{(Wx)^2\,dx}{2EI} = \frac{W^2}{2EI}\int_0^l x^2\,dx$$

$$= \frac{W^2 l^3}{6EI}$$

$$\frac{dU}{dW} = \frac{2Wl^3}{6EI} = \frac{Wl^3}{3EI}$$

$\frac{dU}{dx}$

Ques: find the deflection at the free ends of a cantilever which carries a uniformly distributed load & pt load on free end.



$$\left[M_{xx} = \frac{-wx^2}{2} - Wx\right]$$

$$U = \int_a^b \frac{(M_{xx})^2\,dx}{2EI}$$

$$\frac{\partial U}{\partial W} = \int_a^b \frac{M}{EI}\frac{\partial M}{\partial W}\,dx = \delta_A$$

$$\delta_A = \frac{\partial U}{\partial W_A}$$

$=$

W=0

$$\delta_A = \frac{\partial U}{\partial W} = \int_0^l \frac{M}{EI}(-x)\,dx$$

$\frac{wx^2}{2}$

$\frac{wx^2+2Wx}{2}$

$$= \frac{1}{EI}\int_0^l \left(\frac{-wx^2}{2} - Wx\right)(-x)\,dx$$

$$= \frac{1}{EI}\left(-\frac{wx^2}{2} - Wx\right)_0^l (-x) = \frac{1}{EI}\Big($$

Now assuming dummy Moment

$\therefore M_{xx} = -\frac{wx^2}{2} - Wx - M_A$

$$\theta_A = \frac{\partial U}{\partial M_A} = \int_0^l \frac{M}{EI}\left(\frac{\partial M_x}{\partial M_A}\right)dx$$

$$= \int_0^l \frac{M}{EI}\frac{\partial}{\partial M_A}\left(\frac{-wx^2}{2} - Wx - M_A\right)dx$$

$$= \int_0^l \frac{M}{EI} \times \left(\frac{-wx^2}{2} - Wx - 1\right)dx$$

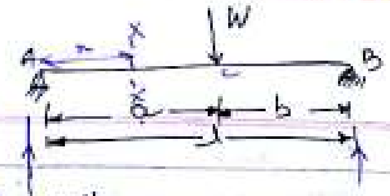
$$= \frac{M}{EI}\int_0^l \left(\frac{-wx^2}{2} - Wx - 1\right)dx$$

$$\theta = \frac{M}{EI}\Big($$

$$\theta = \int_0^l \left(\frac{wx^3 - Wx - M_A}{EI}\right)^0 (-1)\,dx$$

$$= \frac{1}{EI}\left(\frac{w x^3}{3} - \frac{Wx^2}{2}\right)_0^l (-1)$$

A Simply supported beam (AB) of length (L) carrying a concentrated load (W) at a distance (a) from end (A) & (b) from (B). Deduce expression for deflection at under the load. Also find the deflection if the load is at mid ~~stand~~ span.



$R_A = \frac{Wb}{l}$ $\qquad R_B = \frac{Wa}{l}$

$$~~M_{xx} = Wa - \frac{Wa}{l}x\,l - \frac{Wa}{l}(a+b)~~$$

(A) $M_x = \frac{Wbx}{l}$

(B) $M = \frac{Wax}{l}$

$$\delta_c = \int_0^a \frac{\left(\frac{Wbx}{l}\right)^2 dx}{2EI} + \int_0^b \frac{\left(\frac{Wax}{l}\right)^2 dx}{2EI}$$

$$= \int_0^a \frac{(Wbx)^2 dx}{l^2 EI} + \int_0^b \frac{(Wax)^2 dx}{l^2\, 2EI}$$

$$= \frac{W^2b^2}{l^2 2EI}\left(\frac{x^3}{3}\right)_0^a + \frac{W^2a^2}{l^2(2EI)}\left(\frac{x^3}{3}\right)_0^b$$

$$= \frac{W^2b^2}{l^2 2EI}\left(\frac{a^3}{3}\right) + \frac{W^2a^2}{2l^2EI}\left(\frac{b^3}{3}\right)$$

$$~~= \int_0^a \frac{M_A}{EI}\frac{bx}{L}dx + \int_0^b \frac{M_B}{EI}\frac{ax}{L}dx~~$$

$$= \frac{W^2a^3b^2}{6l^2EI} + \frac{W^2a^2b^3}{6l^2EI}$$

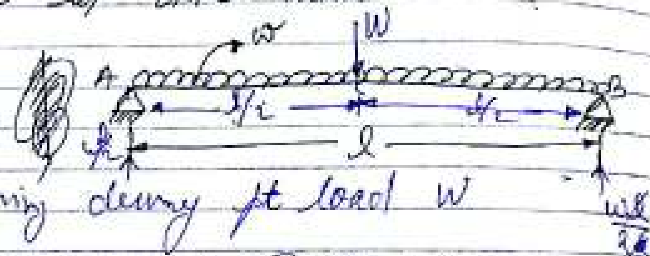
$$\delta_c = \frac{W^2}{6l^2EI}\left(a^3b^2 + a^2b^3\right)$$

When $a = b = l/2$

$$\delta_c = \frac{W^2}{6l^2EI}\left(\left(\frac{l}{2}\right)^3\left(\frac{l}{2}\right)^2 + \left(\frac{l}{2}\right)^2\left(\frac{l}{2}\right)^3\right)$$

$$= \frac{W^2}{6l^2EI}$$

Ques-> Determine the max. deflection of a simply supported beam of span (L) carrying a load of (w) per unit length using strain energy method also find the slope at the ends.



=> Let assuming dummy pt load W

$M_A = \frac{wl}{2}(x)$, $M_B = \frac{wl}{2}(x)$

$\delta_c = \int_0^{l/2}$

$R_A \times l - wl \times \frac{l}{2} - \frac{wl}{2} = 0$

$\boxed{R_A = \frac{w + wl}{2}}$

$R_A + R_B = w + wl$

$\frac{w+wl}{2} + R_B = w + wl$

$R_B = (w + wl) - \left(\frac{w+wl}{2}\right)$

$\boxed{R_B = \frac{w+wl}{2}}$

$M_A = \frac{w+wl}{2}x - \frac{wx^2}{2}$

$U = \int_0^l \frac{\left(\left(\frac{w+wl}{2}(x)\right)^2 - \frac{wx^2}{2}\right)^2}{2EI} dx$

$\delta_c = \int_0^{l/2} \frac{M_A}{EI} \frac{x}{2} dx$

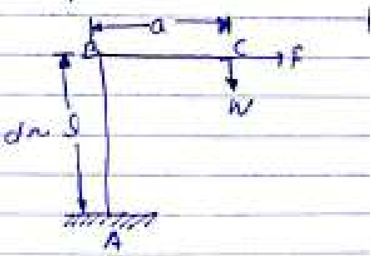
$= \int_0^{l/2} (wx + wl\ldots$

$= \int_0^{l/2} \frac{(wx + wlx - wx^2)^2 \frac{x}{2} dx}{EI}$

$= \int_0^{l/2} \left(\frac{wx^2 + w^2 l^2 x^2 - w^2 x^4}{EI}\right) \frac{x}{2} dx$

$= \frac{1}{2EI} \int_0^{l/2}$

Ques-> ~~A right circular bar~~ A rigid cantilever ~~frame~~ beam as shown in the figure carries a load W at the load. Assuming a const. value of EI, determine the vertical & horizontal displacement at the free end C.

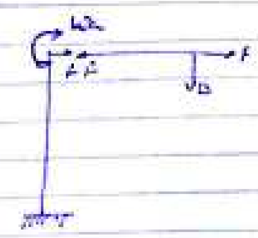


$\delta = \int_0^a \frac{(-Wx)}{EI}(-x)\,dx + \int_0^l \frac{(-Wa)}{EI}(-a)\,dx$

$= \int_0^a \frac{Wx^2}{EI} dx + \int_0^l \frac{Wa^2}{EI} dx$

$= \frac{W}{EI}\left(\frac{x^3}{3}\right)_0^a + \frac{Wa^2}{EI}(l)$

$\delta_c = \frac{Wa^3}{3EI} + \frac{Wa^2 l}{EI}$



$\int \frac{-lcWl}{D}$

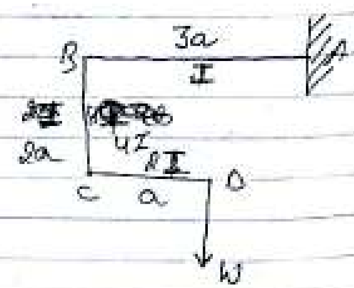
$$\delta = \int_0^a \frac{(-wx)(x)}{EI}\,dx + \int_a^l \frac{(-wa - Fx)}{EI}(-x)\,dx$$

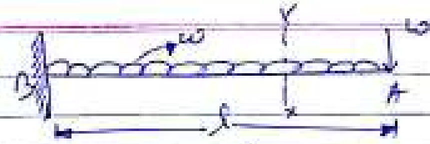
$$= \int_0^a \frac{wx^2}{EI}\,dx + \int_0^l \cancel{\frac{wax + Fx^2}{EI}}$$

$$\delta = \int_0^l \frac{(-wa - Fx)}{EI}(-x)\,dx$$

$$= \int_0^l$$

Ques) A bent cantilever frame as shown in figure carries a load W at the free end D. Determine the ~~and~~ vertical displacement of the free end. In the figure ~~small~~ a and I denote the length & moment of inertia respectively





$$M_{x-x} = \frac{wx^2}{2} + Wx$$

$$U = \int_a^b \frac{M^2 dx}{2EI}$$

$$\delta_A = \frac{\partial U}{\partial W_A}$$

$$U = \int_0^l \frac{\left(\frac{wx^2}{2} + Wx\right)^2}{2EI}\,dx$$

$$= \frac{1}{2EI}\int_0^l \left(\frac{wx^2}{2} + Wx\right)^2 dx$$

$$= \frac{1}{2EI}\left[\frac{\left(\frac{wx^2}{2} + Wx\right)^3}{3}\right.$$

$$U = \int_0^l \frac{\frac{wx^2}{2} + Wx}{2EI}\,dx$$

$$+ \frac{wx^3}{6} + \frac{Wx^2}{2}\Big]_0^l$$

$$= \frac{1}{2EI}\left(\frac{wx^3}{6} + \frac{Wx^2}{2}\right)_0^l \qquad U = \frac{1}{2EI}\Big[$$

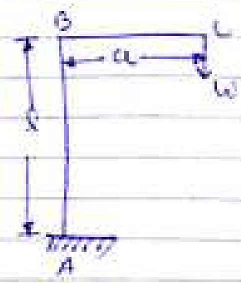
$$U = \frac{1}{2EI}\left(\frac{wl^3}{6} + \frac{Wl^2}{2}\right) \qquad \frac{d}{dW}\left(\frac{wx^2}{2} + Wx\right)$$

$$= \frac{x^2}{2} + x$$

$$= \frac{x^2 + 2x}{2}$$

$$\delta_A = \frac{\partial U}{\partial W_A} =$$

Ques ⇒ A bent cantilever frame as shown in figure. Carries a load 'w' at free end 'c' find the vertical displacement of the free end.

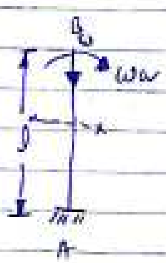


$M_{xx} = wx$

$\delta_{y} = \frac{\partial U}{\partial w} = \int_0^a \frac{M}{EI}\frac{\partial M}{\partial w}\,dx$

$= \int_0^a \frac{(wx)}{EI}\cdot(x)\,dx = \int_0^a \frac{wx^2}{EI}dx = \frac{w}{EI}\left[\frac{x^3}{3}\right]_0^a$

$\boxed{\delta_1 = \frac{wa^3}{3EI}}$



$M_{x-x} = wa$

$\delta_2 = \int_0^l \frac{(wa)}{EI}\frac{\partial}{\partial w}(wa)\,dx$

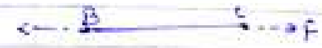
$= \int_0^l \frac{wa}{EI}\times a\,dx = \int_0^l \frac{wa^2}{EI}dx$

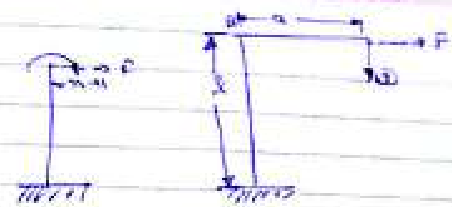
$\cancel{\delta_f = \frac{w}{EI}\left[\frac{a^3}{3}\right]}$ $= \frac{wa^2}{EI}\int_0^l 1.dx$

$\boxed{\delta_2 = \frac{wa^2 l}{EI}}$

$\delta_T = \delta_1 + \delta_2 = \frac{wa^3}{3EI} + \frac{wa^2 l}{EI} = \frac{wa^2}{EI}\left(\frac{a}{3} + l\right)$

for horizontal distribution





$M_{xx} = wa + Fx$

$\frac{\partial M_{xx}}{\partial w} = x$

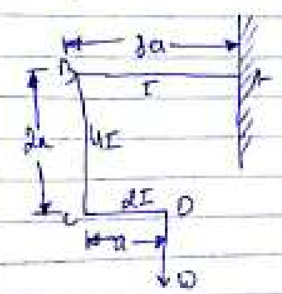
$\delta = \int_0^l \frac{M}{EI}\frac{\partial M}{\partial w}dx = \int_0^l \frac{(wa + Fx)}{EI}\cdot x\,dx$

$\delta = \frac{1}{EI}\int_0^l (wax + Fx^2)dx = \frac{1}{EI}\left[\frac{wax^2}{2} + \frac{Fx^3}{3}\right]_0^l$

$\delta = \frac{1}{EI}\left[\frac{wal^2}{2} - \frac{Fl^3}{3}\right]^0 = \frac{wal^2}{2EI}$

$\boxed{\delta = \frac{wal^2}{2EI}}$

Ques ⇒ A bent cantilever frame as shown in the figure. consider a load 'w' at the free end 'D'. find the vertical displacement of the free end. In this fig. 'a' & E. denote length & moment of inertia respectively.



$M_{x-x} = wx$

$\frac{\partial M_{x-x}}{\partial W} = x$

$\delta_1 = \int_0^a \frac{M}{EI}\frac{\partial M}{\partial W}dx = \int_0^a \frac{Wx}{EI} \cdot x\, dx$

$= \frac{W}{EI}\int_0^a x^2 dx = \frac{W}{EI}\left[\frac{x^3}{3}\right]_0^a$; $\boxed{I = 2I}$

$\boxed{\delta_1 = \frac{Wa^3}{6EI}}$

$M_{x-x} = Wa$ & $\frac{\partial M_{x-x}}{\partial W} = a$

$\delta_2 = \int_0^{2a} \frac{(Wa)}{E \times I} \times (a)\, dx$

$\delta_2 = \frac{Wa^2}{4EI}\int_0^{2a} 1\, dx \Rightarrow \frac{Wa^2}{4EI}\left[x\right]$

$\delta_2 = \frac{Wa^2}{4EI} \times 2a \Rightarrow \boxed{\delta_2 = \frac{Wa^3}{2EI}}$

$M_{x-x} = \underbrace{Wa}_{const.} + Wx$

$\frac{\partial M_{x-x}}{\partial W} = x$

$\delta_3 = \int_0^{3a} \frac{-(Wa+Wx)}{EI} \times x\, dx$

$= \int_0^{3a} \frac{-W(ax+x^2)}{EI} dx$

$= \frac{W}{EI}\int_0^{3a} (ax+x^2)\, dx$

$= \frac{W}{EI}\left[\frac{ax^2}{2} + \frac{x^3}{3}\right]_0^{3a}$

$\delta_3 = \frac{W}{EI}\left[9a^3 - \frac{9a^3}{2}\right]$

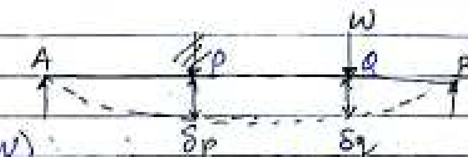
$\boxed{\delta_3 = \frac{9Wa^3}{2EI}}$

$\delta_T = \delta_1 + \delta_2 + \delta_3$

Maxwell's Reciprocal Deflection Theorem

The deflection of any pt (P) resulting from the application of the load at any other pt (Q) is same as the deflection of Q resulting from the application of same load at P.

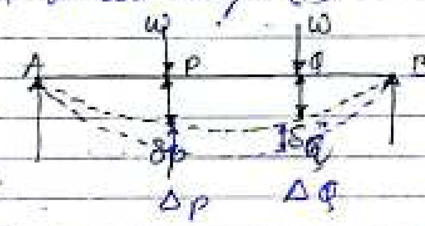
Proof



In the fig. a structure AB is applied by a load (W) at any pt (Q), due to this deflection at pt (P) is δp & δq.

$\boxed{\text{Then work done on the structure} = \frac{1}{2} W\delta q}$ --- (I)

Now, apply another load W at a pt (P)



Due to applied load W on pt P, there will be additional deflection ΔP & ΔQ respectively.

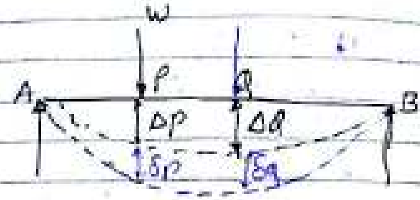
Additional work done $= \underbrace{\frac{1}{2}W\Delta p}_{gradual} + \underbrace{W\Delta q}_{sudden}$ --- (II)

Then for the total work done on this type of loading is equal to –

Total W/D $= \frac{1}{2}W\delta q + \frac{1}{2}W\Delta p + W\Delta q$ ---- (III)

Now, again load the structure in a different way.

First apply a load W at a pt (P). So, that the deflection at a pt P & Q will be Δp & Δq.



work done on the structure $= \frac{1}{2} \times W\Delta p + \Delta q$ --- (iv)

Again apply another load W at a pt Q.

Additional work $= \frac{1}{2} W\delta q + W\Delta p$ --- (v)

Therefore the total W.D on this type of loading.

$T.W/D = \frac{1}{2} W\delta q + \frac{1}{2} \cancel{\delta\Delta p + \delta\Delta q}$ --- (vi)

$= \frac{1}{2} W\delta q + \frac{1}{2} W\Delta p + W\delta p$

Therefore, as the load on the structure is same in both the cases, the work done should also be same.

So, equating (i) & (ii)

$\frac{1}{2} W\delta q + \frac{1}{2} W\delta p + W\Delta q = \frac{1}{2} W\Delta p + \frac{1}{2} W\delta q + W\delta q$

$\boxed{\Delta q = \delta q}$

Therefore, it means deflection at Q due to load W at (P) is equal to deflection at (P) due load (W) at Q.

$\boxed{\Delta q = \delta p}$

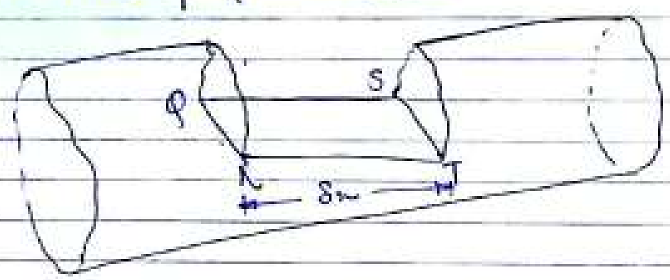
Date
11/12/2017

UNIT-II

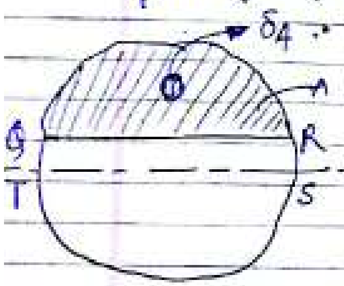
* Shear Stress In Beam *

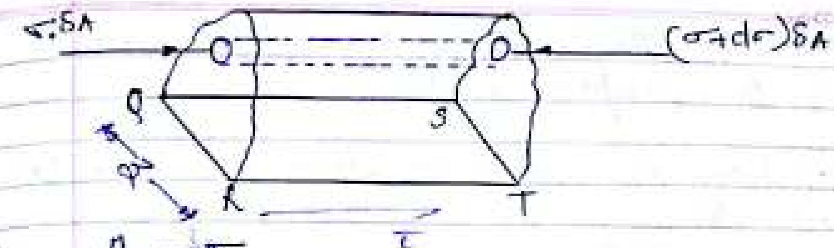
The shear force in a beam at any cross section ~~sets up~~ sets up shear stress on transverse section, the magnitude of which varies across the section. In the analysis it is assumed that the shear stress is uniform across the width & does not affect the distribution of bending stress.

Variation of Shear Stress



Let a beam under transversal loading. To find out the shear stress on an element consider a small block [QRTS] of length (δx) of the beam cut horizontally





$\frac{M}{I} = \frac{\sigma}{y}$

$\sigma = \frac{My}{I} \Rightarrow \boxed{\sigma.\delta A = \frac{My}{I}.\delta A}$ ---- ①

Force on the left side

$\boxed{(\sigma+d\sigma)\delta A = \left(\frac{M+dM}{I}\right).\delta A}$ ---- ⑪

Force on right side

⇒ These force, the net force applied on the elemental area towards left -

② - ①

$\frac{(M+\delta M)}{I}y.\delta A - \frac{M}{I}y\delta A = (\sigma+d\sigma)\delta A - \sigma\delta A$

$d\sigma\,\delta A = \frac{dMy}{I}.\delta A$ --- ③ ← Net force on 'dA'

Total force applied on the gross area of the block equal to

$\left[\int\frac{\delta M}{I}.y.\delta A\right]$ --- ④

This force on the block tends to slide the block towards left. This is resisted by horizontal shear force on the surface area of the block QRTS.

If τ is the average shear stress value at the surface, then –

Therefore, the net shear force on area PRT will be –

= Avg. Shear stress × Area of QRT

$= \tau \times (z\,dx)$ ---- (5)

(5) = (4)

$$\int \frac{\delta M}{I} \cdot y \cdot \delta A = \tau \times (z\,dx)$$

$$\tau = \left(\frac{\delta M}{\delta x}\right) \frac{1}{Iz} \int y\, dA$$

$F \rightarrow$ shear force
$z \rightarrow$ constant
$y \rightarrow$ variable

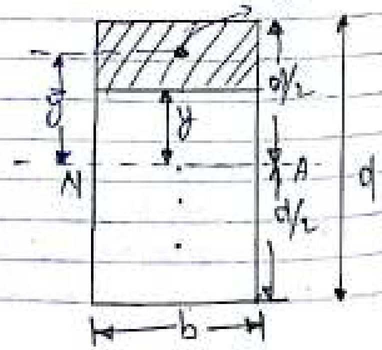
$$\boxed{\tau = \frac{F \cdot A\bar{y}}{Iz}}$$

Shear Stress in Rectangular Section

$$\tau = \frac{FA\bar{y}}{zI}$$

$$\bar{y} = \frac{1}{2}\left(\frac{d}{2} - y\right) + y$$

$$A = \left(\frac{d}{2} - y\right) b$$



$$\tau = \frac{F\left(d/2 - y\right) b \times \left[\frac{1}{2}\left(\frac{d}{2} - y\right) + y\right]}{zI}$$

$$= \frac{F\left(\frac{d}{2} - y\right) b \times \left\{\frac{1}{2}\left(\frac{d}{2} - y\right) + y\right\}}{b \times \frac{bd^3}{12}}$$

$$= \frac{\left(Fd/2 - Fy\right) b \times \left[\left(\frac{d}{4} - y/2\right) + y\right]}{b \times \frac{bd^3}{12}}$$

$$= \frac{\left(Fdb/2 - Fyb\right)\left(d/4 - y/2 + y\right)}{\frac{b^2 d^3}{12}}$$

$$= \frac{\left(\frac{Fdb}{2} - Fyb\right)\left(\frac{d}{4} + y/2\right)}{\frac{b^2 d^3}{12}}$$

$$= \frac{\left(\frac{Fd^2 b}{8} + \frac{Fdby}{4} - \frac{Fybd}{4} - \frac{Fy^2 b}{2}\right) 12}{b^2 d^3}$$

$$= \frac{\cancel{b}\left(\frac{Fd^2}{8} + \frac{Fdy}{4} - \frac{Fyd}{4} - Fy^2/2\right) 12}{b^2 d^3}$$

$$= \frac{\cancel{\frac{F}{2}}\left(\frac{d^2}{4} + \cancel{\frac{dy}{2}} - \cancel{\frac{yd}{2}} - y^2\right)\cancel{12}}{bd^3}$$

$$= \frac{6F\left[\left(d^2/4 + \cancel{yd/2} - \cancel{yd/2}\right) - y^2\right]}{bd^3}$$

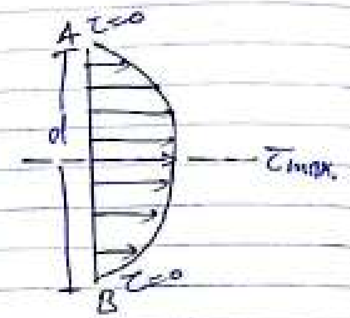
$$= \frac{6F\left(d^2/4 - y^2\right)}{bd^3}$$

$$\boxed{\tau = \frac{6F}{bd^3}\left(\frac{d^2}{4} - y^2\right)}$$

Case-I) $y = 0$

$$\tau = \frac{6F}{bd^3}\left(\frac{d^2}{4} - 0\right) \Rightarrow \tau = \frac{3Fd^2}{2bd^3}$$

$$\boxed{\tau_{max} = \frac{3F}{2bd}}$$



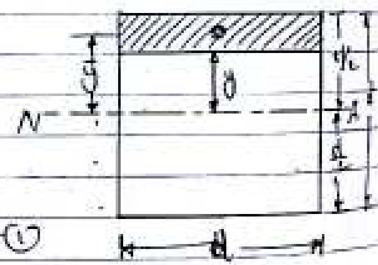
$\tau_{max} = \frac{3}{2}$ mean.

Case-II) $y = d/2$

$$\boxed{\tau = 0}$$

Shear Stress in Square Section

$$\tau = \frac{FA\bar{y}}{ZI} \quad \text{---} \quad (A)$$



$$\bar{y} = \frac{1}{2}\left(d/2 - y\right) + y$$

$$A = \left(\frac{d}{2} - y\right) \times d = \frac{d^2}{2} - yd \quad \text{--- (I)}$$

$$\bar{y} = \frac{d}{4} - \frac{y}{2} + y = \frac{d}{4} + y/2 \quad \text{--- (II)}$$

Putting (I) & (II) in eqn. (A)

$$\tau = \frac{F\left(\frac{d^2}{2} - yd\right)\left(\frac{d}{4} + \frac{y}{2}\right)}{d \times \frac{d d^3}{12}}$$

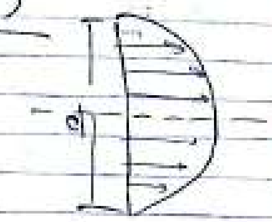
$$\tau = \frac{F\left(\frac{d^3}{8} + \frac{d^2y}{4} - \frac{yd^2}{4} - \frac{y^2d}{2}\right)}{d^5/12}$$

$$\tau = \frac{12F\left(\frac{d^3}{8} + \frac{d^2y}{4} - \frac{yd^2}{4} - \frac{y^2d}{2}\right)}{d^5}$$

$$\tau = \frac{6F\left(d^3/4 + \frac{d^2y}{2} - \frac{yd^2}{2} - y^2d\right)}{d^5}$$

$$\tau = \frac{6F\cancel{d}\left(d^2/4 + \cancel{dy/2} - \cancel{yd/2} - y^2\right)}{d^{\cancel{5}}4}$$

$$\boxed{\tau = \frac{6F}{d^4}\left(\frac{d^2}{4} - y^2\right)}$$



If $y = 0$

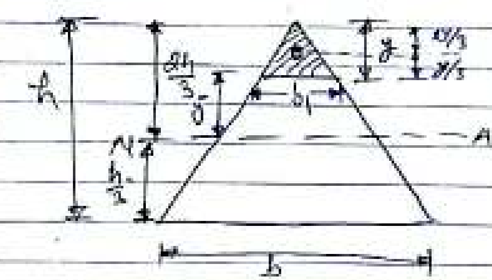
$$\tau = \frac{\overset{3}{\cancel{6}}F}{d^{\cancel{4}2}}\left(\frac{d^2}{4}\right) = \frac{3F}{2d^2}$$

$\tau = \frac{3}{2}$ mean.

3/2/15

Shear Stress in Triangle Section

$$A = \frac{1}{2}[b_1 y]$$



$$\bar{y} = \left(\frac{2h}{3} - \frac{2y}{3}\right)$$

$$Z = b_1$$

$$I_{N.A} = \frac{bh^3}{36}$$

$$\tau = \frac{FA\bar{y}}{ZI}$$

$$\tau = \frac{F \times \left(\frac{1}{2} \times b_1 \times y\right)\left(\frac{2h}{3} - \frac{2y}{3}\right)}{b_1 \times \frac{bh^3}{36}}$$

$$= F\left(\frac{b_1hy}{3} - \frac{b_1y^2}{3}\right) \times \frac{36}{b_1 \times bh^3} = F\cancel{b_1}\left(\frac{hy}{3} - \frac{y^2}{3}\right) \times \frac{36}{b_1 \times bh^3}$$

$$\tau = F\left(\frac{hy}{3} - \frac{y^2}{3}\right) \times \frac{36}{bh^3}$$

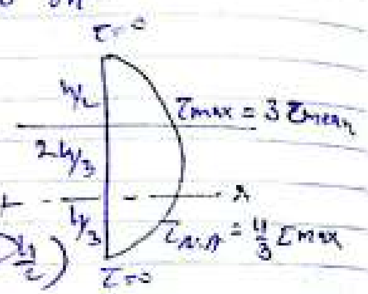
$$\boxed{\tau = \frac{12 F y}{b h^3}(h - y)}$$

$\Rightarrow$ at $y = \frac{2h}{3}$

$$\tau = \frac{4}{3}\tau_{mean} = \frac{4}{3}\frac{F}{bh^3}$$

$\Rightarrow$ at $y = 0$

$$\tau = 0$$

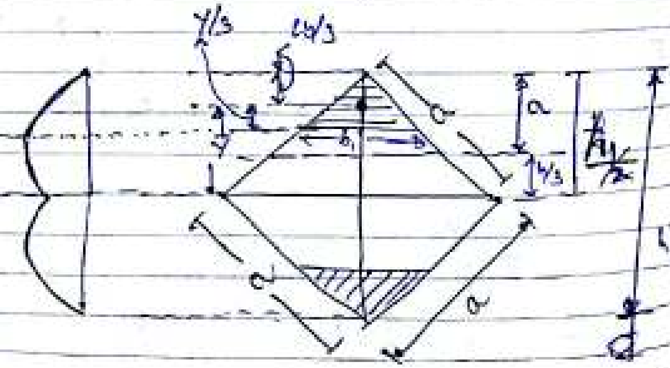


$\Rightarrow$ at $\frac{h}{2} = y$

$\boxed{\tau = 3\,\tau_{mean}}$ $\tau = \frac{12 F y}{bh^3}\left(h - \frac{h}{2}\right)$

$$= \frac{12 F y}{b h^2} \times K = \frac{6 F y}{}$$

③ Square X-Section with Diagonal



Unetary method

$y \Rightarrow b_1$

$\frac{?}{} - \frac{b_1}{y}$

$\frac{h_1}{2} - \frac{b_1 h_1}{2y}$

$$I_{NA} = 2\left[\frac{1}{12} b\left(\frac{h_1}{2}\right)^3\right]$$

$$A = \frac{1}{2} b_1 y$$

$$\bar{y} = \left[\frac{h_1}{2} - \frac{2y}{3}\right]$$

$$z = b_1$$

$$\tau = \frac{F A \bar{y}}{z I}$$

$$\tau = \frac{F\left(\frac{1}{2} b_1 y\right)\left(\frac{h_1}{2} - \frac{2y}{3}\right)}{b_1 \times b\left(\frac{h}{2}\right)^3 \times \frac{1}{6}}$$

$$= \frac{F\left(\frac{1}{2} b_1 y\right)\left(\frac{h_1}{2} - \frac{2y}{3}\right)}{b_1 \times b\frac{h_1^3}{4} \times \frac{1}{6}} \times \quad = \frac{F\left(\frac{1}{2} b_1 y\right)\left(\frac{h_1}{2} - \frac{2y}{3}\right)}{\frac{b_1 \times b h_1^3}{24}}$$

$$= F\left(\tfrac{1}{2} b_1 y\right)\left(\tfrac{h_1}{2} - \tfrac{2y}{3}\right) \times \frac{24}{b_1 \times b h_1^3}$$

$$= F\left(\tfrac{y}{2}\right)\left(\tfrac{h_1}{2} - \tfrac{2y}{3}\right) \times \frac{24}{b h_1^3}$$

$$= F\left(\frac{h_1 y}{4} - \frac{2y^2}{6}\right) \times \frac{24}{b h_1^2}$$

$$= \frac{24 F}{b h_1^3}\left(\frac{h_1 y}{4} - \frac{y^2}{3}\right) = \frac{24 F y}{b h_1^3}\left(\frac{h_1}{4} - \frac{y}{3}\right)$$

for max.

$$\frac{\partial \tau}{\partial y} = 0$$

$$\frac{24 F}{b h^2} \quad \frac{24 F}{b h_1^2}\left(\frac{h_1}{4} - \frac{2y}{3}\right) = 0$$

$$\frac{h_1}{4} - \frac{2y}{3} = 0$$

$$\frac{2y}{3} = \frac{h_1}{4}$$

$$2y = \frac{3h_1}{4}$$

$$y = \frac{3h_1}{8}$$

$$\boxed{y = \frac{3}{8} h_1}$$

At $y = \frac{3}{8} h_1$ (for τ_{max})

at centre $y = h/2$ (70) to check τ_{max} ... [illegible]

$$\tau_{max} = \frac{24F\left(\frac{3}{8}h_1\right)}{bh_1^3}\left(\frac{h_1}{4} - \frac{\frac{3}{8}h_1}{3}\right)$$

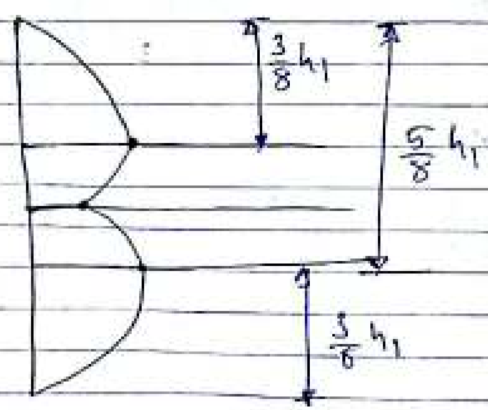
$$= \frac{9Fh_1}{bh_1^3}\left(\frac{h_1}{4} - \frac{h_1}{8}\right)$$

$$= \frac{9Fh_1^2}{bh_1^3}\left(\frac{1}{8}\right)$$

$$= \frac{9F}{8bh_1}$$

$$= \frac{9}{8} \times \frac{F}{A}$$

$$\boxed{\tau = \frac{9}{8}\tau_{mean}}$$

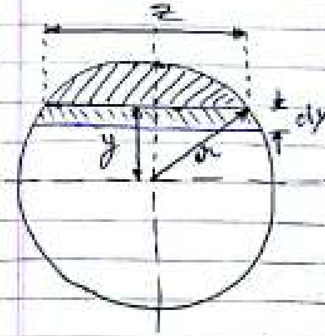


$$\tau_{max} = \frac{9F}{4bh_1}$$

$$= \frac{9}{8}\left(\frac{F}{bh_1/2}\right)$$

$$= \frac{9}{8}\tau_{mean}$$

(4) Circular X-Section



$\tau_{max} = \frac{4}{3}\tau_{mean}$

$$\frac{z}{2} = \sqrt{r^2 - y^2}$$

$$z = 2\sqrt{r^2 - y^2}$$

$$z^2 = 4(r^2 - y^2)$$

By differentiation

$$\boxed{2z\,dz = 4(-2y\,dy)}$$

$$\boxed{y\,dy = -\frac{z\,dz}{4}} \quad \text{--- (1)}$$

Area of the el. strip $= z.dy$

→ Moment of elementary area about neutral axis

$$= z.dy.y$$

For whole circle about

$$A\bar{y} = \int_y^{2} z \cdot dy \cdot y$$

$$A\bar{y} = \cancel{z}\left(\frac{y^2}{2}\right)_y^{2} \quad \cancel{z}\, z\left(\frac{x^2}{2} - \frac{y^2}{2}\right)$$

$=$

$$\boxed{A\bar{y} = \frac{z^3}{12}}$$

$$\tau = \frac{F A\bar{y}}{z I} = \frac{F\left(\frac{z^3}{12}\right)}{z \cdot \frac{\pi d^4}{64}}$$

$$\tau = F\left(\frac{z^2}{\frac{12}{3}}\right) \times \frac{\cancel{64}\,16}{\cancel{z}\times\pi d^4} = \frac{16 F z^2}{3\pi d^4}$$

$$\tau = \frac{16 F z^2}{3\pi (2r)^4} = \frac{16 F z^2}{3\pi \times 16 \times r^4}$$

$$\tau = \frac{F z^2}{3\pi r^4}$$

$$\tau = \frac{F\left(4(r^2 - y^2)\right)}{3\pi r^4} = \frac{4}{3}\frac{F(r^2-y^2)}{\pi r^4}$$

$z = 2r$

$y = 0$

$x = y$

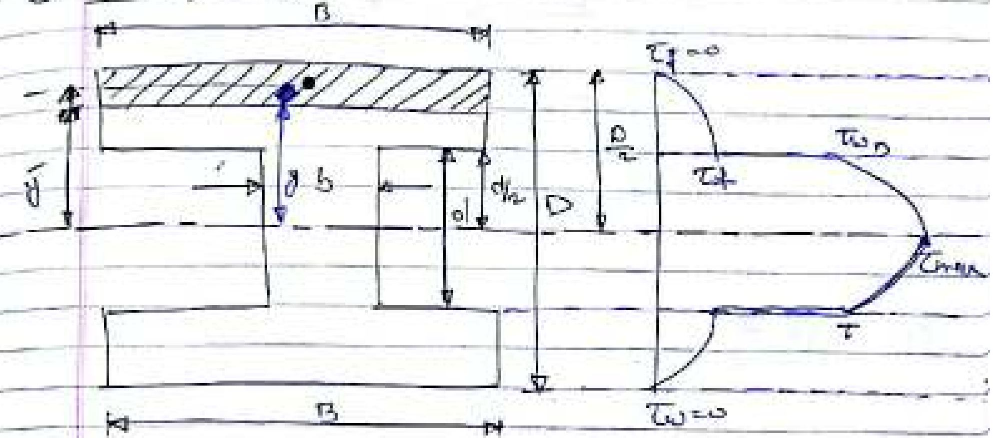
Case-I, at $y = 0$

$$\boxed{\tau_{max} = \frac{4}{3}\tau_{mean}}$$

when $r = y$

$$\boxed{\tau = 0}$$

⑤ I- Section



$$\tau_f = \frac{F A\bar{y}}{B I}$$

$$A = \left(\frac{D}{2} - y\right)B$$

$$\bar{y} = \frac{1}{2}\left(\frac{D}{2} - y\right) + y$$

$$\tau_f = \frac{F\left[\left(\frac{D}{2} - y\right)B\right]\left[\frac{1}{2}\left(\frac{D}{2} - y\right) + y\right]}{B I}$$

$$= \frac{F\left(\frac{BD}{2} - yB\right)\left(\frac{D}{4} - \frac{y}{2} + y\right)}{B I}$$

$$= \frac{F\left(\frac{BD}{2} - yB\right)\left(\frac{D}{4} - \frac{y}{2}\right)}{B I}$$

$$\boxed{\tau_f = \frac{F}{2I}\left[\frac{D^2}{4} - y^2\right]}$$

(i) $y = \frac{D}{2}$, $(\tau_f)_A = 0$

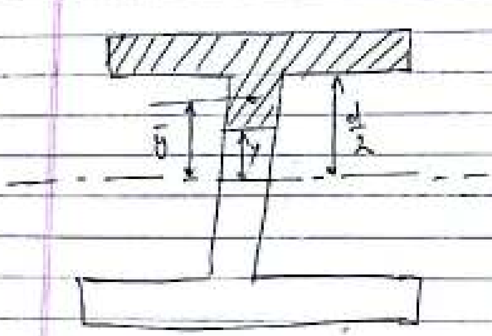
$$\tau_f = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{D}{2}\right)^2\right]$$

$$\boxed{\tau_f = 0}$$

(ii) $y = \frac{d}{2}$,

$$\tau_f = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{d}{2}\right)^2\right]$$

$$(\tau_f)_B = \frac{F}{8I}\left[D^2 - d^2\right]$$



$$A = \left(\frac{d}{2} - y\right)b$$

$$\bar{y} = \frac{1}{2}\left[\frac{d}{2} - y\right] + y$$

$$\tau_w = \frac{F}{bI}\left[A\bar{y}\,(\text{flange}) + A\bar{y}\,(\text{web})\right]$$

$$\tau_w = \frac{F}{bI}\left[\left(\frac{D}{2} - y\right)B \times \left[\frac{1}{2}\left(\frac{D}{2} - y\right) + y\right] + \left(\frac{d}{2} - y\right)b\left(\frac{1}{2}\left(\frac{d}{2} - y\right) + y\right)\right]$$

$$= \frac{F}{bI}\left\{\left(\frac{DB}{2} - yB\right)\left(\frac{D}{4} - \frac{y}{2} + y\right) + \left(\frac{db}{2} - yb\right)\left(\frac{d}{4} - \frac{y}{2} + y\right)\right\}$$

$$\tau_w = \frac{F}{bI}\left\{\left(\frac{BD}{2} - yB\right)\left(\frac{D}{4} + \frac{y}{2}\right) + \left(\frac{db}{2} - yb\right)\left(\frac{d}{4} + \frac{y}{2}\right)\right\}$$

$$= \frac{F}{bI}\Big\{$$

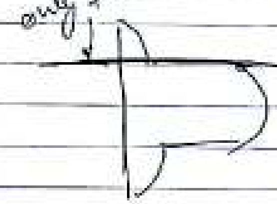
$$\boxed{\tau_w = \frac{F}{8I}\left[\frac{B}{b}\left(D^2 - 4y^2\right) + \left(d^2 - 4y^2\right)\right]}$$ **

(i) at $\boxed{y = d/2}$

$$\tau_w = \frac{F}{8I}\left[\frac{B}{b}\left(D^2 - d^2\right)\right]$$

** $\boxed{\tau_w = \tau_f \times \frac{B}{b}}$

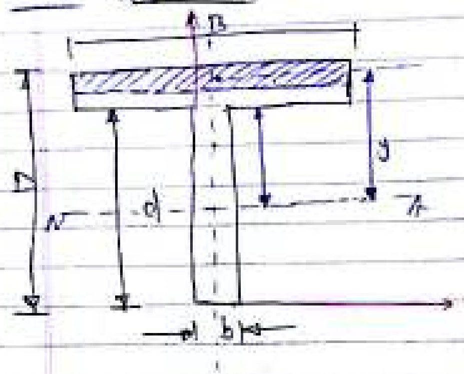
only for this line



(ii) at $y = 0$

$$\tau_w = \frac{F}{8I}\left\{\frac{B}{b}\left(D^2\right) + \left(d^2\right)\right\}$$

⑥ T- Section ②



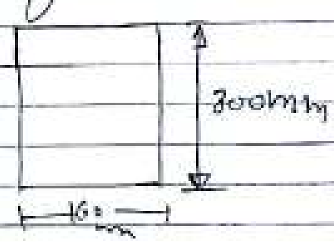
$$\bar{y} = \frac{A_1\bar{x}_1 + A_2\bar{y}_2}{A_1 + A_2}$$

Ques→ A 400mm ~~×~~ ~~200~~, 40mm I beam is subjected to a shear force of 10kN. Find the transverse shear stress

Ques→ A beam of rectangular cross section 160mm wide & 300mm deep is of 4m span & is laid dead with a central pt load of 50 kN. Determine the bending & shear stresses at the top, 100mm & 40mm from the neutral axis of the section and at the neutral axis. Consider the bending moment at the mid cross section of the beam. Also find the principle planes & principle stress at these points & plot the variation along the section.

Solution→ $I = \frac{bd^3}{12}$

$I = \frac{160 \times (300)^2}{12}$



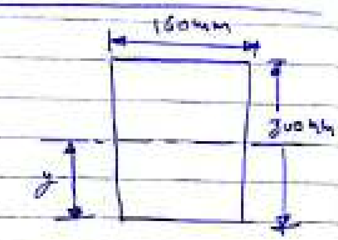
Ques→ A beam has a triangular cross section having base 40mm and altitude 60mm. The section is subjected shear force of 30 kN. Find the max. shear stress in the cross section. (IT)

Ques→ Derive an equation for shear distribution over a circular ~~cross~~ section & also draw shear stress diagram (5)

$\frac{M}{I} = \frac{\sigma}{y}$

$\boxed{y = \frac{d}{2}}$ max. condn.

Ques→ Shear stress distribution over triangular section. (2.5)

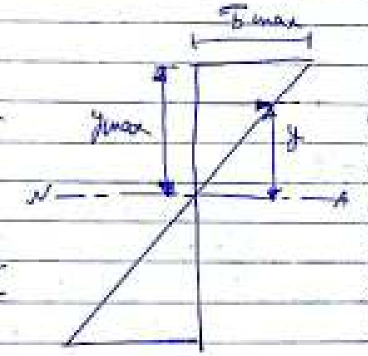


$\frac{M}{I} = \frac{\sigma}{y}$

$M = \frac{Wl}{4} = 50 \text{ kNm}$

$\sigma_b = \frac{MY}{I} = \frac{50 \times 10^3 \times 150 \times 10^{-3}}{\frac{160 \times 300^3}{12}}$

$\boxed{\sigma_b = 20.83 \text{ MPa}}$



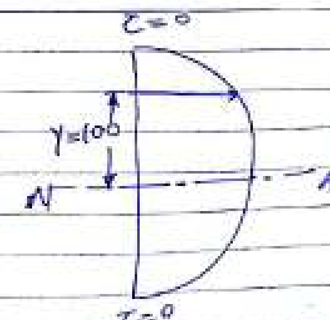
$\tau = \frac{3}{2} \times \frac{50 \times 10^3}{160 \times 300 \times 10^{-6}}$

$\boxed{\tau = 0.016 \text{ MPa}}$

$\sigma_{1,2} = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$

$\boxed{\sigma_1 = 20.83}$, //

$\boxed{\sigma_2 = 0}$



$\sigma_{1,2} = \frac{1}{2}\left\{(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4(\tau_{xy})^2}\right.$

$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$2\theta = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$

$\boxed{\theta = 0}$

$\sigma_b = \frac{My}{I}$ 80

$\boxed{\sigma_b = 0}$ $\because y = 0 \rightarrow$ (bending stress at N/A)

$\sigma_b (\text{at } 100mm) = (\sigma_b)_{max} \times \frac{y}{y_{max}}$

$= \frac{26.53 \times 10^6 \times 100 \times 10^{-3}}{150 \times 10^{-3}}$

$= 17.82 \text{ MPa}$

$\sigma_b (\text{at } 140mm) = 52.05 \text{ MPa}$

$\tau = \frac{6F}{bd^3}\left[\frac{d^2}{4} - y^2\right]$

at 100mm

$\tau = \frac{6 \times 50 \times 10^3}{100 \times 300^3 \times 10^{-12}}\left[\frac{300^2}{4} - 100^2\right] \times 10^{-6}$

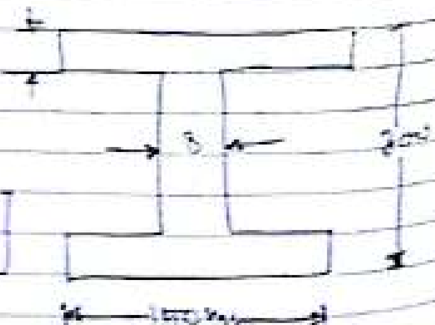
$\tau = 0.0000014 \text{ MPa}$

=|2|2017

A 300 x 150 mm I beam having 12 mm thick flange and 8 mm thick web is subjected to a shear force of 150 kN at a particular section. Determine the value of max. shear in the flange. also find the ratio of max. shear stress to mini. shear stress in the web.

$I = \frac{150 \times 300^3}{12} - 2\left(\frac{71 \times 276^3}{12}\right)$

$= 55709184$



$\tau_w = \frac{F}{8I}\left[\frac{B}{b}(D^2 - 4y_1^2) - (d^2 - 4y^2)\right]$

$= \frac{150 \times 10^3}{8 \times 55709184}\left[\frac{150}{8}(150^2 - 4\right.$

$(\tau_w)_0 = \frac{F}{8I}\left[\frac{150}{8}(300^2 - 4 \times 138^2) + (276^2 - 4 \times 138^2)\right]$ 81

$= \frac{150 \times 10^3}{8 \times 55709184}$ 15.4072

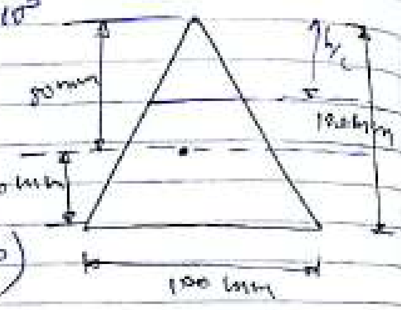
8.92

$\boxed{x = 1.2}$

max → 5 MPa
4.44 → N.A.

Ques: A beam is a triangular cross section with a base 100mm, height 120 mm, the lower surface being horizontal at a section ... draw the dist. of the shear stress in the beam.

$\tau = \frac{12F y}{b h^3}(h-y)$, $F = 20\times10^3$



$\frac{2\times120}{3}$

$\tau_{max} = \frac{12F \times 60}{(100)(120)^3}(120-60)$

$= \frac{12\times20\times10^3\times60}{100\times(120)^3}(120-60)$

$= 5\ MPa$

$\tau_{NA} = \frac{12\times20\times10^3\times80}{100\times(120)^3}\cdot(120-80)$

$= 4.44\ MPa$

13/2/2017

THICK CYLINDER :-

⇒ Lame's theory

Assumptions :-

① The material of the cyl. is homogeneous & isotropic

(ii) The material is stressed within elastic limit

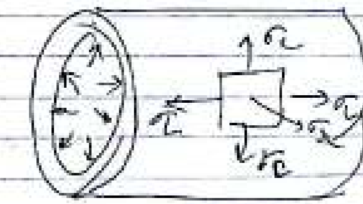
(iii) Plane section normal to the longitudinal axis remains plane after the application of pressure

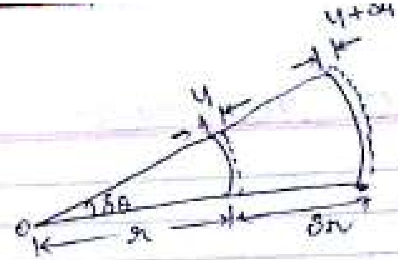
(iv) Young's modulus is same as in tension & compression

(v) All fibers are free to expand or contract under the action of forces irrespective of action of adjacent fiber

When a thick cylinder is acted upon internal radial pressure at the surfaces, three principle stress comes into the picture.

① Radial pressure (P) [compression]
② Circumferential (σ_c) [tensile]
③ Longitudinal (σ_L) [tensile]





→ Strain in longitudinal direction.

$$e_l = \frac{\sigma_l - \mu(\sigma_c - p)}{E} \quad [-ve \to \text{due to compression}]$$

longitudinal stress →

$$eE = \sigma_l - \mu(\sigma_c - p)$$

Diffⁿ it w.r.t 'r' –

$$0 = \frac{d\sigma_l}{dr} = \mu\left(\frac{d\sigma_c}{dr} - \frac{dp}{dr}\right)$$

$$\frac{d\sigma_l}{dr} = \mu\left(\frac{d\sigma_c}{dr} - \frac{dp}{dr}\right) \text{ ---- (1)}$$

Radial strain = $\frac{\text{Increase in } (\delta r)}{\text{change in } r}$

$$= \frac{[(u+\delta u)-u]}{\delta r} = \frac{du}{dr}$$

Radial stress = $E\frac{du}{dr}$

Radial strain = $\frac{-p-\mu(\sigma_c+\sigma_l)}{E}$

$$= -p-\mu(\sigma_c+\sigma_l)$$

$$E\frac{du}{dr} = -p-\mu(\sigma_c+\sigma_l) \text{ ..... (2)}$$

→ Circumferential (hoop) strain. = $\frac{2\pi(r+u) - 2\pi r}{2\pi r}$

$$= \frac{u}{r}$$

Circumferential stress = $E\frac{u}{r}$

Circumferential strain = $\frac{\sigma_c - \mu(\sigma_l - p)}{E}$

$$= \sigma_c - \mu(\sigma_l - p)$$

$$E\frac{u}{r} = \sigma_c - \mu(\sigma_l - p) \text{ ---- (3)}$$

Differentiating the above equation.

$$E\frac{du}{dr} = \sigma_c - \mu[\sigma_l - p] + r\left[\frac{d\sigma_c}{dr} - \mu\frac{d\sigma_l}{dr} + \mu\frac{dp}{dr}\right]$$

equating with eqⁿ (2)

$$-[p + \mu(\sigma_l + \sigma_c)] = \sigma_c - \mu(\sigma_l - p) + r\left[\frac{d\sigma_c}{dr} - \mu\frac{d\sigma_l}{dr} + \mu\frac{dp}{dr}\right] \text{ ----- (3)}$$

$$(p+\sigma_c)(1+\mu) + r\left[\frac{d\sigma_c}{dr} - \mu\frac{d\sigma_l}{dr} + \mu\frac{dp}{dr}\right] = 0 \text{ ----- (4)}$$

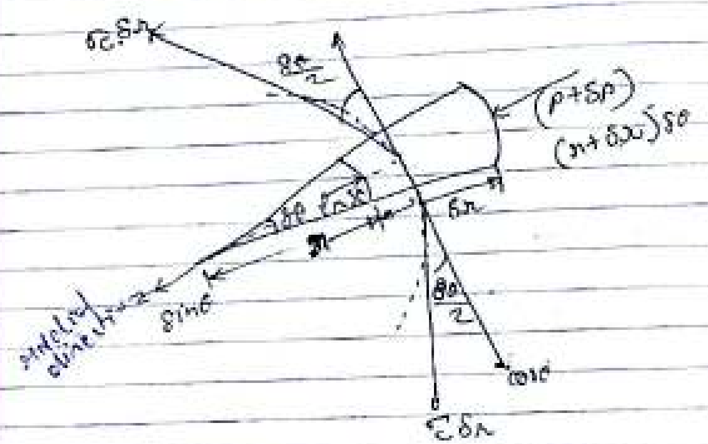
Now eqⁿ (1) & (4)

$$(p+\sigma_c)(1+\mu) + r\left[\frac{d\sigma_c}{dr} - \mu\left(\mu\left(\frac{d\sigma_c}{dr} - \frac{dp}{dr}\right)\right) + \mu\frac{dp}{dr}\right] = 0$$

we get -

$$(p+\sigma_c)(1+\mu) + r\left[\frac{d\sigma_c}{dr}(1-\mu^2) + \mu^2\frac{dp}{dr} + \mu\frac{dp}{dr}\right] = 0$$

$$(p+\sigma_c) + r\left[\frac{d\sigma_c}{dr}(1-\mu) + \mu\frac{dp}{dr}\right] = 0 \quad \text{--- (5)}$$



Radial equilibrium eqn :-

Outer radial force on inner face
$= p(r\delta\theta)$

Inward radial force on outer face
$= (p+\delta p)(r+\delta r)\delta\theta$

Inward radial component of circumferential force
$= 2\sigma_c(\delta r \cdot 1)\sin\left(\frac{\delta\theta}{2}\right)$

$= \sigma_c \,\delta r\, \delta\theta$ ---- $\left[\sin\left(\frac{\delta\theta}{2}\right) = \frac{\delta\theta}{2}\right]$

equilibrium equation.

$$(p+\delta p)(r+\delta r)\delta\theta - p(r\cdot\delta\theta) + \sigma_c\,\delta r\cdot\delta\theta = 0$$

$$\delta\theta\, pr + p\delta r\delta\theta + \delta p r\delta\theta + \delta p\delta r\delta\theta - pr\cdot\delta\theta + \sigma_c\,\delta r\cdot\delta\theta = 0$$

$$\boxed{p + \sigma_c + r\cdot\frac{dp}{dr} = 0} \quad \text{--- (6)}$$

Solving eqn (5) & (6)

$$p + \sigma_c = -r\frac{dp}{dr}$$

Putting in eqn (5)

$$-r\frac{dp}{dr} + r\left[\frac{d\sigma_c}{dr}(1-\mu) + \mu\frac{dp}{dr}\right] = 0$$

$$-r\frac{dp}{dr} + r\frac{d\sigma_c}{dr}(1-\mu) + r\mu\frac{dp}{dr} = 0$$

$$-\frac{dp}{dr} + \frac{d\sigma_c}{dr}(1-\mu) + \mu\frac{dp}{dr} = 0$$

$$\boxed{\frac{d\sigma_c}{dr} - \frac{dp}{dr} = 0}$$

Let, $\sigma_c - p = \text{constant} \rightarrow 2a$

or

$$\boxed{\sigma_c = p + 2a} \quad \text{--- (7)}$$

From equation (6) & (7)

$$p + p + 2a + r\frac{dp}{dr} = 0$$

$$2p + 2a + \frac{r\,dp}{dr} = 0$$

$$2p + \frac{r\,dp}{dr} = -2a$$

$\frac{r^2 dp}{dr} + P2r$

$r\left[\frac{r\,dp}{dr} + 2p\right]$

$$\frac{d}{dr}[pr^2] = -2ar$$

$$d[Pr^2] = -2ar\,dr$$

$$pr^2 = -ar^2 + B$$

$$\boxed{P = -a + \frac{B}{r^2}}$$

$$4B = b$$

$$\boxed{P = -a + \frac{b}{d^2}} \quad \text{-----} \; (2)$$

From (1) & (2)

$$\sigma_c = \left(-a + \frac{b}{d^2}\right) + 2a$$

$$\boxed{\sigma_c = a + \frac{b}{d^2}} \quad \text{----} \; (3)$$

Lamé's end

⇒ If the cyl. is acted upon by an internal pressure P_i and diameter d_i and the external pressure P_o & diameter d_o

The radial stress at the surfaces must be equal to applied stress therefore we can say that from eq^n. (2) -

$$\left.\begin{aligned} P_i &= -a + \frac{b}{d_i^2} \\ P_o &= -a + \frac{b}{d_o^2} \end{aligned}\right\} \quad \text{-----} \; (A)$$

$$\therefore P_i - P_o = \frac{b}{d_i^2} - \frac{b}{d_o^2}$$

$$= b\left[\frac{1}{d_i^2} - \frac{1}{d_o^2}\right]$$

$$b = \frac{d_i^2 d_o^2}{d_o^2 - d_i^2}[P_i - P_o]$$

$$P_i = -a + \frac{b}{d_i^2}$$

$$P_i = \frac{d_o^2}{d_o^2 - d_i^2}(P_i - P_o) - a$$

$$\therefore a = \frac{d_o^2}{d_o^2 - d_i^2}(P_i - P_o) - P_i$$

Using eq^n (2) and putting the value of a, b we get!

$$P = \frac{d_o^2}{d_o^2 - d_i^2}(P_i - P_o) - P_i + \frac{\frac{d_i^2 d_o^2}{d_o^2 - d_i^2}(P_i - P_o)}{d^2}$$

$$P = \frac{P_o d_o^2 - P_i d_i^2 + (d_i^2 d_o^2 / d^2)(P_i - P_o)}{d_o^2 - d_i^2} \quad \text{-----} \; (3)$$

Similarly

$$\sigma_c = a + \frac{b}{d^2}$$

$$\sigma_c = \frac{d_o^2}{d_o^2 - d_i^2}(P_i - P_o) - P_i + \frac{d_i^2 d_o^2}{(d_o^2 - d_i^2)d^2}[P_i - P_o]$$

$$= P_i - P_o \} \; d_o$$

$$\sigma_c = \frac{P_i d_i^2 - P_o d_o^2 + (d_i^2 d_o^2 / d^2)\cdot(P_i - P_o)}{d_o^2 - d_i^2} \quad \text{----} \; (C)$$

Max. Shear stress $= \frac{1}{2}[\sigma_c + p]$

$$= \frac{1}{2}\left[\frac{p_i d_i^2 - p_o d_o^2 + (d_i^2 d_o^2 / d^2)\cdot(p_i - p_o)}{d_o^2 - d_i^2} + \right.$$

$$\frac{p_o d_o^2 - p_i d_i^2 + (d_i^2 d_o^2 / d^2)(p_i - p_o)}{d_o^2 - d_i^2}$$

$$= \frac{d_i^2 d_o^2 (p_i - p_o)}{(d_o^2 - d_i^2)\, d^2} \quad \text{---} \quad (a)$$

x — x — x — x — x — x

⇒ For internal pressure only, $\boxed{d = d_i}$ $\boxed{d = d_o}$

$$p = \frac{p_o d_o^2 - p_i d_i^2 + (d_i^2 d_o^2 / d_i^2)(p_i - p_o)}{d_o^2 - d_i^2}$$

$$p = \frac{p_o d_o^2 - p_i d_i^2 + d_o^2 (p_i - p_o)}{d_o^2 - d_i^2}$$

For internal pressure only means p_o external pressure will not be consider so the generalized eqn. becomes

$p_o = 0$

$$\therefore\ p = \frac{d_o^2 - d^2}{d_o^2 - d_i^2} \times \frac{d_i^2}{d^2} \cdot p_i$$

⇒ For internal pressure only.
i.e. $d = d_i$

$$p = \frac{p_o d_o^2 - p_i d_i^2 + \left(\frac{d_i^2 d_o^2}{d^2}\right)(p_i - p_o)}{d_o^2 - d_i^2}$$

$$= \frac{p_o d_o^2 - p_i d_i^2 + \left(\frac{\cancel{d_i^2} d_o^2}{\cancel{d_i^2}}\right)(p_i - p_o)}{d_o^2 - d_i^2}$$

$$= \frac{p_o d_o^2 - p_i d_i^2 + d_o^2 (p_i - p_o)}{d_o^2 - d_i^2}$$

$$= \frac{\cancel{p_o d_o^2} - p_i d_i^2 + p_i d_o^2 - \cancel{p_o d_o^2}}{d_o^2 - d_i^2}$$

$$= \frac{p_i (d_o^2 - d_i^2)}{d_o^2 - d_i^2}$$

⇒ For external pressure only. i.e. $d = d_o$

$$p = \frac{p_o d_o^2 - p_i d_i^2 + \left(\frac{d_i^2 \cancel{d_o^2}}{\cancel{d_o^2}}\right)(p_i - p_o)}{d_o^2 - d_i^2}$$

$$= \frac{p_o d_o^2 - \cancel{p_i d_i^2} + \cancel{p_i d_i^2} - p_o d_i^2}{d_o^2 - d_i^2}$$

$$= \frac{p_o (d_o^2 - d_i^2)}{d_o^2 - d_i^2}$$

⇒ For internal pressure only means (p_o) external pressure will not be consider.
So, the generalized equation becomes a use-val

$\therefore P_o = 0$

$$\cancel{P = \frac{P_o d_o^2 - P_i d_i^2 + (d_i^2 d_o^2/d^2)(P_i - P_o)}{d_o^2 - d_i^2}}$$

$$\cancel{P = \frac{-P_i d_i^2 + (d_i^2 d_o^2)(P_i)}{d^2(d_o^2 - d_i^2)} = \frac{P_i d_i^2 (d_o^2 - d_i^2)}{d^2(d_o^2 - d_i^2)}}$$

$\boxed{P_i = 0}$

$$\cancel{P = \frac{P_o d_o^2 - P_i d_i^2 + (d_i^2 d_o^2/d^2)(P_i - P_o)}{d_o^2 - d_i^2}}$$

$$\cancel{P = \frac{P_o d_o^2 + (d_i^2 d_o^2)(-P_o)}{d^2(d_o^2 - d_i^2)} = \frac{P_o d_o^2 - (d_i^2 d_o^2)(P_o)}{d^2(d_o^2 - d_i^2)}}$$

$P = P_o d_o^2 \Big($

$P_o = 0$

$$P = \frac{P_o d_o - P_i d_i^2 + \left(\frac{d_i^2 d_o^2}{d^2}\right)(P_i - P_o)}{d_o^2 - d_i^2}$$

$$= \frac{-P_i d_i^2 + \left(\frac{d_i^2 d_o^2}{d^2}\right)(P_i)}{d_o^2 - d_i^2}$$

$$= \frac{-P_i d_i^2 d^2 + (d_i^2 d_o^2)(P_i)}{d^2(d_o^2 - d_i^2)}$$

$$= \frac{P_i d_i^2 (d_o^2 - d^2)}{d^2(d_o^2 - d_i^2)}$$

$P_i = 0$

$$P = \frac{P_o d_o^2 - P_i d_i^2 + \left(\frac{d_i^2 d_o^2}{d^2}\right)(P_i - P_o)}{d_o^2 - d_i^2}$$

$$= \frac{P_o d_o^2 d^2 - d_i^2 d_o^2 P_o}{d^2(d_o^2 - d_i^2)}$$

$$= \frac{P_o d_o^2 (d^2 - d_i^2)}{d^2(d_o^2 - d_i^2)}$$

Similarly for σ_c

$\boxed{d = d_i}$

$$\sigma_c = \frac{P_i d_i^2 - P_o d_o^2 + \left(\frac{d_i^2 d_o^2}{d_i^2}\right)(P_i - P_o)}{d_o^2 - d_i^2}$$

$$= \frac{d_i^2 P_i d_i^2 - d_i^2 P_o d_o^2 + d_o^2 d_i^2 P_i - P_o d_o^2 d_i^2}{(d_o^2 - d_i^2)\, d_i^2}$$

$$= \frac{P_i d_i^2 + P_i d_o^2 - 2 P_o d_o^2}{d_o^2 - d_i^2}$$

$$\frac{P_i(d_i^2 + d_o^2)}{d_o^2 - d_i^2} = \frac{-2 P_o d_o^2}{d_o^2 - d_i^2} \quad \ldots \quad [P_o = 0]$$

$\boxed{d = d_o}$

$$\sigma_c = \frac{P_i d_i^2 - P_o d_o^2 + \left(\frac{d_i^2 d_o^2}{d_o^2}\right)(P_i - P_o)}{d_o^2 - d_i^2}$$

$$= \frac{P_i d_i^2 - P_o d_o^2 + P_i d_i^2 - P_o d_i^2}{d_o^2 - d_i^2}$$

$$\frac{-P_o(d_o^2 + d_i^2)}{d_o^2 - d_i^2} = \frac{2 P_i d_i^2 - P_o(d_o^2 + d_i^2)}{d_o^2 - d_i^2}$$

$$= \frac{2 P_i d_i^2}{d_o^2 - d_i^2} \quad \ldots \quad [P_o = 0]$$

$\boxed{P_i = 0}$

$$\sigma_c = \frac{P_i d_i^2 - P_o d_o^2 + \left(\frac{d_o^2 d_i^2}{d^2}\right)(\cancel{P_i} - P_o)}{d_o^2 - d_i^2}$$

$$\sigma_c = \frac{-P_o d_o^2 d^2 - d_o^2 d_i^2 P_o}{(d_o^2 - d_i^2) d^2}$$

$$= \frac{-P_o d_o^2 (d^2 + d_i^2)}{(d_o^2 - d_i^2) d^2}$$

$\boxed{P_o = 0}$

$$\sigma_c = \frac{P_i d_i^2 - \cancel{P_o d_o^2} + (d_o^2 d_i^2/d^2)(P_i - \cancel{P_o})}{d_o^2 - d_i^2}$$

$$\sigma_c = \frac{d^2 (P_i d_i^2) + P_i d_o^2 d_i^2}{(d_o^2 - d_i^2) d^2}$$

$$= \frac{P_i d_i^2 (d^2 + d_i^2)}{(d_o^2 - d_i^2) d^2}$$

⇒ for external pressure only
$P_i = 0$

$$P = \frac{P_o d_o^2 + (d_i^2 d_o^2/d^2)(-P_o)}{d_o^2 - d_i^2}$$

~~$P = P_o d_o^2 + d$~~ $\quad P = \frac{P_o d_o^2}{d^2}\left(\frac{d_o^2 - d_i^2}{d_o^2 - d_i^2}\right)$

⇒ σ_c for external
~~P_i~~ $P_i = 0$

$$\sigma_c = \frac{P_i d_i^2 - P_o d_o^2 + (d_i^2 d_o^2/d^2)(P_i - P_o)}{d_o^2 - d_i^2}$$

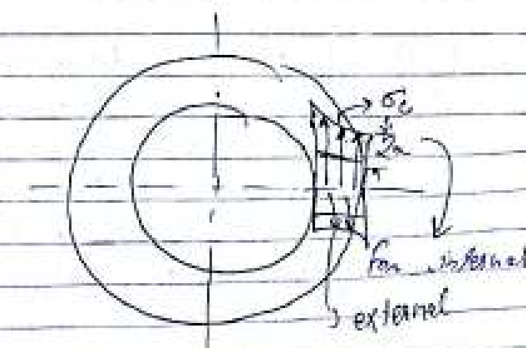
$$\sigma_c = \frac{-P_o d_o^2 + (d_i^2 d_o^2/d^2)(-P_o)}{d_o^2 - d_i^2}$$

$d = d_i$

$$\sigma_c = \frac{-2 P_o d_o^2}{d_o^2 - d_i^2}$$

$d = d_o$

$$\sigma_c = -P_o\left[\frac{d_o^2 + d_i^2}{d_o^2 - d_i^2}\right]$$



Scanned by CamScanner

Max. Shear stress $= \frac{1}{2}[\sigma_c + p]$

$\boxed{d = d_i}$ $\boxed{d = d_o}$

$$M.S.S = \frac{d_i^2 d_o^2 (p_i - p_o)}{(d_o^2 - d_i^2)\cancel{d_i^2}} = \frac{d_o^2(p_i - p_o)}{d_o^2 - d_i^2}$$

$d = d_o$

$$M.S.S. = \frac{d_i^2 \cancel{d_o^2}(p_i p_o)}{(d_o^2 - d_i^2)\cancel{d_o^2}} = \frac{d_i^2(p_i - p_o)}{d_o^2 - d_i^2}$$

⇒ Now for longitudinal stress for a cyl. with closed ends, can be obtained from the equation

$$\sigma_L \frac{\pi}{4}(d_o^2 - d_i^2) = p_i \frac{\pi}{4} d_i^2$$

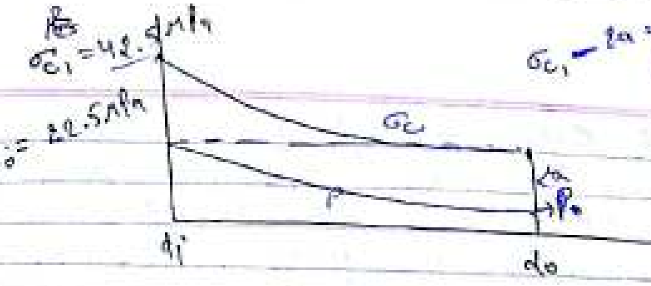
$$\sigma_L = \frac{p_i d_i^2}{d_o^2 - d_i^2}$$

Ques) A pipe of 100 mm external diameter and 20 mm thickness carries water at a pressure of 20 MPa. Determine the max. & min. intensity of hoop stress in the section of pipe, also plot the variation of hoop & radial stress across the thickness of the pipe.

Ans)
$d_o = 100\,mm$
$t = 20\,mm$
$p_i = 20 \times 10^6\,Pa$

$d_i = 100 - (20)2$
$= 60\,mm$

$d_i = 0$
$d_o = 0$



$\sigma_{c_i} - 2a = b$

$$\cancel{\sigma_i} = \cancel{-\frac{2 p_o d_o^2}{d_o^2 - d_i^2}} = -2c$$

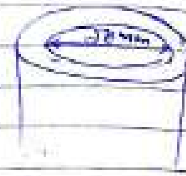
$$\sigma_c = \frac{(d_i^2 + d_o^2)p_i}{(d_o^2 - d_i^2)} = \frac{(60^2 + 100^2)20}{100^2 - 60^2} = 42.5\,MPa.$$

$$(\sigma_c)_o = \frac{2 \times p_i d_i^2}{d_o^2 - d_i^2} = \frac{2 \times 20\,MPa \times (60\,mm)^2}{(100^2 - 60^2)\,mm^2}$$

$= 22.5\,MPa.$

14/2/2017

Que-1 → Find the thickness of cyl. of a hydraulic ram of 50 mm dia (internal), ~~to~~ withstand of $P_i = 30\,MPa$. The allowable tensile stress is limited to 45 MPa & allowable shear stress to 40 MPa.



$P_i = 30\,MPa$

$d_i = 50\,mm$

$$M.S.S. = \frac{d_o^2(P_i - P_o)}{d_o^2 - d_i^2} = d_o^2(30\,MPa -$$

$$\sigma_c = \frac{(d_i^2 + d_o^2)P_i}{d_o^2 - d_i^2}$$

$$45 \times 10^6 = \frac{(50 + d_o^2)\,30}{d_o^2 - 30^2}$$

$$45 = \frac{1500 + 30 d_o^2}{d_o^2 + 900}$$

$45(d_o^2 + 900) = 1500 + 30 d_o^2$

$45 d_o^2 + 40500 = 900 + 30 d_o^2$

$45 d_o^2 - 30 d_o^2 = 900 - 40500$

$15 d_o^2 = -39600$

$d_o^2 = -2640$

$d_o = 51.38$

$\boxed{d_o = 111.8\,mm}$

$$\tau = \frac{P_i d_o^2}{d_o^2 - d_i^2} = \frac{30 \times d_o^2}{d_o^2 - 50^2}$$

$40(d_o^2 - 2500) = 30 d_o^2$

$40 d_o^2 - 100{,}000 = 30 d_o^2$ $\boxed{d_o = 100}$

Lagan f(d_i, d_o)

Thickness

$$= \frac{d_o - d_i}{2} = \frac{111.8 - 50}{2} = 30.9$$

Que-2 → A thick cyld of $d_o = 200\,mm$, $140\,mm = d_i$ is subject $P_i = 40\,MPa$ & external pressure of 0 MPa. Determine the max. shear stress in the material of cylinder at the inside diameter.

$$P = -a + \frac{b}{d^2}$$

$$\sigma_c = a + \frac{b}{d^2}$$

$$P_i = -a + \frac{b}{d_i^2} \qquad P_o = -a + \frac{b}{d_o^2}$$

$P_i =$

$$40\,MPa = -a + \frac{b}{(140)^2} \qquad 0 = -a + \frac{b}{(200)^2}$$

$960000 = -a + b$

~~$5600 = -a + b$~~

$-a + b = 784000$

$-\cancel{4}(960000\,\cancel{4}) + b = 784000$

$$\tau = \frac{1}{2}(\sigma_c + P)$$

$$\tau_{d_i} = \frac{1}{2}(\sigma_c + P_i)$$

$$= \frac{1}{2}(-8.527 + 40)$$

$\boxed{\tau_{d_i} = 31.37}$

$$\sigma_{c_i} = a + \frac{b}{d_i}$$

$$= -8.53 + \frac{61440}{(140)^2}$$

$= -8.527$

$= 22.74$

$\boxed{MPa = N/mm^2}$

100

Q No. A Thick cyl. has $d_i = 120$ mm, $d_o = 150$ mm, subjected at $p_o = 9$ MPa. Find the $p_i = ?$ which can be applied in the max. given not exceed 30 MPa. Draw the curve show the variation of hoop & radial stress through the material of the cylinder.

$p_o = -a + \frac{b}{d_o^2}$ | $\sigma_{c_1} = a + \frac{b}{d_1^2}$ | $a = 3$, $b = 539000$

$9\,MPa = -a + \frac{b}{(150\,mm)^2}$ | $30 = a + \frac{b}{(120)^2}$

$\cancel{-a+b = 1890}$ | $\cancel{a+b = 360c}$

$9\,MPa = -a + \frac{b}{(150)^2}$ | $432000 = a+b$

$291600 = -a+b$

$291600 = -a + (432000 - a)$

$9\,MPa = -a + \frac{b}{(150)^2}$ | $30 = a + \frac{b}{(120)^2}$

$29\,1600 = -32400a + b$ | $432000 = 14400a + b$

$432000 - 14400a = b$

$291600 = 32400a + (432000 - 14400a)$

$291600 = (32400)a + 432000 - 14400a$

$291600 - 432000 = -111800\,a$

$-140400 = -111800\,a$ | $\boxed{312120 = 14400a + b}$

$312120 - 14400a = b$

$9\,MPa = -a + \frac{b}{(150)^2}$ | $104976 + 32400a = 312120 - 14400a$

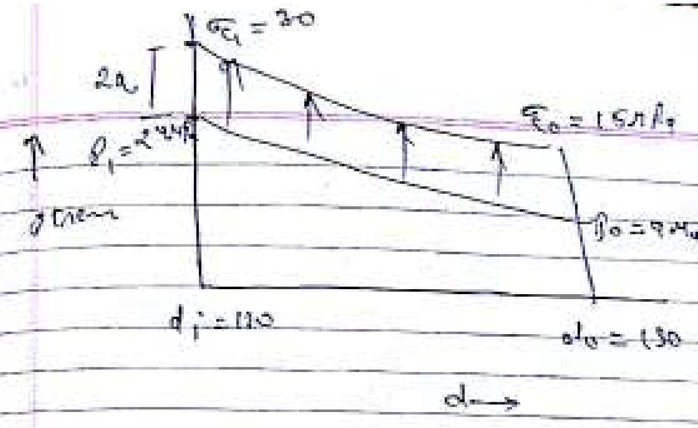
$104976 - 312120 = -14400a - 32400a$

$-207144 = -46800\,a$

$\boxed{p_1 = -a + \frac{b}{d_1^2}}$

$\boxed{p_1 = 24}$

101



Q2) A Thick cylinder of inner dia 60 mm, $d_o = 100$ mm subjected $p_i = 64$ MPa. Find the hoops stress developed in the cylinder.

$d_i = 60$ mm
$d_o = 100$ mm
$p_i = 64$ MPa

$\sigma_c = \frac{(d_1^2 + d_o^2)}{d_o^2 - d_1^2} \times p_i$

$= \frac{(60)^2 + (100)^2}{(100)^2 - (60)^2} \times 64$

$= 136$ MPa

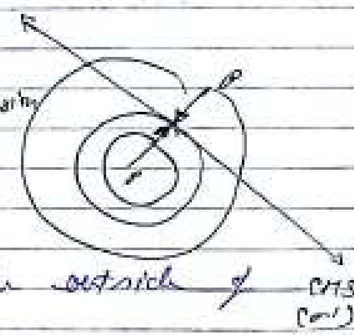
16/8/2017

Compound tubes

As we know hoop stress is max. at the inner radius of the thick cy. tube. The material at the outside of the tube is not stressed to its limit. To even out the stresses a tube may be made of two parts, one shrunk on the other by heating it to the required temp. This makes the inner tube in compression and outer tube in tension. On applying the internal pressure a tensile hoop stress is super imposed on the compressive or shrinkage stress of the inner tube.

CHS
(σ) (tension)

CHS
Inner (σ') [compression]



⇒ CHS (σ) [tension] at the outside of the inner tube

⇒ CHS (σ') [compression] at inside of the outer tube

Initially, the inside diameter of the outer tube is smaller than the outside dia of the inner tube.

On heating the outer tube, this is increased and only made equal to that. On cooling both the diameter are reduced by the same amount.

Thus the analysis is made by calculating the stress due to shrinkage in each component. Thus the final diameter of the outer tube increases from the original and that of inner tube decreases. On applying the inner internal pressure the stress are calculated by assuming two tube as a single tube if they are made up of same material.

Due to shrinkage let us assume,

d = dia of the common surface of inner & outer tube

p = radial pressure at the common surface

σ = compr. hoop stress at the outside of the inner tube

σ' = tensile hoop stress at the inside of the ~~inner~~ outer tube

E = young's modulus of the material of inner tube.

E' = young's modulus of material of outer tube

$$\epsilon_x = \frac{\sigma_x - \mu\sigma_y}{E}$$

μ = poisson's ratio of the material of inner tube

μ' = poisson ratio of the material of outer tube.

⇒ Decrease in dia of inner tube = Strain × d

$$= \frac{-\sigma - (\mu(-p))}{E} \times d \quad \text{--- (1)}$$

⇒ Increase in dia. of outer tube = Strain × d

$$= \frac{\sigma' - (\mu(-p))}{E} \times d$$

$$= \frac{\sigma' + \mu' p}{E'} \times d \quad \text{--- (2)}$$

⇒ Difference of diameter before shrinkage (for different material)

(1) − (2)

$$-\left(\frac{\sigma + \mu p}{E}\right) \times d = \left(\frac{\sigma' + \mu' p}{E'}\right) \times d$$

$$d\left\{\left[\frac{-\sigma + \mu p}{E}\right] - \left[\frac{\sigma' + \mu' p}{E'}\right]\right\}$$

for same material

$$\left(\frac{-\sigma + \mu p}{E}\right) \times d = \left(\frac{\sigma' \times \mu p}{E}\right) \times d$$

$$(-\sigma - \sigma')\frac{d}{E} = 0$$

Internal Pressure Only.

dia	Radial Stress (compression)	Circumferential Stress (tension)
d	$\frac{d_o^2 - d^2}{d_o^2 - d_i^2} \cdot \frac{d_i^2}{d^2} \cdot p_i$	$\frac{d_o^2 + d^2}{d_o^2 - d_i^2} \cdot \frac{d_i^2}{d^2} \cdot p_i$
d_i	p_i	$\frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} \cdot p_i$
d_o	0	$\frac{2d_i^2}{d_o^2 - d_i^2} \cdot p_i$

External Pressure Only :-

dia	Radial Stress (compression)	Circumferential stress (tension)
d	$\frac{d^2 - d_i^2}{d_o^2 - d_i^2} \cdot \frac{d_o^2}{d^2} \cdot p_o$	$-\left(\frac{d^2 + d_i^2}{d_o^2 - d_i^2} \cdot \frac{d_o^2}{d^2}\right) p_o$
d_i	0	$-\left(\frac{2d_o^2}{d_o^2 - d_i^2}\right) \times p_o$
d_o	p_o	$-\left(\frac{d_o^2 + d_i^2}{d_o^2 - d_i^2}\right) p_o$

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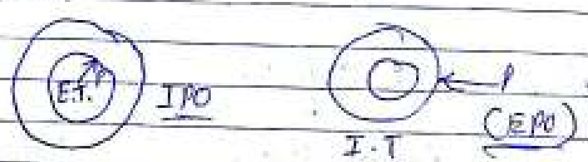
Q.no> A compound cy. is formed by shrinking one tube to another. The inside & outside dia of the outer tube having 120 mm & 180 mm & of the inner tube having 60 mm & 120 mm respectively. After shrinking the radial pressure at the common surface is 30 MPa. If the cyl. is subjected to internal pressure of 30 MPa. Determine the final stress set up at various surface of the cylinder. What is the resultant radial pressure at the common surface.

Solution>



$\boxed{\text{I.P.O}}$

Outer tube → 120 mm, → 180 mm

Inner tube → 60 mm, → 120 mm

$$(\sigma_c)_{60} = \frac{d_o^2 + d_i^2}{d_o^2 - d_i^2}$$

$$= -\left[\frac{2d_o^2}{d_o^2 - d_i^2}\right] P_o = -\left[\frac{2(120)^2}{(120)^2 - (60)^2}\right] 30$$

$$= -80$$

$$(\sigma_c)_{120} = -\left[\frac{d_o^2 + d_i^2}{d_o^2 - d_i^2}\right] P_o$$

$$= -50$$

$\boxed{\text{E.P.O}}$

$$(\sigma_c)_{120} = 78 \text{ MPa} = \frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} \times P_i$$

$$(\sigma_c)_{180} = 48 \text{ MPa} = \frac{2d_i^2}{d_o^2 - d_i^2} \times P_i =$$

When combined the tube

$\boxed{\text{I.P}}$

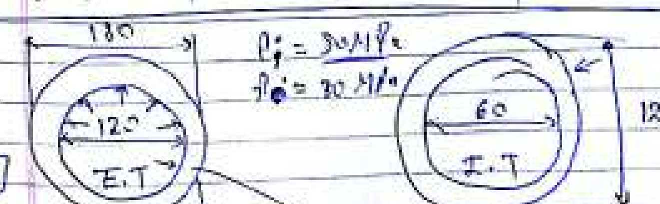
$d_i = 60, \quad d_o = 180$

$$(\sigma_i)_{60} = \frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} \cdot P_i = \frac{180^2 + 60^2}{180^2 - 60^2} \times 30 = 100$$

$$(\sigma_o)_{180} = \frac{2d_i^2}{d_o^2 - d_i^2} \times P_i = \frac{2(60)^2 \cdot 30}{180^2 - 60^2} = 20$$

$(\sigma_c)_{60} = 100 \text{ MPa}, \quad (\sigma_c)_{180} = 20 \text{ MPa}$

	Inner tube		Outer tube	
	60 mm	120 mm	120 mm	180 mm
Shrinkage stress	-80	-50	75	41
Stress due to pressure	100	32.5	32.5	20
Final stress	20	17.5	110.5	68



$\boxed{\text{IPO}}$ $\boxed{\text{EP.O}}$

$$(\sigma_i)_{120} = \frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} \cdot P_i = \frac{(180)^2 + (120)^2}{(180)^2 - (120)^2} \times 30$$

$$= 78 \text{ MPa}$$

$$(\sigma_o)_{180} = \frac{2d_i^2}{d_o^2 - d_i^2} \times P_i = \frac{2(120)^2}{180^2 - 120^2} \times 30$$

$$= 48 \text{ MPa}$$

$$(\sigma_i)_{60} = -\left(\frac{2d_o^2}{d_o^2-d_i^2}\right)\times P_o = -\left(\frac{2(120)^2}{120^2-60^2}\right)\times 30$$
$$= -80 \text{ MPa}$$

$$(\sigma_o)_{120} = -\left(\frac{d_o^2+d_i^2}{d_o^2-d_i^2}\right)P_o = -\left(\frac{120^2+60^2}{120^2-60^2}\right)\times 30$$
$$= -50 \text{ MPa}$$

Calc. $\sigma_c = \frac{2d_i^2}{d_o^2-d_i^2}\times P_i$

$$(\sigma_c)_{120} = \frac{2(60)^2}{120^2-60^2}\times 80$$

$$\sigma_c = \frac{2d_i^2}{d_o^2-d_i^2}\times P_i =$$

$$\sigma_c = P_i d_i^2 \left(\frac{d^2+d_o^2}{d_o^2-d_i^2}\right)d^2$$

$=$

$d_o = 130$
$d_i = 60$
$d = 120$

$$P = \left(\frac{d_o^2-d^2}{d_o^2-d_i^2}\right)\frac{P_i d_i^2}{d^2}$$

$$= \left(\frac{130^2-120^2}{130^2-60^2}\right)\frac{80\times 60^2}{120^2} = 12.5 \text{ MPa}$$

~~Net for~~ Resultant pressure on the common surface $= 12.5+30$
$= 42.5$ (compressive)

Q. A tube 96 mm diameter is used to the reinforce a tube of 48 mm internal dia & 72 mm outer dia. The compound tube is made to withstand an internal pressure of 60 MPa. The shrinkage allowance is such that final max. stress in each tube is same. Determine this stress and plot a diagram to show the variation of hoops stress in the two tubes. Also calculate the initial diff. of diameter before the shrinkage. $E = 208$ GPa

Inner tube pressure on outer.

48, 72

$$(\sigma_c)_i = -\left(\frac{2d_o^2}{d_o^2-d_i^2}\right)\times P_o$$
$$= -\left(\frac{2\times 72^2}{72^2-48^2}\right)P$$
$$= -3.6P$$

$$(\sigma_c)_o = -\left(\frac{d_o^2+d_i^2}{d_o^2-d_i^2}\right)P_o = -\left(\frac{72^2+48^2}{72^2-48^2}\right)P$$
$$= 2.6P$$

Outer tube, Internal pressure.

72, 96

$$(\sigma_c)_o = \frac{2d_i^2}{d_o^2-d_i^2}P = \frac{2(72)^2}{96^2-72^2}$$
$$= 2.57P$$

$$(\sigma_c)_i = \frac{d_o^2+d_i^2}{d_o^2-d_i^2}\times P = 3.57P$$

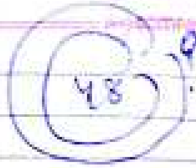
After Reinforcement

I. -22.3 -16.1 $0 = 55.6$ 140.7

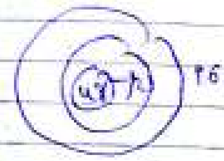
Due to internal pressure

$$(\sigma_c)_{48} = \frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} \times P_i \quad 96_o$$

$$= 100 \text{ MPa}$$



$$(\sigma_c)_{72} = \cancel{\frac{2d_i^2}{d_o^2 - d_i^2} \times P_i}$$



$$= \frac{d_o^2 + d^2}{d_o^2 - d_i^2} \cdot \frac{d_i^2}{d^2} \cdot P_i$$

$$= \frac{96^2 + 72^2}{96^2 - 48^2} \cdot \frac{48^2}{72^2} \cdot P_o$$

$$= 55.6 \text{ MPa}$$

Resultant stress

$100 - 22.3 = 77.7$

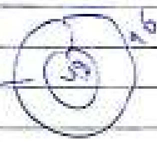
$55.6 - 16.1 = 39.5$

$35.6 + 22.1 = 77.7$

$-41 + 15.7 = 24.1$

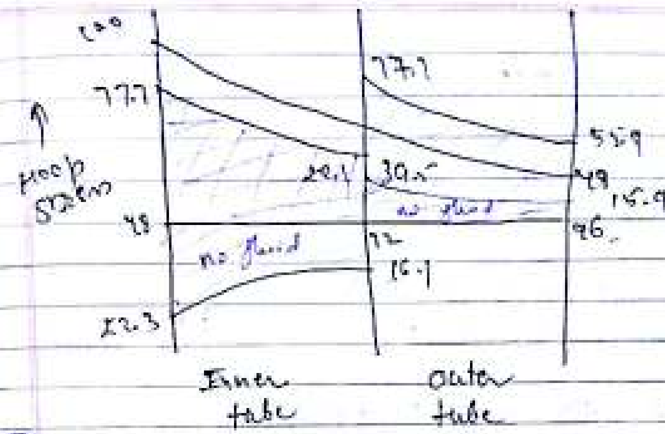
$$(\sigma_c)_{96} = \frac{2d_i^2}{d_o^2 - d_i^2} \times P_i$$

$$= 40 \text{ MPa}$$



$$-369 + 100 = 3.57P + 55.6$$

$$P = 0.19 \text{ MPa}$$

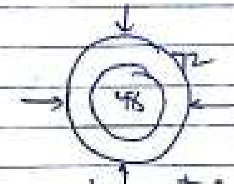


E.P.O.

$$\cancel{1/2}\ (\sigma_c)_{d_i=48} = -\left(\frac{2d_o^2}{d_o^2 - d_i^2}\right) P_o$$

$$= -\left(\frac{2 \times 72^2}{72^2 - 48^2}\right) P_o$$

$$= -3.6 P_o \text{ MPa}$$



inner tube

$$(\sigma_c)_{d_o=72} = -\left(\frac{d_o^2 + d_i^2}{d_o^2 - d_i^2}\right) P_o = -\left(\frac{72^2 + 48^2}{72^2 - 48^2}\right) P_o$$

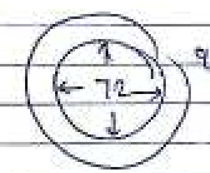
$$= 2.6 P_o \text{ MPa}$$

$\Rightarrow$ I. P.O

$$(\sigma_c)_{d_i=72} = \frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} \times P_i$$

$$= \frac{96^2 + 72^2}{96^2 - 72^2} \times P_i$$

$$= 3.57 P_i \text{ MPa}$$



External tube

$$(\sigma_c)_{d_o=96} = \frac{2d_i^2}{d_o^2 - d_i^2} \times P_i = \frac{2 \times 72^2}{96^2 - 72^2} \times P_i$$

$$= 2.57 P_i \text{ MPa}$$

I.P.

$$(\sigma_c)_{48} = \frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} \times P_i$$

compound

$$= \underline{100 \text{ MPa.}}$$

$$(\sigma_c)_{d_o = 96} = \frac{2d_i^2}{d_o^2 - d_i^2} \times P_i = \underline{40 \text{ MPa}}$$

$$-3.6P_o + 100 = 3.57P + 40$$
$$80 = 6.17P$$

$$\boxed{-3.6P_o + 100 = 3.57P + 50.6}$$

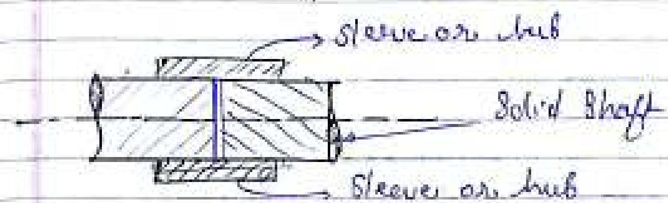
$$(\sigma_c)_{72} = \frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} \cdot \frac{d_i^2}{d^2} \cdot P_i$$

$$= \frac{96^2 + 72^2}{96^2 - 48^2} \times \frac{48^2}{72^2} \times 60$$

$$= \underline{55.50 \text{ MPa}}$$

20/3/2017

Hub on the Shaft/Sleeve on the Shaft.



The shaft is subjected to an external pressure. Let σ_c & σ_r be the hoop & radial stresses at the radius r. The following equation –

For radial pressure $(P) = -a + \frac{b}{r^2}$

and

hoop stress $(\sigma_c) = a + \frac{b}{r^2}$

Let, $b = 0$ for $r = 0$

$\therefore P = \sigma_c$ The stresses cannot be infinite at the centre of the solid shaft there

for $b = 0$, So $\boxed{\sigma_c = a = -p}$

which shows that the hoop stress is numerically equal to the radial stress & both are in compression.

Parameters –

d = diameter of the common surface of shaft & hub.

= inner dia of hub

= outer dia of the shaft

$P \rightarrow$ radial pressure at the common surface

$\sigma \rightarrow$ compressive hoop stress at the outside dia of the shaft.

$\sigma' \rightarrow$ tensile hoop stress at the inside dia of the hub.

$E \rightarrow$ Young's modulus of the material of the shaft.

$E' \rightarrow$ young's modulus of the material of the hub

$\mu \rightarrow$ poisson's ratio of the material of the shaft.

$\mu' \rightarrow$ poisson's ratio of the material of the hub.

δd = strain × d

$= \frac{\sigma - \mu p}{E} \times d$ --- decrease in shaft dia.

(both σ & p are compressive)

$$\delta d = \frac{-\sigma + \mu p}{E} \times d \quad \text{--- (1)}$$

~~In crease~~;

Increase in hub dia = strain × d

$$= \frac{\sigma' - \mu' p}{E'} \times d \quad \text{--- (2)}$$

$$= \frac{\sigma' + \mu' p}{E'} \times d$$

$\Rightarrow$ Difference of diameter before shrinkage --

$$= \frac{-\sigma + \mu p}{E} \times d - \left(\frac{\sigma' - \mu' p}{E'}\right) \times d$$

$$= d\left(\frac{-\sigma + \mu p}{E} - \frac{\sigma' - \mu' p}{E'}\right)$$

$$= d\left[-\left(\frac{\sigma - \mu p}{E}\right) - \left(\frac{\sigma' + \mu' p}{E'}\right)\right]$$

$$= d\left[\frac{\sigma - \mu p}{E} + \frac{\sigma' + \mu' p}{E}\right] \text{ in compression}$$

$\sigma = -p$

$\sigma = p$ / $= \left[\frac{\sigma' + \mu' p}{E'} + \frac{p - \mu p}{E}\right] \times d$ --- (3)

for internal diameter

$$= d_i\left[\frac{p(1-\mu)}{E} + \frac{\sigma' + \mu' p}{E'}\right]$$

I.P.O of d_i

$$= d_i\left[\frac{p(1-\mu)}{E} + \frac{\frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} \cdot p \times \mu' p}{E}\right]$$

$$= d_i p\left[\frac{1-\mu}{E} + \frac{\frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} + \mu'}{E'}\right] \text{ --- (4)}$$

If hub and shaft are of same material

for d_i

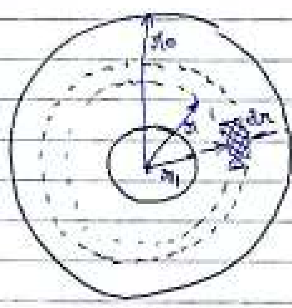
$= \frac{d_i p}{E}$ $d_i p \{ 4($

$$= \frac{d_i p}{E}\left[(1-\mu) + \frac{d_o^2 + d_i^2}{d_o^2 - d_i^2} + \mu\right]$$

$$= \frac{2p d_i d_o^2}{E\,(d_o^2 - d_i^2)} \quad --- \quad (5)$$

Date 21/2/2017

THICK SPHERICAL SHELL



Let u be the radial shift at the unstrained radius r, r becomes $r+u$ after straining.

Similarly $u+\delta u$ be the radial shift at unstrained radius $r+\delta r$

Radial strain = $\dfrac{\text{Increase in } \delta r}{\text{change in } r}$

$$= \frac{[(u+\delta u) - u]}{\delta r} = \frac{du}{dr}$$

or

Radial strain = $\dfrac{-p - \mu\sigma - \mu\sigma}{E}$

$$= -p - \mu(2\sigma)$$

$$E\frac{du}{dr} = -p - \mu\sigma - \mu\sigma$$

Circumferential (hoop) strain = $\dfrac{2\pi(r+u) - 2\pi r}{2\pi r}$

$$= \frac{u}{r}$$

Circumferential stress = $E\dfrac{u}{r}$

Circumferential strain = $\dfrac{\sigma - \mu(\sigma - p)}{E}$

$$\frac{Eu}{r} = \sigma - \mu\sigma + \mu p$$

Now differentiating

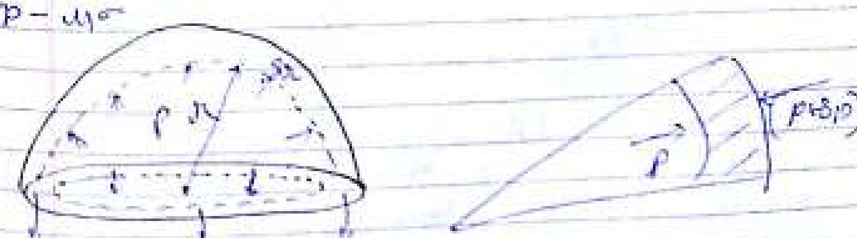
$$E\frac{u}{dr} = \sigma - \mu(\sigma - p) + r\left[\frac{d\sigma}{dr} - \mu\frac{d\sigma}{dr} + \mu\frac{dp}{dr}\right]$$

$$-p - \mu\sigma - \mu\sigma = \sigma - \mu(\sigma - p) + r\left[\frac{d\sigma}{dr} - \mu\frac{d\sigma}{dr} + \mu\frac{dp}{dr}\right]$$

$$-p - 2\mu\sigma = \sigma - \mu\sigma + \mu p + r\left[\frac{d\sigma}{dr}(1-\mu) + \mu\frac{dp}{dr}\right]$$

~~$-p - 2\mu\sigma$~~ =

$-p - \mu\sigma$



Equilibrium eq.

$$-\,8\pi r\,\delta r + (p+\delta p)\pi(r+\delta r)^2 = p\pi r^2$$

$$2\sigma r\delta r + (p+\delta p)(r^2+\delta r^2+2r\delta r) = pr^2$$

$$2\sigma r\delta r + \cancel{pr^2} + p\delta r^2 + 2pr\delta r + \delta p r^2 + \delta p\delta r^2 + \delta p\, 2r\delta r = \cancel{pr^2}$$

$$2\sigma r\delta r + 2p r\delta r + \delta p r^2 = 0$$

$$(\sigma+p)2\delta r = -\delta p r$$

By putting the value of $\sigma + p$ ~~$\delta\sigma = \delta p$~~ we get

$$\boxed{\frac{d\sigma}{dr} - \frac{1}{2}\frac{dp}{dr} = 0}$$

By integrating

$$\boxed{\sigma - \frac{1}{2}p = \text{constant} = A}$$

$$\boxed{\sigma = A + \frac{p}{2}}$$

$$\sigma + p = -\frac{r}{2}\left(\frac{\delta p}{\delta r}\right)$$

$$\left(A + \frac{p}{2}\right) + p = \frac{r}{2}\left(\frac{\delta p}{\delta r}\right)$$

$$\frac{2A+p}{2} + p = \frac{r}{2}\left(\frac{\delta p}{\delta r}\right)$$

$$3p + r\frac{dp}{dr} = -2A$$

we can write the above eqn as -

$$\frac{1}{r^2}\left[\frac{d}{dr}(pr^3)\right] = -2A$$

$$\frac{d}{dr}(pr^3) = -2Ar^2$$

Integrating above eq^n.

$$pr^3 = -2A\frac{r^3}{3} + B$$

$$p = -\frac{2}{3}A + \frac{B}{r^3}$$

Let, $a = \frac{2}{3}A$ $\quad$ $8B = b$, $B = \frac{b}{8}$

$8B = b$

then $\boxed{p = -a + \frac{b}{d^3}}$

$$p = -a + \frac{b/8}{r^3}$$

$$= -a + \frac{b/8}{(d/2)^3}$$

$$= -a + \frac{b/8}{d^3/8}$$

$$\sigma = A + \frac{p}{2}$$

$$= A + \frac{(-a + b/d^3)}{2}$$

$$= A + \frac{\left(\frac{-ad^3+b}{d^3}\right)}{2}$$

$$= A + \frac{(-ad^3+b)}{2d^3}$$

$$\boxed{\sigma = a + \frac{b}{2d^3}}$$

Let d_i & d_o be the inner & outer diameter with respect to the pressure P_i & P_o respectively

Then $P_i = -a + \frac{b}{d_i^3}$

$$P_o = -a + \frac{b}{d_o^3}$$

$$a = \frac{b}{d_i^3} - P_i$$

$$P_i = -\left(\frac{b}{d_i^3} - P_i\right) + \frac{b}{d_i^3}$$

$$= -\frac{b}{d_i^3} + P_i + \frac{b}{d_i^3}$$

$=$

$$\boxed{b = \frac{d_i^3 d_o^3 (p_i - p_o)}{d_o^3 - d_i^3}}$$

$$\boxed{a = \frac{p_i d_i^3 - p_o d_o^3}{d_o^3 - d_i^3}}$$

$$p = -a + \frac{b}{d^3} = -\left(\frac{p_i d_i^3 - p_o d_o^3}{d_o^3 - d_i^3}\right) + \frac{\frac{d_i^3 d_o^3 (p_i - p_o)}{d_o^3 - d_i^3}}{\frac{d^3}{1}}$$

$$= \frac{-p_i d_i^3 + p_o d_o^3}{d_o^3 - d_i^3} + \frac{d_i^3 d_o^3 (p_i - p_o)}{d^3 (d_o^3 - d_i^3)}$$

$$= \cancel{\frac{d_o^3 - d_i^3}{}}$$

$$\frac{1}{d_o^3}.$$

$$\boxed{p = \frac{p_o d_o^3 - p_i d_i^3 + (d_i^3 d_o^3 / d^3)(p_i - p_o)}{d_o^3 - d_i^3}}$$

$$\sigma = a + \frac{b}{2d^3}$$

$$= \frac{p_i d_i^3 - p_o d_o^3}{d_o^3 - d_i^3} +$$

$$\sigma_c = \frac{p_i d_i^3 - p_o d_o^3 + (d_i^3 d_o^3 / 2d^3)(p_i - p_o)}{d_o^3 - d_i^3}$$

$\boxed{\text{I. } p_o = 0}$

$$p = \frac{p_i d_i^3 \left(\frac{d_i^3 d_o^3}{d^3}\right) p_i}{d_o^3 - d_i^3}$$

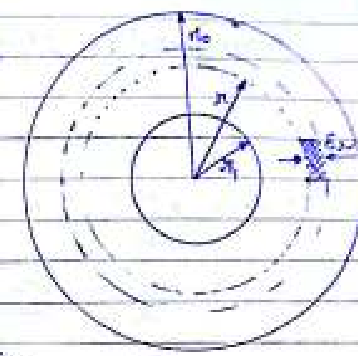
p_i

p_o

$\frac{d_o}{d_i} \to 0$

THICK SPHERICAL SHELL ($\sigma_c = \sigma_z$)

Let 'u' be the radial shift at the unstrained radius r, r becomes r+u after straining.



Similarly, u+δu be the radial shift at a ~~unstr~~ unstrained radius r+δr.

$\Rightarrow$ Radial strain = $\frac{\text{Increase in } \delta x}{\text{change in } x}$

$$= \frac{[(u+\delta u) - u]}{\delta x} = \frac{du}{dx} \quad \text{------ (i)}$$

$e = \frac{\delta l}{l}$

$\sigma = \frac{P}{A}$

$\frac{P}{A} = \frac{E \delta l}{l}$

Radial stress = $E \cdot \frac{du}{dx}$

$\Rightarrow$ Radial strain can also be written as–

$$\text{Radial strain} = \frac{-p - \mu(\sigma + \sigma)}{E} \quad \text{---} [\sigma_c = \sigma_z]$$

$$= \frac{-p - 2\mu\sigma}{E} \quad \text{------ (ii)}$$

From (i) & (ii) we can written as–

$$\frac{du}{dx} = \frac{-p - 2\mu\sigma}{E}$$

$$E \frac{du}{dx} = -p - 2\mu\sigma \quad \text{------ (iii)}$$

$\Rightarrow$ Circumferential (hoop) strain = $\frac{2\pi(x+u) - 2\pi x}{2\pi x} = \frac{2\pi x + 2\pi u - 2\pi x}{2\pi x}$

$$= \frac{2\pi u}{2\pi x} = \frac{u}{x} \quad \text{---- (iv)}$$

→ Circumferential stress $= E.\frac{u}{r}$

Circumferential strain $= \frac{\sigma - \mu(\sigma - p)}{E}$

$= \frac{\sigma - \mu\sigma + \mu p}{E}$ ---- (v)

From eq^n (v) & (iv)

$$\frac{u}{r} = \frac{\sigma - \mu\sigma + \mu p}{E}$$

$$E\frac{u}{r} = \sigma - \mu\sigma + \mu p \text{ ----- (vi)}$$

Now differentiating the equation (vi) w.r.t. r.

$$Eu = r(\sigma - \mu\sigma + \mu p)$$

$$E\frac{du}{dr} = \sigma - \mu\sigma + \mu p + r\left(\frac{d\sigma}{dr} - \mu\frac{d\sigma}{dr} + \mu\frac{dp}{dr}\right)$$

---- (vii)

Using eq^n (vii) and (iii), we get

$$-(p + 2\sigma\mu) = \sigma - \mu(\sigma - p) + r\left(\frac{d\sigma}{dr} - \mu\frac{d\sigma}{dr} + \mu\frac{dp}{dr}\right)$$

$$p + 2\mu\sigma + \sigma - \mu(\sigma - p) + r\left[\frac{d\sigma}{dr}(1-\mu) + \mu\frac{dp}{dr}\right] = 0$$

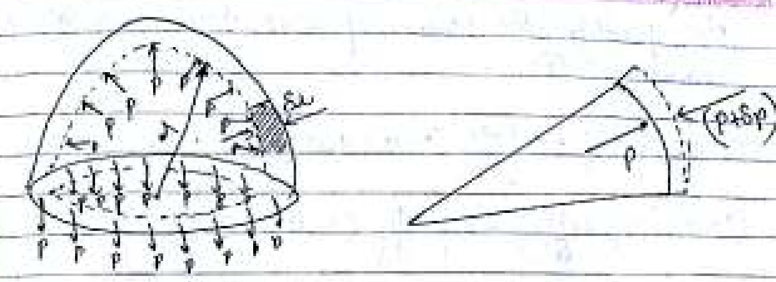
$$p + 2\mu\sigma + \sigma - \mu\sigma + \mu p + r\left[\frac{d\sigma}{dr}(1-\mu) + \mu\frac{dp}{dr}\right] = 0$$

$$p + \mu p + \sigma + \mu\sigma + r\left[\frac{d\sigma}{dr}(1-\mu) + \mu\frac{dp}{dr}\right] = 0$$

$$(1+\mu)p + (1+\mu)\sigma + r\left[\frac{d\sigma}{dr}(1-\mu) + \mu\frac{dp}{dr}\right] = 0$$

$$(1+\mu)(p+\sigma) + r\left\{\frac{d\sigma}{dr}(1-\mu) + \mu\frac{dp}{dr}\right\} = 0$$

------- (8)



Now, the equilibrium equation can be written as:-

$$\sigma 2\pi r\delta r + (p+\delta p).\pi.(r+\delta r)^2 = p\pi r^2$$

$$\sigma 2\pi r\delta r + \pi(p+\delta p)(r^2 + (\delta r)^2 + 2\delta r.r) = p\pi r^2$$

$$\sigma 2\pi.r.\delta r + \pi(pr^2 + p\delta r^2 + 2p\delta r.r + \delta pr^2 + \delta p\delta r^2 + 2\delta p\delta r.r) = p\pi r^2$$

$$\cancel{\pi}(2\sigma r.\delta r + pr^2 + p\delta r^2 + 2p\delta r.r + \delta pr^2 + \delta p\delta r^2 + 2\delta p\delta r.r) = p\cancel{\pi}r^2$$

$$2\sigma r.\delta r + p\delta r^2 + 2p\delta r.r + \delta pr^2 + \cancel{\delta p\delta r^2} + 2\delta p\delta r.r = \cancel{pr^2} - pr^2$$

Now neglecting the term containing $\delta p\delta r$ due to it smallest value-
we get

$$2\sigma r.\delta r + 2p\delta r.r + \delta pr^2 = 0$$

$$(\sigma + p)2\delta r.r = -\delta pr^2$$

$$\sigma + p = -\frac{\delta pr^2}{2\delta r.r}$$

$$\sigma + p = -\frac{1}{2}\times\left(\frac{\delta p}{\delta r}\right)(r)$$

$$\sigma + p = -\frac{r}{2}\left(\frac{\delta p}{\delta r}\right) \text{ ----- (9)}$$

Now, putting the value of $\sigma + p$ from eqn (9) in equation (7)

$$(1+\mu)(p+\sigma) + r\left[\frac{d\sigma}{dr}(1-\mu) + \mu\frac{dp}{dr}\right] = 0$$

$$(1+\mu)\left(-\frac{r}{2}\times\frac{\delta p}{\delta r}\right) + r\left[\frac{d\sigma}{dr}(1-\mu) + \mu\frac{dp}{dr}\right] = 0$$

$$-\frac{r}{2}(1+\mu)\frac{dp}{dr} + r(1-\mu)\frac{d\sigma}{dr} + r\mu\frac{dp}{dr} = 0$$

$$r\left\{-\frac{(1+\mu)}{2}\frac{dp}{dr} + (1-\mu)\frac{d\sigma}{dr} + \mu\frac{dp}{dr}\right\} = 0$$

$$\frac{dp}{dr}\left(\frac{-1-\mu}{2}+\mu\right) + (1-\mu)\frac{d\sigma}{dr} = 0$$

$$\frac{dp}{dr}\left(\frac{-1-\mu+2\mu}{2}\right) + (1-\mu)\frac{d\sigma}{dr} = 0$$

$$\frac{dp}{dr}\left(\frac{-1+\mu}{2}\right) + (1-\mu)\frac{d\sigma}{dr} = 0$$

$$-\frac{dp}{2\,dr}(1-\mu) + (1-\mu)\frac{d\sigma}{dr} = 0$$

$$(1-\mu)\left\{-\frac{dp}{2dr} + \frac{d\sigma}{dr}\right\} = 0$$

$$\boxed{\frac{d\sigma}{dr} - \frac{1}{2}\frac{dp}{dr} = 0} \quad \text{----- (10)}$$

By integrating the above eqn, we get -

$$\sigma - \frac{1}{2}p = \text{constant} = A$$

$$\boxed{\sigma = A + \frac{1}{2}p} \quad \text{----- (11)}$$

Now putting the value of σ in eqn (9)

$$\sigma + p = -\frac{r}{2}\left(\frac{\delta p}{\delta r}\right)$$

$$\left(A + \frac{p}{2}\right) + p = -\frac{r}{2}\left(\frac{\delta p}{\delta r}\right)$$

$$A + \frac{p}{2} + p = -\frac{r}{2}\left(\frac{\delta p}{\delta r}\right)$$

$$\frac{2A + p + 2p}{2} = -\frac{r}{2}\left(\frac{\delta p}{\delta r}\right)$$

$$2A + 3p = -r\left(\frac{\delta p}{\delta r}\right)$$

$$3p + r\left(\frac{\delta p}{\delta r}\right) = -2A$$

We can write the above equation as -

$$\frac{1}{r^2}\left[\frac{d}{dr}\cdot pr^3\right] = -2A$$

$$\frac{d}{dr}(pr^3) = -2Ar^2$$

Integrating the above equation -

$$pr^3 = -2A\times\frac{r^3}{3} + B$$

$$\boxed{p = -\frac{2A}{3} + \frac{B}{r^3}}$$

Let, $a = \frac{2A}{3}$ and $8B = b$

$B = b/8$

$$\therefore p = -a + \frac{b/8}{(d/2)^3}$$

$$\boxed{p = -a + \frac{b}{d^3}} \quad \text{----- (12)}$$

From equation (ii) & (iii)

$$\sigma = A + \frac{1}{2} \times P$$

$$\sigma = A + \frac{1}{2}\left(-a + \frac{b}{d^3}\right)$$

$$\sigma = \frac{3a}{2} + \frac{1}{2}\left(-a + \frac{b}{d^3}\right) \quad \text{------} \left[a = \frac{2A}{3}\right]$$

$$\sigma = \frac{3a}{2} - \frac{a}{2} + \frac{b}{2d^3}$$

$$\sigma = \frac{\cancel{2}a}{\cancel{2}} + \frac{b}{2d^3}$$

$$\boxed{\sigma = a + \frac{b}{2d^3}} \quad \text{------} \; (13)$$

Let d_i and d_o be the inner and outer diameters with respect to pressure P_i and P_o respectively.

Then,

$$P_i = -a + \frac{b}{d_i^3} \quad \text{and} \quad P_o = -a + \frac{b}{d_o^3}$$

$$P_i - P_o = -a + \frac{b}{d_i^3} - \left(-a + \frac{b}{d_o^3}\right)$$

$$= -\cancel{a} + \frac{b}{d_i^3} + \cancel{a} - \frac{b}{d_o^3}$$

$$= \frac{b}{d_i^3} - \frac{b}{d_o^3}$$

$$= b\left(\frac{1}{d_i^3} - \frac{1}{d_o^3}\right)$$

$$= b\left(\frac{d_o^3 - d_i^3}{d_i^3 d_o^3}\right)$$

$$\boxed{b = \frac{(d_i^3 d_o^3)(P_i - P_o)}{d_o^3 - d_i^3}}$$

$$P_i = -a + \frac{b}{d_i^3} = -a + \frac{\frac{(d_i^3 d_o^3)(P_i - P_o)}{d_o^3 - d_i^3}}{d_i^3}$$

$$P_i = -a + \frac{(d_i^3 d_o^3)(P_i - P_o)}{d_i^3(d_o^3 - d_i^3)}$$

$$= -a + \frac{d_i^3 d_o^3 . P_i - d_i^3 d_o^3 . P_o}{d_i^3(d_o^3 - d_i^3)}$$

$$= -a + \frac{\cancel{d_i^3}(P_i d_o^3 - P_o d_o^3)}{\cancel{d_i^3}(d_o^3 - d_i^3)}$$

$$P_i = -a + \frac{P_i d_o^3 - P_o d_o^3}{d_o^3 - d_i^3}$$

$$\boxed{a = \frac{P_i d_o^3 - P_o d_o^3}{d_o^3 - d_i^3} - P_i}$$

$$a = \frac{P_i d_o^3 - P_o d_o^3 - P_i(d_o^3 - d_i^3)}{d_o^3 - d_i^3}$$

$$P = -a + $$

$$a = \frac{\cancel{P_i d_o^3} - P_o d_o^3 - \cancel{P_i d_o^3} + P_i d_i^3}{d_o^3 - d_i^3}$$

$$P = -$$

$$\boxed{a = \frac{P_i d_i^3 - P_o d_o^3}{d_o^3 - d_i^3}}$$

Now, putting the value of a & b in general eqn of P

$$P = -a + \frac{b}{d^3}$$

$$P = -\left(\frac{P_i d_i^3 - P_o d_o^3}{d_o^3 - d_i^3}\right) + \frac{\frac{d_i^3 d_o^3 (P_i - P_o)}{d_o^3 - d_i^3}}{d^3}$$

$$p = \frac{-p_i d_i^3 + p_o d_o^3}{d_o^3 - d_i^3} + \frac{d_i^3 d_o^3 (p_i - p_o)}{d^3 (d_o^3 - d_i^3)}$$

$$\boxed{p = \frac{p_o d_o^3 - p_i d_i^3 + (p_i - p_o) d_i^3 d_o^3 / d^3}{d_o^3 - d_i^3}}$$ General equation of p

Now, $\sigma = a + \dfrac{b}{2d^3}$

$$\sigma = \frac{p_i d_i^3 - p_o d_o^3}{d_o^3 - d_i^3} + \frac{d_i^3 d_o^3 (p_i - p_o) / (d_o^3 - d_i^3)}{2d^3}$$

$$\sigma = \frac{p_i d_i^3 - p_o d_o^3}{d_o^3 - d_i^3} + \frac{d_i^3 d_o^3 (p_i - p_o) / 2d^3}{(d_o^3 - d_i^3)}$$

$$\boxed{\sigma = \frac{p_i d_i^3 - p_o d_o^3 + (p_i - p_o) d_i^3 d_o^3 / 2d^3}{d_o^3 - d_i^3}}$$ General eqn of σ

Cases

(I) For internal pressure only $(p_o = 0)$

$\Rightarrow$ For radial pressure

$$\boxed{p = \frac{-p_i d_i^3 + (p_i)(d_i^3 d_o^3)/d^3}{(d_o^3 - d_i^3)}}$$

At $d = d_i$

$$p = \frac{-p_i d_i^3 + (p_i)(d_i^3 d_o^3)/d_i^3}{(d_o^3 - d_i^3)}$$

$$p = \frac{-p_i d_i^3 + p_i d_o^3}{d_o^3 - d_i^3} = \frac{p_i (d_o^3 - d_i^3)}{d_o^3 - d_i^3}$$

$$\boxed{p = p_i}$$

At $d = d_o$

$$p = \frac{-p_i d_i^3 + (p_i)(d_i^3 d_o^3)/d_o^3}{d_o^3 - d_i^3}$$

$$p = \frac{-p_i d_i^3 + p_i d_i^3}{d_o^3 - d_i^3} = 0$$

$$\boxed{p = 0}$$

$\Rightarrow$ For hoop stress

$$\boxed{\sigma = \frac{p_i d_i^3 + (p_i)(d_i^3 d_o^3)/2d^3}{(d_o^3 - d_i^3)}}$$

At $d = d_i$

$$\sigma = \frac{p_i d_i^3 + (p_i)(d_i^3 d_o^3)/2d_i^3}{(d_o^3 - d_i^3)}$$

$$\sigma = \frac{p_i d_i^3 + p_i d_o^3/2}{d_o^3 - d_i^3} = \frac{2(p_i d_i^3) + p_i d_o^3}{2(d_o^3 - d_i^3)}$$

$$\sigma = \frac{p_i (2d_i^3 + d_o^3)}{2(d_o^3 - d_i^3)}$$

At $d = d_o$

$$\sigma = \frac{p_i d_i^3 + (p_i)(d_i^3 d_o^3)/2d_o^3}{(d_o^3 - d_i^3)}$$

$$\sigma = p_i d_i^3 + p_i d_i^3/2$$

$$\sigma = \frac{2 p_i d_i^3 + p_i d_i^3}{2(d_o^3 - d_i^3)}$$

$$\sigma = \frac{3 p_i d_i^3}{2(d_o^3 - d_i^3)}$$

(II) For external pressure only ($P_i = 0$)

⇒ For radial pressure

$$\boxed{P = \frac{-P_o d_o^3 + P_o \times d_i^3 d_o^3 / d^3}{(d_o^3 - d_i^3)}}$$

At $d = d_i$

$$P = \frac{P_o d_o^3 - P_o d_o^3}{d_o^3 - d_i^3} \Rightarrow \boxed{P = 0}$$

At $d = d_o$

$$P = \frac{P_o d_o^3 - P_o d_i^3}{d_o^3 - d_i^3} = \frac{P_o \cancel{(d_o^3 - d_i^3)}}{\cancel{(d_o^3 - d_i^3)}}$$

$$\boxed{P = P_o}$$

⇒ For radial stress

$$\boxed{\sigma = \frac{-P_o d_o^3 + P_o \cdot d_i^3 d_o^3 / 2d^3}{(d_o^3 - d_i^3)}}$$

At, $d = d_i$

$$\sigma = \frac{-P_o d_o^3 - P_o d_o^3/2}{d_o^3 - d_i^3}$$

$$\sigma = \frac{-3P_o d_o^3}{2(d_o^3 - d_i^3)}$$

At, $d = d_o$

$$\sigma = \frac{-P_o d_o^3 - P_o d_i^3/2}{(d_o^3 - d_i^3)}$$

$$\sigma = \frac{-P_o(2d_o^3 + d_i^3)}{2(d_o^3 - d_i^3)}$$

⇒ Max. shear stress $= \frac{1}{2}(\sigma_i + P_i)$

$= \frac{1}{2}(\sigma + P)$ at d_i (at I.P. 0)

$$= \frac{1}{2}\left[\frac{P_i(2d_i^3 + d_o^3)}{2(d_o^3 - d_i^3)} + P_i\right]$$

$$= \frac{1}{2}\left[\frac{P_i(2d_i^3 + d_o^3) + P_i \cdot 2(d_o^3 - d_i^3)}{2(d_o^3 - d_i^3)}\right]$$

$$= \frac{1}{2}\left[\frac{\cancel{2P_i d_i^3} + P_i d_o^3 + 2P_i d_o^3 - \cancel{2P_i d_i^3}}{2(d_o^3 - d_i^3)}\right]$$

$$= \frac{1}{2}\left[\frac{3P_i d_o^3}{2(d_o^3 - d_i^3)}\right]$$

$$\boxed{\tau_{max} = \frac{3P_i d_o^3}{4(d_o^3 - d_i^3)}}$$

Date 23/2/2017

Ques A spherical shell of 60 mm inside diameter has to be withstand an internal pressure of 25 MPa. Find the thickness of the shell if the max. tensile stress is to be 75 MPa.

Solution $d_i = 60$ mm

$p_i = 25$ MPa

$\sigma = 75$ MPa

$d_o = ?$

$$p_i = -a + \frac{b}{d_i^3}$$

$$25 \times 10^6 = -a + \frac{b}{(60)^3}$$

$$\sigma = a + \frac{b}{2d_i^3}$$

$$75 = a + \frac{b}{2(60)^3}$$

$75 = 432000a + b$

$32400000 = 432000a + b$

$32400000 - 432000a = b$

$a = 41.6$

$b = 14400000$

$$p_o = -a + \frac{b}{d_o^3}$$

$$0 = 41.6 + \frac{14400000}{d_o^3}$$

$0 = d_o^3(41.6) + 14400000$

$\boxed{d_o = 70.21}$

$$t = \frac{d_o - d_i}{2} = \frac{70.21 - 60}{2}$$

$t = 5.105$

Q → A thick spherical shell of 40 cm inside diameter is subjected to an internal pressure of 15 K/cm². Determine the thickness of the shell if the permissible stress in the shell material is 80 K/cm²

$d_i = 40$ cm

$$p_i = \frac{15\,K}{cm^2} = \frac{15\,kg}{cm^2} \times 9.81 = 147.15\ \frac{N}{cm^2}$$

$$\boxed{\frac{kg}{cm^2} \times 9.81 = \frac{N}{cm^2}}$$

$\sigma_{c_1} = 80\ K/cm^2 = 80 \times 9.81 = 784.8\ N/cm^2$

$\boxed{p_o = 0}$

$$p_i = -a + \frac{b}{d_i^3}$$

$$147.15 = -a + \frac{b}{(40)^3}$$

$$\sigma = a + \frac{b}{2(d_i)^3}$$

$$784.8 = a + \frac{b}{2(40)^3}$$

$\boxed{a = 147.14}$ $\boxed{b = \cancel{0.0127}}$ 18835200

$$p_o = -a + \frac{b}{d_o^3}$$

$$0 = 147.14 + \frac{\cancel{0.0127}\ 18835200}{d_o^3}$$

2D

stress tension

SF, vol

gen. double the

Two TOF (calcul. / Theory)

axial, shear, bending & calculation)

strain energy

Castigl.

Maxwell (derivation, theory)

Shear stress (derivation) I □ Δ

Thick cyl. (Lame's)

Date 6/3/2017

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UNIT-III

Column & Shaft

V+A+C.

A bar or a member of a structure in any position acted upon by a compressive load is known as struts.

However, when a compressive member is in a vertical position & liable to fail by bending or buckling. It may be referred as column. Otherwise they are known as short column which fail by only compression.

The equilibrium of a column or struts is situat similar to that of a rigid body i.e. stable or unstable, neutral.

* Stable condition:-

If a small axial load is applied to a column and a column returns to its original position. It is said to be stable cond^n.

* Neutral equilibrium/ condition or critical & crippling load.

If a load is increased to a value that on its removal at the deflection remain, it is the neutral equilibrium. This load is usually is known as critical or crippling or buckling load.

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* Unstable equilibrium

Any load beyond the neutral condition affect the column and it is unstable condition.

The resistance of a member against bending again its due to its flextural rigidity (EI) as $(I = Ak^2)$

$\therefore (EAk^2)$

For a member there can two moments of inertia and if the least of the considered than it is known

$\therefore$ $\boxed{\frac{l}{k} = \text{slenderness ratio}}$

where, l = length of column

k = ratio of gyration

Euler's theory (for long) struts and column which fail by buckling may exactly analysed by this theory.

Assumption:-

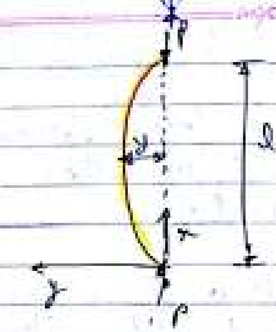
- The column is initially straight.
- Column is uniform throughout
- Line of thrust coincides exactly with the axis of columns.
- The material is homogeneous & isotropic
- The shortening of column due to axial compression is neglegible.

$\boxed{C.R = G_1 os x + G_2 cos x}$

Dat 7/3/2017

$M \to$ & CCW $-ve$; CW $\to$ $+ve$

① Both End Hinged



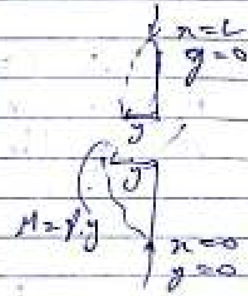
$$\boxed{EI\frac{d^2y}{dx^2} = M}$$

$$EI\frac{d^2y}{dx^2} = -Py$$

$$E.I.\frac{d^2y}{dx^2} + Py = 0$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}\cdot y = 0$$

Let us suppose, $\left[\alpha^2 = \frac{P}{EI}\right]$



$$\boxed{\frac{d^2y}{dx^2} + \alpha^2 y = 0}$$

$C.F. = A\sin\alpha x + B\cos\alpha x$

$\boxed{P.I = 0}$

$y = C.F. + P.I$

$= A\sin\alpha x + B\cos\alpha x$

$\boxed{y = A\sin\alpha x + B\cos\alpha x}$

Boundary cond^n.

① $x = 0,\ y = 0$

$\boxed{B = 0}$

② ~~If x=0~~

$y = 0,\ x = l$

$A\sin\alpha l = 0$

If, we take $\sin\alpha l = 0$

Since $\sin\alpha L = 0$ or $\alpha L \approx 0$

$\sin\pi = 0.05$ tends to zero

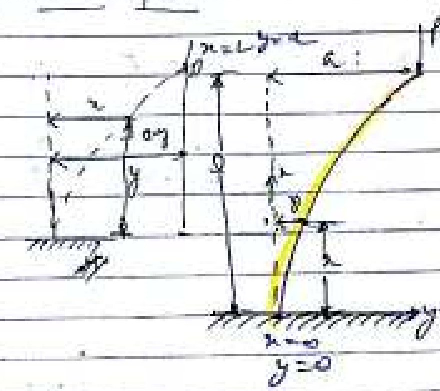
$\therefore \alpha L = \pi$

$\alpha = \frac{\pi}{L}$

$$\therefore \frac{\pi^2}{L^2} = \frac{P}{EI}$$

$$\boxed{P = \frac{\pi^2 EI}{L^2}}$$

② One End fixed other end free.



$$EI\frac{d^2y}{dx^2} = M$$

$$= P(a-y)$$

$$EI\frac{d^2y}{dx^2} = Pa - Py$$

$$\frac{d^2y}{dx^2} = \frac{Pa}{EI} - \frac{Py}{EI}$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 a = \frac{P}{EI}\cdot a \quad --- \left[\alpha^2 = \frac{P}{EI}\right]$$

$C.F = A\sin\alpha x + B\cos\alpha x$

$$\boxed{P.I. = \frac{Pa}{EI\alpha^2}}$$

$$y = A\sin\alpha x + B\cos\alpha x + \frac{Pa}{EI\alpha^2} \quad \text{---} \; (1)$$

Boundary Condn. —

(1) $x = 0, \; y = 0$

$$\boxed{B = -\frac{Pa}{EI\alpha^2} = -a}$$

(2) $x = 0, \; \frac{dy}{dx} = 0 \rightarrow$ slope

$$\boxed{A = 0}$$

(3) $x = L, \; y = a$

$$\boxed{\cos\alpha L = 0}$$

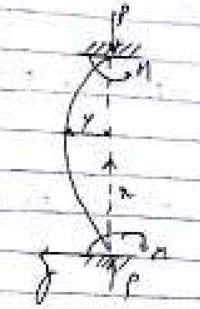
⇒ for $\alpha L = \frac{\pi}{2}$; $\alpha^2 = \frac{P}{EI}$

$$\frac{P^2}{E^2I^2}\cancel{L} = \frac{\pi}{\cancel{2}} \qquad \alpha = \frac{\pi}{2L}, \; \frac{\pi^2}{4L^2} = \frac{P}{EI}$$

$$P = \frac{\pi^2 EI}{4L^2}$$

$$\boxed{P = \frac{\pi^2 EI}{(2L)^2}}$$

(5) Both end fixed



$$EI\frac{d^2y}{dx^2} = -Py + M$$

$$\frac{d^2y}{dx^2} = \frac{-Py}{EI} + \frac{M}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M}{EI}$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \frac{M}{EI}$$

$$\boxed{y = A\sin\alpha x + B\cos\alpha x + \frac{M}{EI\alpha^2}}$$

Boundary condn.

(1) $x = 0, \; y = 0$

$$\boxed{B = -\frac{M}{EI\alpha^2}}$$

$$B = -\frac{M}{P} \quad \text{----} \quad \alpha^2 = \frac{P}{EI}$$

(2) $x = 0, \; \frac{dy}{dx} = 0$

$$\frac{dy}{dx} = A\alpha\cos\alpha x - B\alpha\sin\alpha x$$

$$0 = A\alpha$$

$$\boxed{A = 0}$$

(3) $x = l, \; y = 0$

$$0 = A\sin\alpha l + B\cos\alpha l + \frac{M}{EI\alpha^2}$$

$B \neq 0$

$\cos\alpha L = 1$

$\alpha L = 2\pi$

$\alpha = \frac{2\pi}{L}$

$\alpha^2 = \frac{4\pi^2}{L^2}$

$\frac{P}{EI} = \frac{4\pi^2}{L^2}$

$P = \frac{4\pi^2 EI}{L^2}$

$$\boxed{P = \frac{\pi^2 EI}{(L/2)^2}}$$

④ One end fixed, other end hinged

$$-Py + R(l-x) = EI\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \frac{R}{EI}(l-x)$$

$$y = A\sin\alpha x + B\cos\alpha x + \frac{R}{EI\alpha^2}(l-x)$$

Boundary condⁿ —

① $x=0\,;\; y=0$

$$B = -\frac{R}{EI\alpha^2}(l)$$

$$B = \frac{-Rl}{P}$$

② $x=0,\; \frac{dy}{dx}=0$

$$A = \frac{R}{P\alpha}$$

③ $x=l\,;\; y=0$

$$0 = \frac{R}{P\alpha}\sin\alpha l - \frac{Rl}{P}\cos\alpha l$$

$$\boxed{\tan\alpha l = \alpha l}$$

4.49 rad

$$\boxed{\alpha l = 4.49}$$

$$\alpha = \frac{4.49}{l}$$

$$\alpha^2 = \frac{P}{EI}$$

$$\frac{(4.49)^2}{l^2} = \frac{P}{EI}$$

$$P = \frac{(4.49)^2 EI}{l^2}$$

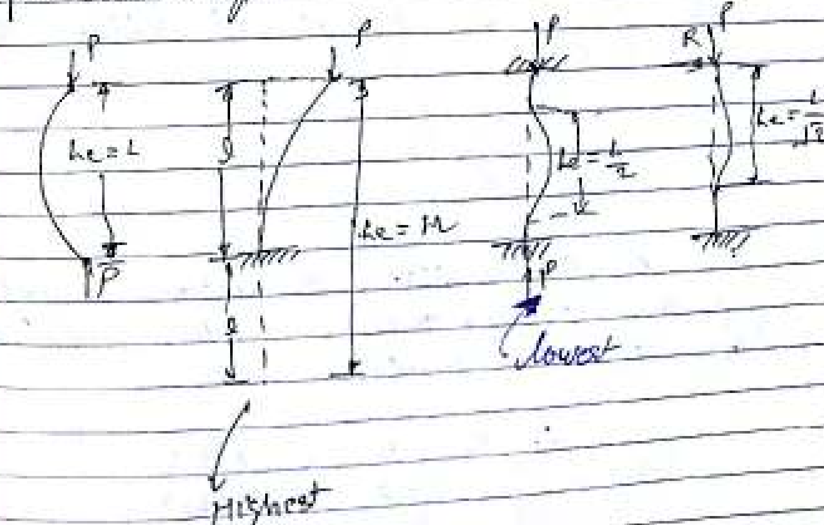
$$P = \frac{20.16 \times EI}{l^2}$$

$$\pi^2 = (3.14)^2$$

$$= \frac{\pi^2 EI}{l^2} \times 2.04$$

$$\boxed{P = \frac{\pi^2 EI}{\left(\frac{l}{\sqrt{2}}\right)^2}}$$

Equivalent length



* Limitation of Euler's theory :-

i) Initially in column there always been a eccentricity present in its curvature.

ii) It is noted that no strength property of material exist in the formula. The only property involve is young's modulus (E) which represent the stiffness characteristics of the material.

Thus due to imperfection, in practice a column suffers a deflection before the crippling load which increases with the load.

As a result a bending moment acts on the column that causes the failure before the euler load is reached & the failure is by stressed rather than by buckling.

$$\boxed{\sigma_c = \frac{P}{A}}$$

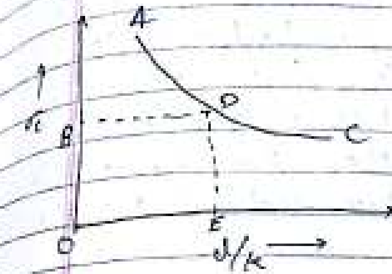
For the critical stress σ_c which is defined as an average stress over the cross section for a standard case

$$\therefore \sigma_c = \frac{P_e}{A} = \frac{\pi^2 EI}{A L_e^2}$$

$$= \frac{\pi^2 E (Ak^2)}{A L_e^2}$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/k)^2}$$

$\frac{L_e}{k}$ = Slenderness ratio



⇒ In the graph ADC plotted for critical stress vers slenderness ratio & it is known as euler's curve.

OB ⇒ yield criteria

OB ⇒ plastic stage

The curve is entirely defined by the magnitude of E.

At higher value of l/k for long column the value of critical stress falls rapidly. This condition of this rapidness cannot be improved by taking higher steel strength, which has higher modulus of ~~elastic~~ elasticity.

⇒ OB represent the yield stress of the material, obviously the euler formula can't applied if l/k is less than OE as below of this value the material will become plastic and will not follow hooke's law.

Example :- for steel σ_c = 320 MPa &
E = 205 GPa.

$$\left(\frac{l}{k}\right)_E = \frac{\pi^2 E}{\sigma_c} = \frac{\pi^2 \times 205\ GPa}{320\ MPa}$$

$$= 79.5 \approx 80\ MPa$$

For short column (l/k = 20-50)
" medium " (l/k = 50-80)
" long " (l/k = 80 → 120)

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Ques ⇒ A for 4 m long hollow alloy tube with dia (d_i = 36 mm & d_o = 48 mm) respectively, it elongate by 3 mm under the application of tensile force of 50 kN. Determine the buckling load for the tube when it is used as a column with a FOS of 5.

$$P_e = \frac{\pi^2 EI}{L^2}$$

$$\delta = \frac{PL}{AE}$$

$$E = \frac{50\times10^3\times4000}{\frac{\pi}{4}(48^2-36^2)\times3}$$

$$= \cancel{23.39}\ 252626.9$$

$$I = \frac{\pi}{64}(48^4-36^4) = 178128.3035$$

$$= \cancel{178128.30}$$

$$P_e = \boxed{1.846}\ \text{Ans.}$$

$$P_e = \frac{\pi^2\times23.39\times178128.30}{4\times10^3}$$

$$I = \frac{\pi}{64}(48^4-36^4)$$

$$= 178128.3035$$

$$\delta = \frac{PL}{AE}$$

$$E = \frac{PL}{AE}$$

$$= \frac{50\times10^3\times4000}{3\times\frac{\pi}{4}(48^2-36^2)}\ N/mm^2$$

$$= 84208.9646\ N/mm$$

$$P_e = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2\times84208.9646\times178128.3035}{(4000\ mm)^2}\ \frac{N\,mm}{mm^2}$$

$$= \cancel{1850.55\ N/mm}\ 9252.9\ N/mm$$

$$= \cancel{1.85\ kN/mm}$$

$$P_e = \frac{9252.9}{5} = 1850.55\ N/mm$$

$$= 1.85\ kN/mm$$

$$I_{xx} = I_{G_1} + A_1h^2 \qquad h = (y_1 - \bar{y})$$

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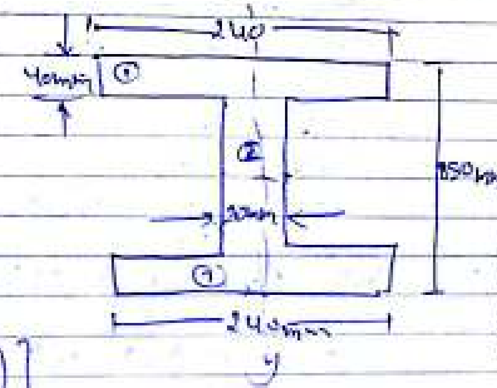
Ques ⇒ A simply supported beam of I-section as shown in the fig deflects 12 mm when subjected to a uniformly distributed load 50 kN/m length. Determine the safe load if the beam is used as a column with both end fixed. Use eulers formula with FOS 5 & E = 205 GPa.



$$I_{xx} = \frac{2}{12}\left(40(240)^3\right)$$

$$I_{xx} = \frac{240\times(880)^3}{12}$$

$$= 1.36\times10^{10}$$

$$I_{xx} = 1.76\times10^{10} - \left[2\left(\frac{800\times10^3}{12}\right)\right]$$

$$= 1.76\times10^{10} - \left[2\left(\frac{110\times800^3}{12}\right)\right] = 4413424.8\ mm^4$$

$$I_{yy} = \frac{880\times240^3}{12} - 2\left[\frac{800\times10^3}{12}\right]$$

$$I_{x_1} = \bar{I}_x + A(\ldots)^2$$

$$\boxed{l = 11.7\ m}\ \text{Ans}$$

$$S.L = \frac{3\omega l^3}{384\ EI_{xx}}$$

** Ques: Derive an equation for buckling applied on column with both end hinged. (6)

Ques: A hollow alloy tube 5 m long with dia 40mm & 25mm respectively was found to extend 6.4mm under a tensile load 6 tons. Find the buckling load for the tube when used as a column with both end hinged.

$d_i = 25mm$

$d_o = 40mm$

load $(P) = 6 \times 10^3$

$\delta = 6.4mm$

both ~~type~~ hinged $(l) = L$

$I = \frac{\pi}{64}(40^4 - 25^4) = \cancel{53.70}$

$= 106.488 \times 10^{3}$

$\delta = \frac{PL}{AE} = \frac{6000 \times 9.81 \times 5000}{\frac{\pi}{4}(40^2 - 25^2) \times E} = 6.4$

$\cancel{E = \frac{PL}{A\delta}}$

$E = 60050.3843$ MPa

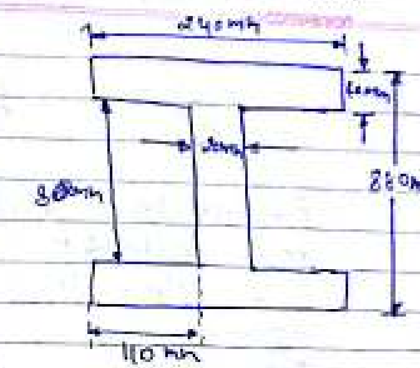
$P_e = \frac{\pi^2 EI}{L^2}$

$= \frac{\pi^2 \times 60050.38483 \times 106.488 \times 10^3}{(5000)^2}$

$= 2.522$ KN

Given - $\delta = 12mm$

$P = 50$ KN/mm



$I_{xx} = \frac{240(880)^3}{12} - 2\left[\frac{110(800)^3}{12}\right]$

$I_{xx} = 4242773333$

$I_{yy} = \frac{880 \times 240^3}{12} - 2\left[\frac{800(110)^3}{12}\right]$

$= 836293333.3$

Given FOS = 5, E = 205 GPa.

$\delta = \frac{PL}{AE} \Rightarrow E = \frac{PL}{\delta A} = \frac{50 \times 10^3 \text{ N/m} \times 880mm}{12mm \times}$

$P_e = \frac{\pi^2 EI}{L^2}$

$\delta = \frac{PL}{AE}$

$\frac{\delta \times AE}{P} = L$

$L = \frac{12mm \times 35200mm^2 \times 205 \times 10^9 Pa}{(50 \times 10^3) N/mm}$

$= 1.7364 \times 10^{12}$ mm

$P_e = \frac{\pi^2 EI}{L^2}$

$= \frac{\pi^2 \times 205 \times 10^9 Pa \times 4242773333}{(1.7364 \times 10^{12})^2 mm^2}$

$= 2.8406 \times 10^{-3}$ N

Safe load $= \frac{28406 \times 10^{-3}}{5}$

$= 5.681 \times 10^{-4}$ N

Date 17/3/2017

- long (buckling) P_{e}^{c} $P = P_e$
- short (crippling) P_c $P = P_c$
- medium

Rankine Formula

$$\boxed{\frac{1}{P} = \frac{1}{P_e} = \frac{1}{P_c}}$$

P = rankines crippling load
P_c = ultimate load for column $= \sigma_c \times A$
P_e = Euler's load for column = For both hinged $= \frac{\pi^2 EI}{L^2}$

For short columns P_e is very large and there $1/P_e$ is small in comparission to $1/P_e$. Thus the crippling load P is practically equal to P_c.

For long columns P_c is very small and there for $1/P_e$ is quite large in comparission to $1/P_c$. Thus the crippling load P is practically equal to P_e.

Rankine-Gordan formula

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_e}$$

$$\frac{1}{P} = \frac{P_c + P_e}{P_e P_c}$$

Case - both hinged.

Put, $P_c = \sigma_c . A$
$P_e = \frac{\pi^2 EI}{l^2}$

$$P = \frac{P_c . P_e}{P_c + P_e}$$

$$P = \frac{(\sigma_c . A)\left(\frac{\pi^2 EI}{l^2}\right)}{(\sigma_c . A) + \left(\frac{\pi^2 EI}{l^2}\right)} = \frac{\sigma_c . A}{1 + \frac{\sigma_c A L^2}{\pi^2 EI}}$$

$$P = \frac{\sigma_c . A}{1 + \frac{\sigma_c}{\pi^2 E}\left(\frac{l}{k}\right)^2}$$

Man moment of inertia $I = Ak^2$

$$\boxed{P = \frac{\sigma_c . A}{1 + a\left(\frac{l}{k}\right)^2}}$$

$$\frac{\sigma_c}{\pi^2 E} = a$$

Ques→ A 3.2m long both fixed end hallow cast iron column has its internal & external dia as 60mm, & 80mm respectively. Determine the rankines crippling load using the value of crussing stress to be 500 MPa & the value of rankine constant (a) = 1/1600

Solution → both end fixed
$\therefore \; l_e = l/2$

$$P = \frac{500 \times 10^6 \times \frac{\pi}{4}(80^2 - 60^2)}{1 + \frac{1}{1600}\left(\frac{3.2/2}{25}\right)^2}$$

$I = \frac{\pi}{64}(80^4 - 60^4)$

$1374446.786 = A k^2$

$\boxed{k = 25}$

$P = \cancel{1.0995}$ $.308$ kN

Cast iron (a) = $1/1600$

$\boxed{\text{working stress} = \frac{\sigma_c}{FOS}}$

Ques 1) A 4m long cast iron hollow column with both end formally fixed supports, and an axial load of 400 KN. The $d_i = 0.6 \times d_o$. Determine the section of column, FOS = 5, Crushing stress 560 MPa. and a = rankine const. (a) = $\frac{1}{1600}$.

$a = \frac{1}{1600}$

$d_i = 0.6 \times d_o$

FOS = 5

$\sigma_c = \frac{560}{4}$ MPa.

$= 140$ MPa (working stress)

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{K}\right)^2}$$

$A = \frac{\pi}{4}\left[(d_o)^2 - (0.6 \times d_o)^2\right]$

$= \frac{\pi}{4}\left[d_o^2 - 0.36 d_o^2\right]$

$= 0.503\, d_o^2$

$$\boxed{K = \frac{I}{A} = \frac{(D^2 + d^2)}{16}}$$

$K = \frac{d_o^2 + 0.6 d_o^2}{16}$

$= \frac{d_o^2 + 0.36 d_o^2}{16}$

$= \frac{1.36 d_o^2}{16}$

$K = 0.085\, d_o^2$

$400 \times 10^3 = \frac{140 \times 10^6 \times 0.503 d_o^2}{1 + \frac{1}{1600}\left(\frac{2}{0.085 d_o^2}\right)^2}$

$400 \times 10^3 = \frac{140 \times 10^6 \times 0.503 d_o^2}{1.0815 d_o^{4/2}}$

1.35

$36400.70415 d_o^2 =$

$D^4 - 7105 D^2 - 117.547 \times 10^6 = 0$

$t^2 - 7105t - 117.547 \times 10^6 = 0$

$\boxed{\begin{array}{l} t = 14961.5 \\ D = 122.3141667 \end{array}}$

$D^2 = 14961.5$

$D = 122.319$

$\boxed{d_1 = 73.39 \text{ mm}}$

* Johnson's Parabolic formula *

$$P = \sigma_c \cdot A \left[1 - b\left(\frac{l_e}{K}\right)^2\right]$$

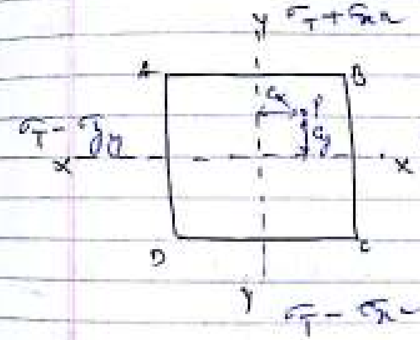
Mild steel, $\sigma_c = 290$ MPa, $b = 0.00003$ for both end hinged or pined

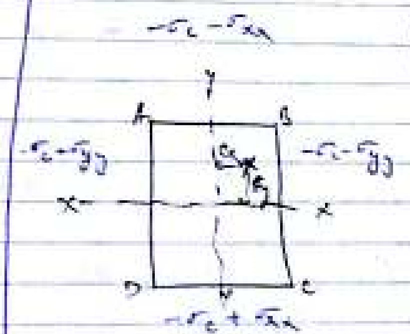
20/3/2017

* Combined Direct & Bending load *

$M_{xx} = P \cdot e_y$

$M_{yy} = P \cdot e_x$





$\sigma_A = -\sigma_c + \sigma_{yy} - \sigma_{xx}$ (Max^m)

$\sigma_B = -\sigma_c - \sigma_{xx} - \sigma_{yy}$

$\sigma_C = -\sigma_c + \sigma_{xx} - \sigma_{yy}$

$\sigma_D = -\sigma_c + \sigma_{yy} + \sigma_{xx}$

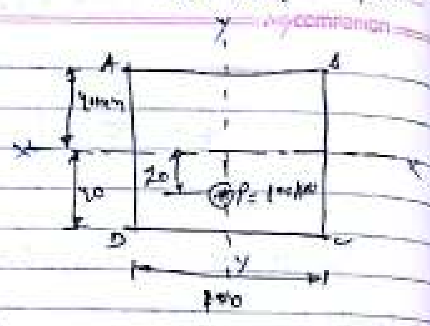
Ques) For the eccentricities of column as shown in the fig. determined the maxim stress developed in the cross section & ratio of Max. stress & axial stress developed in the cross section.

$Z = \frac{bd^3}{12} \times \frac{1}{y}$

$-\sigma_c + \sigma_{xx}$

$\sigma_c = \frac{P}{A} = \frac{-100 \times 10^3}{80 \times 100}$

$= -12.5 \ N/mm^2$



$\sigma_{xx} = \frac{M}{Z} = \frac{100 \times 10^3 \times 20}{\frac{100 \times 80^3}{12} \times \frac{1}{40}}$

$= 18.75 \ N/mm^2$

$-\sigma_c - \sigma_{xx}$

$\sigma_A = -\sigma_c - \sigma_{xx}$
$= (-12.5) - (18.75)$
$= 6.25 \ N/mm^2$

$\sigma_B = -\sigma_c - \sigma_{xx}$
$= -31.25 \ N/mm^2$

$\sigma_c = -\sigma_c + \sigma_{xx}$
$= -31.25 \ N/mm^2$

$\sigma_D = -31.25 \ N/mm^2$

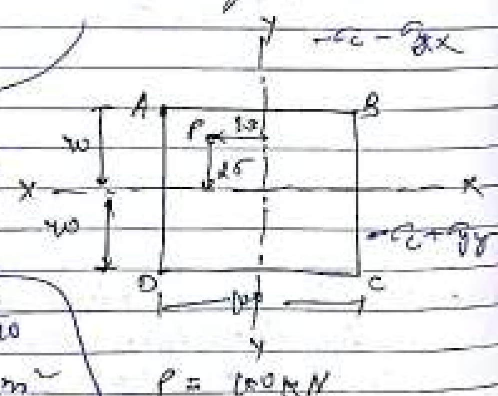
ratio $= \frac{\sigma_c (\text{or } \sigma_D)}{\sigma_{min}} = \frac{-31.25}{-14.5} = 5.2$

Q. Determin the stresses developed at the corners

$-\sigma_c - \sigma_{yx}$

$\sigma_c = \frac{P}{A} = \frac{100 \times 10^3}{80 \times 100}$

$= -12.5 \ N/mm^2$ $-\sigma_c - \sigma_{yy}$



$-\sigma_c + \sigma_{yy}$

$\sigma_{xx} = \frac{M}{Z} = \frac{100 \times 10^3 \times 25}{\frac{100 \times 80^3}{12} \times \frac{1}{40}}$

$= \cancel{150000} \ N/mm^2$
$23.4375 \ N/mm^2$

$P = 100 \ kN$

$-\sigma_c + \sigma_{xx}$

$\sigma_{yy} = \frac{M}{Z} = \frac{100 \times 10^3 \times 20}{\frac{100^3 \times 80}{12 \times 50}} = 15 \ N/mm^2$

$\sigma_A = -\sigma_c - \sigma_{yy} - \sigma_{xx}$
$= -(12.5) - 15 - 23.4375 = -50.93 \ N/mm^2$

$\sigma_B = -\sigma_c - \sigma_{xx} + \sigma_{yy}$
$= -12.5 - 23.45 + 15$ $= -20.935 \ N/mm^2$

$25.9375 \ N/mm^2$

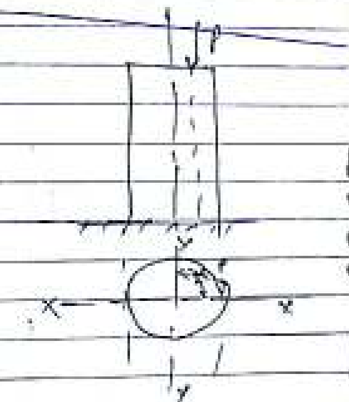
$-4625 \ N/mm^2$

Case

① $\sigma_A > \sigma_{bxx}$

② $\sigma_A < \sigma_{bxx}$

③ $\sigma_A = \sigma_{bxx} = 0$



$-\sigma_c + \sigma_{xx} \le 0$

$-\frac{P}{A} + \frac{M}{Z} \le 0$

$-\frac{P}{A} + \frac{P \cdot e_y}{Z} \le 0$

$-\frac{P}{A} + \frac{P \cdot e_y}{I/y} \le 0$

$-\frac{P}{A} + \frac{P e_y}{\frac{\pi}{32} d^3} \le 0$

$-\sigma_c + \sigma_{xx}$

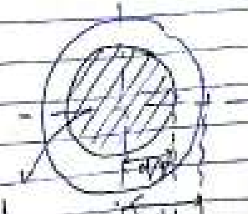
+ve column will fail

$-\sigma_c - \sigma_{xx}$

$\frac{e_y}{\frac{\pi}{32} d^3} \le \frac{1}{\frac{\pi}{4} d^2}$

$\boxed{e_y = \left(\frac{d}{8}\right)} \left(\frac{1}{4}\right)$

Middle 4th Rule



Core or Kernel

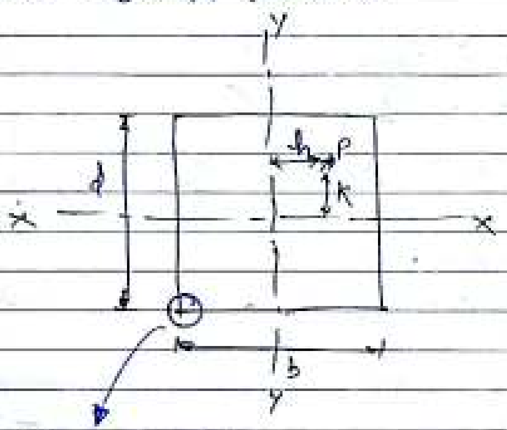
d/8

d/2

$$\left[\therefore \frac{e_{max\,y}}{y} = \frac{d}{2} \times \frac{8}{d} = 4\right]$$

*** Core ***

It is defined as the portion of a cross-section of a column within which the load should not act so that there is no tensile developed on the cross section.



$$-\sigma_c + \sigma_{xx} + \sigma_{yy} \le 0$$

$$\frac{-P}{bd} + \frac{P.k}{\frac{bd^3}{12}} \times \frac{2}{d} + \frac{P.h}{\frac{db^3}{12}} \times \frac{2}{b} \le 0$$

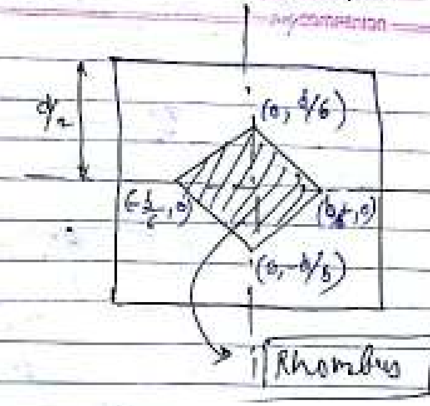
$$-1 + \frac{k}{\frac{d}{6}} + \frac{h}{\frac{b}{6}} \le 0$$

$$\frac{k}{d/6} + \frac{h}{b/6} \le 1$$

let
$h = 0$
$k = d/6$
$k = 0$
$h = b/2$

Let, $h = 0$
$k = d/6$

If
$k = 0$
$h = b/6$



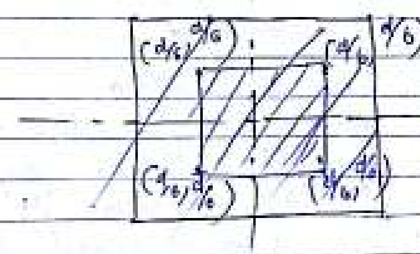
For square —

$d = b$

$$\frac{k}{d/6} + \frac{h}{d/6} \le 1, \quad k + h = \le d/6$$

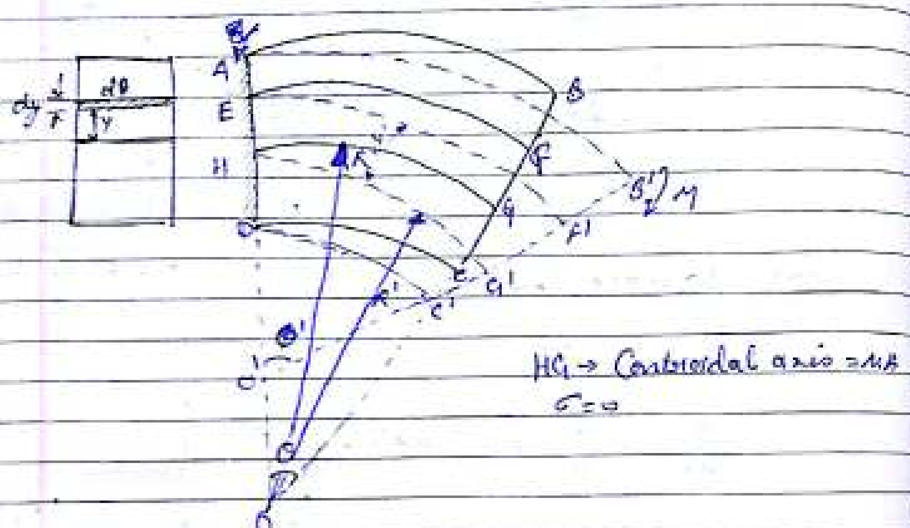
Let
$h = 0$
$k = d/6$

$k = 0$
$h = d/6$



21/8/2017

Stresses in Curved Beam (Winkler - Bach Theory)



Let us suppose as shown in the fig ABCD is a bar initially it is unstrained after applying bending moment 'M' ~~A'B'~~ AB'C'D be the ~~___~~ strained position of the bar.

R = radius of curvature of the centroidal axis HG.

y = distance of the fibre EF from the centroidal layer HG.

R' = radius of curvature of HG'

y' = distance b/w EF' and HG' after strain

θ = original angle substended by the centroidal axis HG and its centre of curvature O

θ' = original angle substended by the centroidal axis HG' and its centre of curvature O'

M = uniform bending moment apply to the beam

$$\boxed{\begin{matrix} M \to +ve\ (R\uparrow) \\ M \to -ve\ (R\downarrow) \end{matrix}}$$

Assumption :-

i) Plane section remains plane after bending.

ii) The material obeys hooke's law.

iii) Radial strain is negligible.

iv) The fibre are free to expand or contract with any constraining effect from the adjacent fibre.

⇒ To find the strain & stress normal to the section consider the fibre EF at a distance y from the centroidal axis

Let σ be the stress in the strained layer EF' under the bending moment 'M'.

⇒ Strain in EF fibre :-

$$e = \frac{EF' - EF}{EF} = \frac{(R'+y')\theta' - (R+y)\theta}{(R+y)\theta}$$

$$e = \frac{(R'+y')\theta'}{(R+y)\theta} - 1 \Rightarrow \boxed{1+e = \frac{(R'+y')\theta'}{(R+y)\theta}} \quad \text{— (1)}$$

⇒ Strain in G.H. (C.A.)

$$e_0 = \frac{R'}{R}\cdot\frac{\theta'}{\theta} - 1$$

$$\boxed{1+e_0 = \frac{R'}{R}\,\frac{\theta'}{\theta}} \quad \text{— (2)}$$

① ÷ ②

$$\frac{1+e}{1+e_0} = \frac{(R'+y')\theta'}{(R+y)\theta'} \times \frac{R\theta}{R'\theta}$$

$$\frac{1+e}{1+e_0} = \frac{R'+y'}{R+y} \times \frac{R}{R'}$$

$$1+e = \frac{(R'+y')R}{(R+y)R'} \times (1+e_0)$$

$$1+e = \frac{R'(1+e_0) + Re_0(R'+y')}{(R+y)R'}$$

$$e = \frac{R'(1+e_0) + Re_0(R'+y')}{(R+y)R'} - 1$$

$$e = \frac{R'(1+e_0) + Re_0(R'+y') - (R+y)R'}{(R+y)R'}$$

$$e = \frac{R' + R'e_0 + RR'e_0 + Re_0y' - RR' - R'y}{(R+y)R'}$$

$$e = \frac{(1+y'/R')(1+e_0) - \left(\frac{1+y}{R}\right)}{(1+y/R)}$$

Now from assumption ③ i.e. $y = y'$

$$\therefore e = \frac{(1+y/R')(1+e_0) - (1+y/R)}{(1+y/R)}$$

$$e = \frac{1+y/R' + e_0 + e_0y/R' - 1 - y/R}{(1+y/R)}$$

In this equation adding and substracting the the term $\frac{e_0y}{R}$

$$e = \frac{1+y/R' + e_0 + e_0y/R' - 1 - y/R + e_0y/R - e_0y/R}{(1+y/R)}$$

$$e = \frac{(1+e_0)\left[\frac{y}{R'} - \frac{y}{R}\right]}{(1+y/R)} + e_0 \quad \text{------ ③}$$

From the fig. it is obvious bending moment of the layer above the centroidal is in tension and below layer is in compression.

$$\sigma = eE$$

$$\sigma = \left(\frac{(1+e_0)\left[y/R' - y/R\right]}{(1+y/R)} + e_0\right) E \quad \text{------ ④}$$

Total force on a section

$$f = \int \sigma . dA$$

$$f = \int \left(\frac{(1+e_0)(y/R' - y/R)}{(1+y/R)} + e_0\right) . E \, dA.$$

$\left(\frac{N}{mm^2} \cdot mm^2 \cdot mm\right)$

$$F = E\left[\int e_0\, dA + (1+e_0)\left(\frac{1}{R_1} - \frac{1}{R}\right) \cdot \int \frac{y}{(1+y/R)}\, dA\right]$$

$$F = E\left[e_0 A + (1+e_0)\left(\frac{1}{R_1} - \frac{1}{R}\right)\int \frac{y}{(1+y/R)}\, dA\right] \quad \text{---} (5)$$

⇒ Now for moment

$$\boxed{\frac{M}{I} = \frac{\sigma}{y}}$$

$$M = \int \sigma y \cdot dA$$

$$M = \int \left(\frac{(1+e_0)(y/R_1 - y/R) + e_0}{(1+y/R)}\right) \cdot E \cdot y \cdot dA$$

$$M = E\left[\int e_0 y\, dA + (1+e_0)\left(\frac{1}{R_1} - \frac{1}{R}\right)\int \frac{y^2}{(1+y/R)}\, dA\right]$$

first moment of plane area w.r.t the centroidal axis is zero

$$\therefore \int y\, dA = 0$$

$$M = E\left[\int (1+e_0)\left(\frac{1}{R_1} - \frac{1}{R}\right)\int \frac{y^2}{(1+y/R)}\, dA\right]$$

Let, $\int \frac{y^2}{1+y/R}\, dA = Ah^2$

where

h^2 = a const for the cross-section of the bar.

Thus, $M = E(1+e_0)\left(\frac{1}{R_1} - \frac{1}{R}\right) Ah^2$

and,

$$\cancel{\int \frac{y}{1+y/R}\, dA = \int y - \frac{y^2}{1+y/R}}$$

Let, $$\int \frac{y}{1+y/R}\, dA = \int \frac{Ry}{R+y}\, dA = \int \left(y - \frac{y^2}{R+y}\right) dA$$

$$= \int \cancel{y\, dA}^{\,0} - \int \frac{y^2}{R+y}\, dA$$

$$= -\int \frac{y^2}{R+y}\, dA$$

$$\therefore = \frac{-1}{R}\int \frac{y^2}{1+y/R}\, dA$$

$$= \frac{-Ah^2}{R}$$

$$F = E\left[e_0 A + (1+e_0)\left(\frac{1}{R_1} - \frac{1}{R}\right)\int \frac{y}{1+y/R}\, dA\right]$$

$$= E\left[e_0 A + (1+e_0)\cancel{\int \frac{Ry}{R+y}}\, \frac{Ah^2}{R}\right]$$

F = 0, because the net force on the curved beam will be zero.

$$E\left[e_0 A - (1+e_0)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\frac{Ah^2}{R}\right] \text{ so}$$

$$\frac{e_0 R}{h^2} = (1+e_0)\left[\frac{1}{R_1} - \frac{1}{R}\right] \quad \text{---} (6)$$

Now, put in the 'M' equation :-

$$M = E(1+e_0)\left[\frac{1}{R_1} - \frac{1}{R}\right] Ah^2$$

$$\therefore M = \cancel{EA} \cdot E \frac{e_0 R}{h^2} \cdot \cancel{Ah^2}$$

$$M = E \cdot e_0 R A$$

$$\therefore \boxed{e_0 = \frac{M}{ERA}}$$

$$\sigma = \left(\frac{(1+e_0)\left(\frac{y}{R_1} - \frac{y}{R}\right)}{(1+y/R)} + e_0\right) E$$

$$= \left[\frac{\left(1+\frac{M}{ERA}\right)\left(\frac{y}{R_1} - \frac{y}{R}\right)}{(1+y/R)} + \frac{M}{ERA}\right] E$$

$$\cancel{\sigma = \frac{M}{AR}\left[1 + \frac{R^2}{h^2}\left(\frac{y}{R+y}\right)\right]}$$

$$= E\left[e_0 + \frac{(1+e_0)\left(\frac{1}{R_1} - \frac{1}{R}\right)y}{1+y/R}\right]$$

$$= E\left[e_0 + \frac{\frac{e_0 R}{h^2} \cdot y}{1+y/R}\right]$$

$$= E e_0\left[1 + \frac{R^2 y / h^2}{R+y}\right]$$

$$= E e_0\left[1 + \frac{R^2}{h^2} \cdot \frac{y}{R+y}\right]$$

$$= \frac{\cancel{E}}{\cancel{E}} \frac{M}{AR}\left[1 + \frac{R^2}{h^2} \cdot \frac{y}{R+y}\right]$$

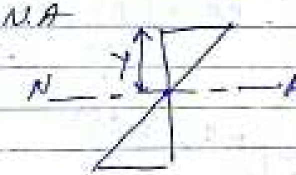
$$\boxed{\sigma = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right]} \quad \text{------ (7)}$$

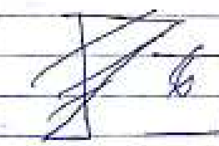
⇒ This equation is only for tensile in nature.

If y is opposite then the eqn. become -

$$\boxed{\sigma = \frac{M}{AR}\left[1 - \frac{R^2 y}{h^2(R+y)}\right]}$$ (compressive in nature)

For finding y at NA





Put $\sigma = 0$

$$\frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right] = 0$$

$$1 + \frac{R^2 y}{h^2(R+y)} = 0$$

$$\frac{R^2 y}{h^2(R+y)} = -1$$

$$R^2 y = -1\left(h^2(R+y)\right)$$

$$R^2 y = -h^2 R - h^2 y$$

$$R^2 y + h^2 y = -h^2 R$$

$$y(R^2 + h^2) = -h^2 R$$

$$\boxed{y = -\frac{h^2 R}{R^2 + h^2}}$$

* Value of h^2 for different Sections *

$$h^2A = \int \frac{y^2}{1+y/R}\, dA$$

$$h^2 = \frac{1}{A}\int \frac{Ry^2}{R+y}\, dA$$

$$h^2 = \frac{R}{A}\left[\int y\,dA^{\,0} - \int R\,dA + \int \frac{R^2\,dA}{R+y}\right]$$

$$= \frac{R}{A}\left[-RA + \int \frac{R^2}{R+y}\, dA\right]$$

$$= \cancel{\frac{R}{A}\left[-RA + \int R^2\right.}$$

$$= -R^2 + R^3\int \frac{dA}{R+y}$$

$$\boxed{h^2 = \frac{R^3}{A}\int \frac{dA}{R+y} - R^2}$$

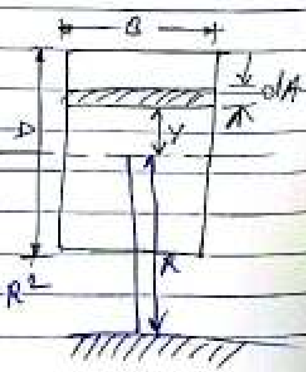
x ———— x ———— x

* Rectangular cross section.

$$h^2 = \frac{R^3}{A}\int_{-D/2}^{D/2} \frac{B\,dy}{R+y} - R^2$$

Area = $B\times D$

$dA = B\,dy$



$$h^2 = \frac{R^3}{BD}\times B\left[\ln(R+y)\right]_{-D/2}^{D/2} - R^2$$

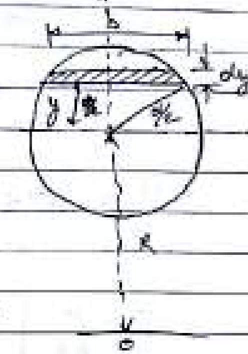
$$h^2 = \frac{R^3}{D}\left[\ln(R+D/2) - \ln(R-D/2)\right] - R^2$$

$$\boxed{h^2 = \frac{R^3}{D}\left[\ln\left(\frac{2R+D}{2R-D}\right)\right] - R^2}$$

24/3/2017

Circular Cross-section

$$\boxed{b = 2\sqrt{\left(\frac{d}{2}\right)^2 - y^2}}$$



$$A = \frac{\pi}{4}(d)^2$$

$dA = b\,dy$

$$h^2 = \frac{R^3}{A}\int \frac{dA}{R+y} - R^2$$

$$= \frac{(d/2)^3}{\frac{\pi}{4}(d)^2}\int_{-d/2}^{d/2} \frac{b\,dy}{(d/2)+y} - \left(\frac{d}{2}\right)^2$$

Expanding the integral by binomial expression we get

$$\left[h^2 = \frac{d^2}{16} + \frac{1}{128}\frac{d^4}{R^2} + \text{———}\right]$$

Ques> As shown in a frame subjected to 2.4 kN, find ① The resultant stress at pt 1 & 2 ② position of neutral axis

$$h^2 = \frac{R^3}{D}\left[\ln\left(\frac{2R+D}{2R-D}\right)\right] - R^2$$

$R = 48$

$D = 48$

$D = 48$

$$= \frac{48^3}{48}\left[\ln\left(\frac{2\times48+48}{2\times48-48}\right)\right]$$

$$h = \frac{48^3}{48}\left[\ln\left(\frac{2\times48+48}{2(48)-48}\right)\right] - 48^2$$

$$h^2 = 227.21\ mm^2$$

$$M_1 = -2.4\times10^3\times(120+48)$$

$$\sigma_{b_1} = \frac{M}{AR}\left[1+\frac{R^2y}{h^2(R+y)}\right]$$

$$= -\frac{403200}{48\times18\times48}\left[1+\frac{(48^2)(24)}{227.202(48+24)}\right]$$

$$= -42.58\ MN/m^2$$

$$\sigma_{b_2} = -\frac{403200}{48\times18\times48}\left[1-\frac{48^2(24)}{227.202(48-24)}\right]$$

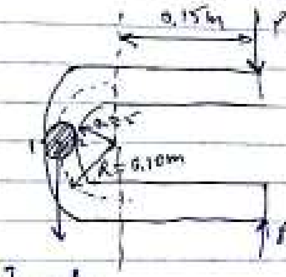
$$= 88.81$$

$$\sigma_D = \frac{2.4\times10^3}{48\times18} = 2.72\ MPa$$

$$\sigma_2 = 88.96 + 2.77 = 91.73$$

$$\sigma_1 = -42.16 + 2.77 = -39.39$$

Q→ The curved member as shown in fig. has a circular cross-section of 0.01 m in dia. If the max. tensile & compressive stresses are 150 MPa & 200 MPa. Determine the value of load P that can be safely carried by the member.



tensile = 150 MPa

compressive = 200 MPa

$$150 = \sigma_D + \sigma_{b_1}$$

$$-200 = \sigma_D + \sigma_{b_2}$$

$$h^2 = \cancel{\frac{R^3}{D}\left\{\ln\left(\frac{2D+D}{2D-D}\right)\right\} - R^2}$$

$$= \frac{d^2}{16} + \frac{1}{128} * \frac{d^4}{R^2}$$

$$M = P\times(0.015+0.05+0.05)$$

$R = 0.10$

$y = 0.05$

$$h^2 = \frac{0.01}{16} + \frac{1}{128}\times\frac{(0.01)^4}{(0.01)^2}$$

$$= 7.03\times10^{-6}$$

$$150 = \frac{-P}{A} \times \frac{M}{AR}\left[1+\frac{R^2y}{h^2(R+y)}\right]$$

$$-200 = \frac{-P}{A} + \frac{M}{AR}\left[1-\frac{R^2y}{h^2(R+y)}\right]$$

$A(150)$

$$\frac{150\times A}{1.26\ P} = P$$

$$150\times A = 1.26P\times P$$

$$0.0942 = 1.26P^2$$

$P = 88.62\ kN$

$46.13\ kN$

safe

① h^2 → calculation

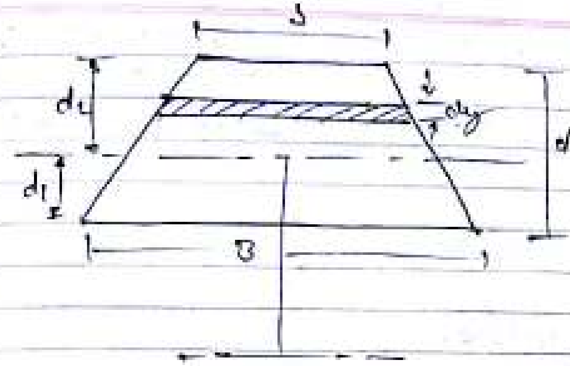
② M ($R^{\uparrow}\downarrow$, -ve in +ve)

③ outside → σ_b ⊕
inside → σ_b ⊖

④ σ_B

$\sigma_T = \sigma_B + \sigma_b$
inner, outer (crosssection)
(-y) (-y)

Date 27/3/2017

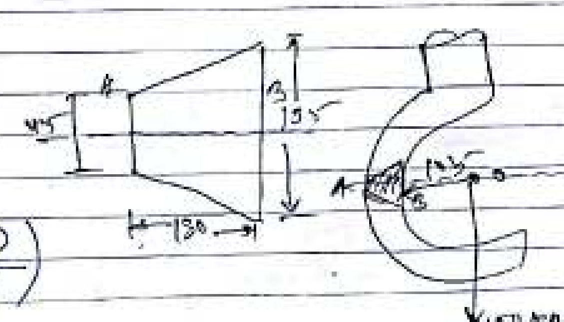


$$A = \left(\frac{B+b}{2}\right) \times d$$

$$d_1 = \frac{d}{3}\left(\frac{B+2b}{B+b}\right), \quad \boxed{d_2 = d - d_1}$$

$$h^2 = \frac{R^3}{A}\left[b \cdot \log_e\left(\frac{R+d_2}{R-d_1}\right) + \left(\frac{B-b}{d}\right)(R+d_2)\log_e\left(\frac{R+d_2}{R-d_1}\right)\right] - R$$

Prob) fig. shows a ~~two~~ crane hook lifting a load of 150 kN. Determine the max. compressive & tensile stresses in the critical section of the crane hook



$$A = \left(\frac{45+135}{2}\right)130$$
$$= 11700\ mm^2$$

$$d_1 = \frac{130}{3}\left(\frac{135+2(45)}{135+45}\right)$$
$$= 75\ mm$$

$$d_2 = d - d_1$$
$$= 180 - 75$$
$$= 105\ mm$$

$R = 66.7$

$d_1 = d - d_1 = 60 - 26.667$
$= 33.334\ mm$

$$h^2 = \frac{R^3}{A}\left[b \cdot \log_e\left(\frac{R+d_2}{R-d_1}\right) + \left(\frac{B-b}{d}\right)(R+d_2)\log_e\left(\frac{R+d_2}{R-d_1}\right) - (B-b)\right] - R^2$$

$$= \frac{(40+\cancel{26.667})^3}{2700}\left(20 \cdot \log_e\left(\frac{40+33.34}{40-26.667}\right) + \left(\frac{80-30}{60}\right)\left(\frac{40+31}{3}\right) \times \log_e\left(\frac{66.7+33.334}{66.7-26.66}\right) - (80-30)\right] - (66.7)^2$$

$= 107.74\,(51.130 + 4.563 - 9\,448.89)$

$h^2 = 309.\ mm^2$

$M = -30 \times 66.73$
$= 2001.9\ N$
$= \cancel{2.001\ KN}$
$= 2001.9\ N\text{-}mm$

$\sigma_D = \frac{P}{A} = \frac{30 \times 10^3}{2700} = \cancel{\ldots}$ 11.11 N-mm

$$(\sigma_B)_{Bending} = \frac{M}{AR}\left[1 - \frac{R^2 y}{h^2(R-y)}\right] \qquad \boxed{95.32\ N/A}$$

$= 0.010772$

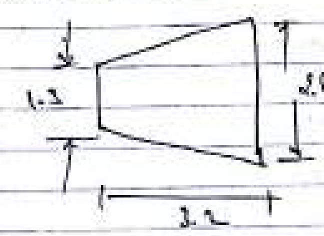
$(\sigma_2)_{bending} = -64.25\ MPa$
$\sigma_D = +11.11\ MPa$

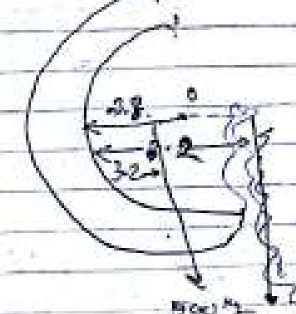
$$(\sigma_A)_{Bending} = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right]$$

$$= \frac{2001.9 \times 10^3}{2700 \times 66.7}\left[1 + \frac{66.7^2\,(33.334)}{319^{?}\,(66.7+33.3)}\right]$$

Q → A crane hook carries a load of 500kg, the line of stroke load of load being at a horizontal distance of 2.2cm from inside edge of a horizontal section through the centre of curvature & the centre of curvature be 3.8cm from the same edge. The horizontal section is a trapezium whose parallel sides are 1.3cm & 2.6cm & height is 3.2 cm. Determine the greatest tensile and compressive stresses in the hook.

Let cross-section is like –





$A = \left(\frac{B+b}{2}\right) \times d$

$= \left(\frac{2.6+1.3}{2}\right) 3.2 = 6.24\ \cancel{\ldots}\ cm^2$

$$y_2 = \frac{d}{3}\left(\frac{B+2b}{B+b}\right) = \frac{3.2}{3}\left(\frac{2.6+2(1.3)}{2.6+1.3}\right) = \begin{matrix}1.422\\ 1.234\end{matrix}\ cm$$

$R = 56.7\,m$

$d_2 = d - d_1 = 60 - 26.667$
$= 33.334\ mm$

$$h^2 = \frac{R^3}{A}\left[b \log_e\left(\frac{R+d_2}{R-d_1}\right) + \left(\frac{B-b}{d}\right)(R+d_2)\log_e\left(\frac{R+d_2}{R-d_1}\right) - (B-b)\right] - R^2$$

$$= \frac{(40+\cancel{26.667})^3}{2700}\left(20 \log_e\left(\frac{40+33.34}{40-26.667}\right) + \left(\frac{80-20}{60}\right)\left(\frac{40+33.3}{30}\right) \times \log_e\left(\frac{66.7+33.334}{66.7-26.66}\right) - (80-30)\right] - (66.7)^2$$

$= 107.74\,(51.130 + 4.863 - 4\,448.89)$

$\simeq 307\ mm^2$

$M = 30 \times 66.73$
$= 2001.9\ N$
$= \cancel{2.001.9\ KN}$
$= 2001.9\ N\text{-}mm$

$\sigma_D = \frac{P}{A} = \frac{30\times10^3}{2700} = \cancel{0.0116}$ 11.11 N-mm

(R) $\text{Bending} = \frac{M}{AR}\left[1 - \frac{R^2 y}{h^2(R-y)}\right]$ $\boxed{95.22\ N/A}$

$= 0.010772\ldots$

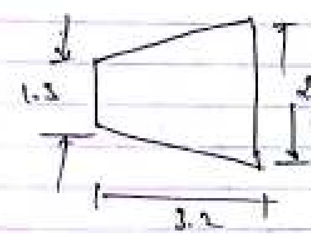
(Sb) bending $= -64.25$ MPa
$\sigma_D = +11.4$ MPa

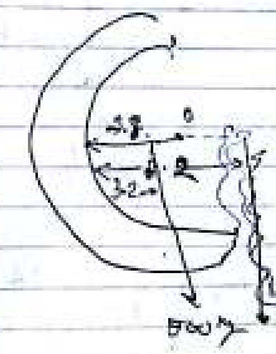
(St) $\text{Bending} = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right]$

$$= \frac{2001.9\times10^3}{2700\times66.7}\left[1 + \frac{66.7^2\,(33.334)}{319^4\,(66.7+33.3)}\right]$$

Q→ A crane hook carries a load of 500kg, the line of stroke load of load being at a horizontal distance of 3.2cm from inner edge of a horizontal section through the centre of curvature & the centre of curvature be 3.0cm from the same edge. The horizontal section is a trapezium whose parallel sides are 1.3cm & 2.6cm & height is 3.2 cm. Determine the greatest tensile and compressive stresses in the hook.

Let cross-section as like —





$A = \left(\frac{B+b}{2}\right)\times d$

$= \left(\frac{2.6+1.3}{2}\right)3.2 = 6.24\ \cancel{mm^2}\ cm^2$

$y = \frac{d}{3}\left(\frac{B+2b}{B+b}\right) = \frac{3.2}{3}\left(\frac{2.6+2(1.30)}{2.6+1.3}\right) = \cancel{1.284}\ 1.422\ cm$

$d_2 = d - d_1$

$= \cancel{3.2 - 1.234}$

$\boxed{\cancel{d_2 = 1.77}}$ $\boxed{d_2 = 1.77}$

$\boxed{R = 3.8 + d_1}$

$$h^2 = \frac{R^3}{A}\left[b \log_e\left(\frac{R+d/2}{R-d_1}\right) + \left(\frac{B-b}{d}\right)(R+d_2) \log_e\left(\frac{R+d/2}{R-d_1}\right) - (B-b)\right] - R^2$$

$$= \frac{(5.222)^3}{6.24}\left[1.3 \log_e\left(\frac{5.22+1.77}{5.22-1.422}\right) + \left(\frac{2.6-1.3}{3.2}\right)(5.22+1.77) \times \log\left(\frac{5.22+1.77}{5.22-1.42}\right) - (2.6-1.3)\right] - (5.22)^2$$

$= 22.79\left[0.7811 + 1.746 - 1.3\right] - 27.2484$

$\boxed{h^2 = 0.773684 \text{ cm}^2}$

$M = \cancel{500 \times 10^3 \times 5.22}$

$= 500 \times 9.81 \times (3.2 + 1.432)$

$= \cancel{22696.8}$

$= 22696.8$ N-cm

$$\sigma_D = \frac{P}{A} = \frac{500 \times 9.81}{6.24 \text{ cm}^2} = 786.05 \text{ N/cm}^2$$

$$(\sigma_r)_{B_0} = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R-y)}\right]$$

$$= \frac{-22696.8}{6.24 \times 5.22}\left(1 + \frac{5.22^2\left(\frac{1.77}{\cancel{5.22}}\right)}{(0.774)^2(5.22+1.77)}\right)$$

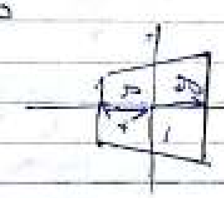
$= \frac{\cancel{87.22\cdot16}}{100} \frac{kN}{cm^2} \times 10^4 \times 100 = -87.22 \times 10^6 \frac{N}{m^2}$

$= 72.28 \times 10^6$ N/mm^2 m^2

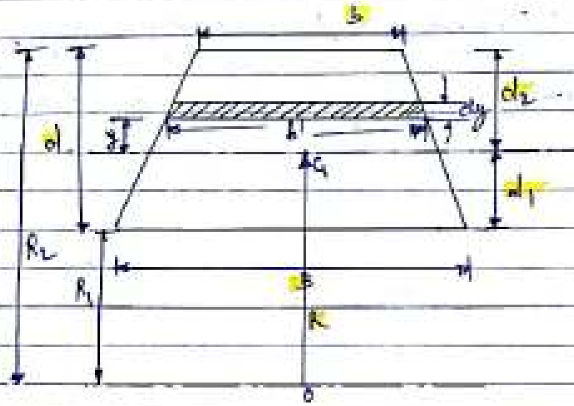
$$(\sigma_B)_B = \frac{-22696.8}{6.24 \times 5.22}\left(1 + \frac{5.22^2 (1.432)}{(0.774)^2 (5.22 - 1.42)}\right)$$

$= \frac{15963.2821}{100} \times \frac{N}{cm^2} \times 100$

$= 159.63 \times 10^6$ N/m^2



(III) **Trapezoidal Section**



Let, $R_2 - y = a$
$dy = da$

$$b' = b + \left(\frac{B-b}{d_1+d_2}\right)(R_2 - a)$$

Area of the strip –

$$dA = b'dy = \left[b + \left(\frac{B-b}{d_1+d_2}\right)(R_2-a)\right]da$$

$$h^2 = \frac{R^3}{A}\int_{R_1}^{R_2}\left[b + \left(\frac{B-b}{d_1+d_2}\right)(R_2-a)\right]\frac{da}{a} - R^2$$

$$= \frac{R^3}{A}\left[\int_{R_1}^{R_2}\frac{b\,da}{a} + \left(\frac{B-b}{d_1+d_2}\right)\int_{R_1}^{R_2}\left(\frac{R_2-a}{a}\right)da\right] - R^2$$

$$= \frac{R^3}{A}\left[b.\,|\log_e a|_{R_1}^{R_2} + \left(\frac{B-b}{d_1+d_2}\right)|R_2\log_e a - a|_{R_1}^{R_2}\right] - R^2$$

$$h^2 = \frac{R^3}{A}\left[b.\log_e\frac{R_2}{R_1} + \left(\frac{B-b}{d_1+d_2}\right)\left\{R_2\log_e\frac{R_2}{R_1} - (R_2-R_1)\right\}\right] - R^2$$

$$= \frac{R^3}{A}\left[b.\log_e\left(\frac{R+d_2}{R-d_1}\right) + \left(\frac{B-b}{d_1+d_2}\right)(R+d_2)\log_e\left(\frac{R+d_2}{R-d_1}\right) - (B-b)\right] - R^2$$

$$\boxed{h^2 = \frac{R^3}{A}\left[b.\log_e\left(\frac{R+d_2}{R-d_1}\right) + \left(\frac{B-b}{d}\right)(R+d_2)\log_e\left(\frac{R+d_2}{R-d_1}\right) - (B-b)\right] - R^2}$$

Where, $\boxed{A = \left(\frac{B+b}{2}\right)d}$

$$\boxed{d_1 = \frac{d}{3}\left(\frac{B+2b}{B+b}\right)}$$

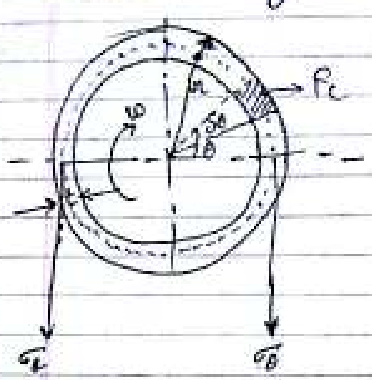
$$\boxed{d_2 = d - d_1}$$

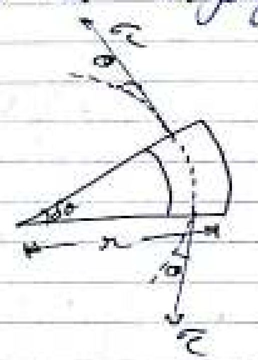
13/8/2017

UNIT-IV

* Stresses due to thin Rotating Ring *

A ring may be considered thin in radial direction if the variation of stresses along the thickness is negligible. & can be ingnore.





Let us considered a small element that is moving with omega velocity.

Let us suppose, r = mean radius
t = thickness of the ring
ρ = Material density of the ring.

Considere centrifugal force on the element per unit length

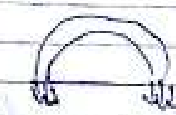
$$F_c = m r \omega^2$$
$$= \rho V r \omega^2 \quad ---- \rho = \frac{m}{V}$$
$$= \rho (r\, d\theta . t . 1)\, r\omega^2$$
$$F_c = \rho r^2 \omega^2 t\, d\theta$$

Total F_c in Vertical direction $= \rho r^2 d\theta . t . \omega^2 . \sin\theta$

If we have a vertical component of single component in ring.

Total F_c/length in upper part of the ring =

$$= 2\int_0^{\pi} \rho r^2 d\theta . t . \omega^2 \sin\theta$$
$$= 4\rho r^2 \omega^2 t \quad ---- (1)$$



Resisting force $= 4\sigma_\theta (t.1) \quad ---- (2)$

equating (1) & (2)

$$4\rho r^2 \omega^2 \cancel{t} = 4\sigma_\theta (\cancel{t}.1)$$
$$\sigma_\theta = \rho r^2 \omega^2$$
$$\boxed{\sigma_\theta = \rho v^2}$$

Q→ The rim of a rotating wheel is 1.2 m in dia. Determine the limiting speed of the wheel and the change in dia. of the wheel. If the max. stress is not to exceed 130 MPa. $\rho = 7700\,kg/m^3$ & $E = 205$ GPa. ~~let the~~ neglect the effects of spockes of the wheel. ~~The Treat~~ the rim to be thin.

$\sigma_\theta = \rho\, r^2 \omega^2$

$170 \times 10^6 = 7700 \times 1.2^2 \times \omega^2$

$\omega = 21.5$

$e = \frac{\delta d}{d}$

$e = \frac{\sigma_0}{t}$

$\frac{\delta d}{d} = \frac{\sigma_0}{t} \Rightarrow \frac{170 \times 10^6}{205 \times 100} = \frac{\delta d}{1.2}$

$\delta d = 0.76$ m

Q→ A fly wheel with the MOI ~~with~~ 800 kgm² & rotates at 300 rpm. If the max. stress is not to exceed 6 MPa, find the thickness of the ring. Take the width of the rim as 150 mm, ρ = 7400 kg/m³. Neglect the inertia of the spokes.

$\sigma_\theta = \rho\, r_0^2 \omega^2$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.42$

$6 \times 10^6 = 7400 \times r_0^2 \times 31.42^2$

$r_0 = 0.9$

mean radius

$MOI = [(2\pi R)\, bt]\, \rho]\, r^2$

~~$800 = [(2\pi r$~~

Let us suppose it would be less than 0.9 & we assuming it 0.85 than

$800 = 2\pi\, 0.85 \times 0.15 \times 7400 \times 0.85^2\, t$

~~$t = 0.01697$~~

$t = 0.07$

$MOI = 2\pi R . w . t\, \rho . R^2$

$R = \frac{R_0 + R_i}{2}$, $t = R_0 - R_i$

$R = \frac{0.85 + R_i}{2}$, $t = 0.85 - R_i$

$800 = 2\pi . \left(\frac{0.85 + R_i}{2}\right) . \frac{}{(31.42)} . (0.85 - R_i) . 7400 \times 0.15^2$

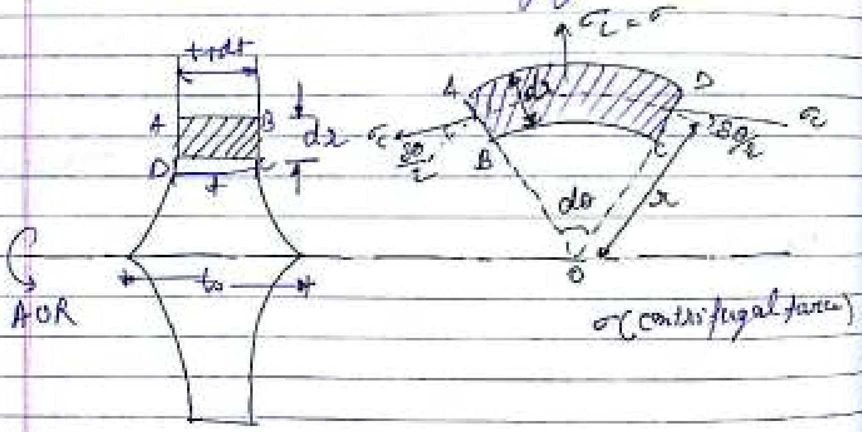
$300 =$

Date 30/3/2017

Disc of Uniform Strength

* A disc of uniform strength is one in which the value of radial & circumferential stresses are equal in magnitude for all the value of radius 'R'.

* This means that disc of uniform strength must have a varying thickness.



Let σ = uniform stresses in the radial & circumferential stresses.

Vol. of ABCD element $= r.d\theta.dr.t$ --- (i)

Centrifugal force of ABCD $= m r \omega^2$

$= \rho V r \omega^2$

$= \rho(r d\theta . dr . t)$ --- (ii)

Radial force of DC $=$ ~~$\sigma r d\theta$~~ $\sigma . A$

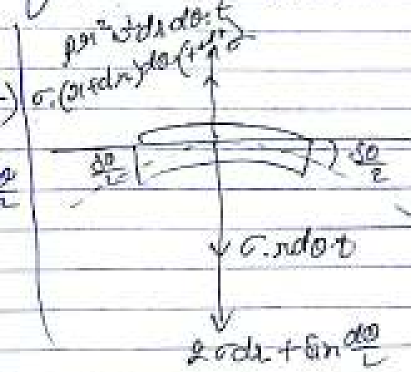
$= \sigma . r d\theta . t$ --- (iii)

Similarly radial force of AB $= \sigma . A$

$= \sigma (r+dr) d\theta (t+dt)$

Circumferential force on BC & BA $= \sigma (dr.t)$ --- (iv)

Resolving all the forces along the radial direction we get -



$\rho r^2 \omega^2 dr . d\theta . t + \sigma (r+dr) d\theta (t+dt)$

$= \sigma . r d\theta . t + 2\sigma . dr . t . \sin \frac{d\theta}{2}$

$\sin \frac{d\theta}{2}$ is very small

$\rho r^2 \omega^2 dr . d\theta . t + \sigma (r+dr) d\theta (t+dt)$

$= \sigma r . d\theta . t + 2\sigma dr . t . \frac{d\theta}{2}$

$\rho r^2 \omega^2 dr . d\theta . t + (\sigma r + \sigma dr) d\theta (t+dt) = \sigma r d\theta . t + 2\sigma dr . t . \frac{d\theta}{2}$

$\rho r^2 \omega^2 dr . d\theta . t + \cancel{\sigma r d\theta . t} + \sigma dr d\theta . dt = \cancel{\sigma r d\theta . t} + \cancel{2\sigma dr . t . \frac{d\theta}{2}}$

~~$\sigma r . d\theta . t = \sigma r . d\theta t$~~ +

$\rho . t . dr . \omega^2 r^2 + \cancel{r t . \sigma} + r dt \sigma$ ~~$t dr . t . \sigma = r t \sigma + t dr \sigma$~~

$\rho t dr . \omega^2 r^2 + r dt \sigma = 0$

$\rho dr \omega^2 r = -\frac{dt \sigma}{t}$

$\frac{\omega^2 \rho}{\sigma} \int r dr = - \int \frac{dt}{t}$

$$\frac{\omega^2 \rho r^2}{2} = -\ln t + \ln A$$

$$\frac{\omega^2 \rho r^2}{2} = \ln \frac{A}{t}$$

$$\frac{A}{t} = (e)^{\frac{\omega^2 \rho r^2}{2}}$$

$$\boxed{t = \frac{A}{e^{\frac{\omega^2 \rho r^2}{2}}}}$$

At, $r=0$, $t=t_0 \Rightarrow A = ??$

$$t_0 = \frac{A}{e^{\frac{\omega^2 \rho r^2}{2\sigma}}}$$

$$\boxed{t_0 = A}$$

$$\boxed{t = \frac{t_0}{e^{\frac{\omega^2 \rho r^2}{2\sigma}}}}$$

Ques A steam turbine rotor is to be designed so that the radial & circumferential stress are to be the same at constant throughout & equal to 90 MN/m², when running at 4000 rpm. If the axial thickness at centre is 20 mm, what is the

thickness at the radius of 400 mm. Assume ρ of rotor is 7800 kg/m³

$\sigma = 70$ MN/m²

$N = 4000$ rpm

$t_0 = 20$ mm

~~t = 400mm~~

$\rho = 7800$ kg/m²

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 4000}{60}$

$= 418.88$

$$t = \frac{\cancel{20mm}\ 0.02}{e^{\frac{(418.88)^2 \times 7800 \times (400)^2}{2 \times 90 \times 10^6}}}$$

$$\boxed{t = 5.93\text{mm}}$$

Date 3/4/2017

Determine the intensities of principle stress in a flat steel disc of uniform thickness having a diameter of ~~1m~~ 1m and rotating at 2400 rpm.

What will be the stresses if the disc has a central hole of 20m diameter. Take poisson ratio 1/3, & density 7850 kg/m³

$$\sigma_r = \frac{c_1}{2} + \frac{c_2}{r^2} - \left(\frac{3+1/m}{8}\right)\rho\omega^2 r^2$$

$$\sigma_c = \frac{c_1}{2} - \frac{c_2}{r^2} + \left(\frac{1+3/m}{8}\right)\rho\omega^2 r^2$$

For solid disc

$$\sigma_r = \left(\frac{3+1/m}{8}\right)\rho\omega^2(R^2 - r^2)$$

$\sigma_c = \left(\frac{1-1/m}{4}\right)\rho\omega^2 r_2^2$

$\omega = \frac{2\pi N}{60}$

$= 251.32$

$c_1 = \left(\frac{3+1/m}{4}\right)\rho\omega^2 r_2^2$

$= \left(\frac{3+3}{4}\right) 7750 \times (251.32)^2 (0.8)^2$

$c_1 = 103.3\ M$

$\sigma_c = \sigma_r = 57.65$

$\sigma_c = \left(\frac{1-3}{4}\right)(7850)(251.32)^2\left(\frac{0.5}{0.6}\right)^2$

$\sigma_c = 991$

$r = 0.8$

$B.C \rightarrow \quad r = 0.1, \quad \sigma_r = 0$

$r = 0.5, \quad \sigma_r = 0$

$\boxed{c_1 = 107.43}$

$\boxed{c_2 = -0.96}$

$c_1 + 200c_2 = 4.132$

$c_1 + 8c_2 = 103.3$

$c_1 = 4.132 - 200c_2$

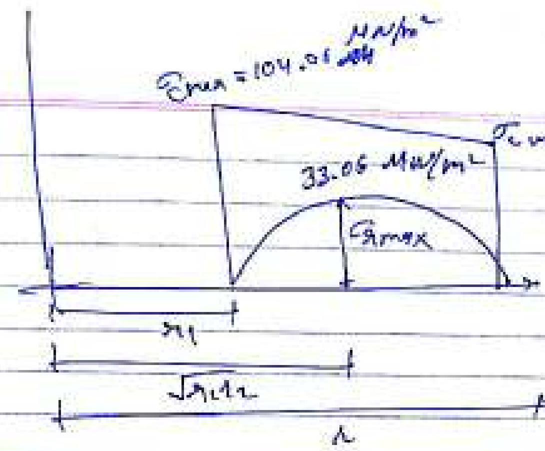
$(4.132 - 200c_2) + 8c_2 = 103.3$

$4.132 - 200c_2 + 8c_2 = 103.3$

$r = 0.1, \quad \frac{c_1}{2} + \frac{c_2}{(0.1)^2} = \left(\frac{3+0.33}{8}\right) \times 7850 \times (251.32)^2 \times (0.1)^2$

$=$

$\sigma_{max} = 104.01\ MN/m^2$



Disc of uniform thickness

$r_2 = 0.5m$

$N = 2400\ r.p.m$

Disc have central hole

$\omega = \frac{2\pi 2400}{60}$

$= 251.32$

$r_1 = 0.1\ m$

$1/3 = 1/m = \mu$

$\rho = 7850\ kg\ m^{-3}$

$\sigma_r = \frac{c_1}{2} + \frac{c_2}{r^2} - \left(\frac{3+1/m}{8}\right)\rho\omega^2 r^2$

$\sigma_c = \frac{c_1}{2} - \frac{c_2}{r^2} - \left(\frac{1+3/m}{8}\right)\rho\omega^2 r^2$

(1) Uniform thickness (solid disc)

Apply boundary condition. $r = r_1 = 0.1, \quad \sigma_r = 0$

$0 = \frac{c_1}{2} + \frac{c_2}{r^2} - \left(\frac{3+0.33}{8}\right)(7850)(0.1)^2 (251.32)^2$

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Date 4/1/2017

E4 ①

Solutions

$$\sigma_r = \frac{C_1}{2} + \frac{C_3}{r^2} - \rho \frac{\omega^2 r^2}{8}\left[\frac{3-2/m}{1-1/m}\right]$$

$$\sigma_c = \frac{C_1}{2} - \frac{C_3}{r^2} - \rho \frac{\omega^2 r^2}{8}\left[\frac{1+2/m}{1-1/m}\right]$$

For solid cyl.

$$\sigma_r = \frac{C_1}{2} - \rho \frac{\omega^2 r^2}{8}\left[\frac{3-2/m}{1-1/m}\right]$$

$$\sigma_c = \frac{C_1}{2} - \rho \frac{\omega^2 r^2}{8}\left[\frac{1+2/m}{1-1/m}\right]$$

$r = r_0$; $\sigma_r = 0$

$$0 = \frac{C_1}{2} - \rho \frac{\omega^2 r_0^2}{8}\left[\frac{3-2/m}{1-1/m}\right]$$

$$0 = \frac{C_1}{2} - 7700 \times \frac{(471.2)^2 \times (0.3)^2}{8}\left(\frac{3-2/3.5}{1-1/3.5}\right)$$

$0 = \frac{C_1}{2} - 80616273.35$

$C_1 = 153058176.93$

$\sigma_r =$

$\boxed{\sigma_x = 66.79}$

$\sigma_c =$

Ques→ A hollow cyl. 200mm external radius & 100mm internal radius is rotating at 3000 rpm. The density of material 7800 kg/m^{-3}, $\mu = 0.3$
① Calculate max. stress in cyl.
② Draw the variation curve radial & hoop stress

$r_o = 200$ mm
$r_i = 100$ mm
$N = 3000$ rpm

$\boxed{\omega = 314.16}$

$$\sigma_r = \frac{c_2}{2} + \frac{c_3}{r^2} - \rho \cdot \frac{\omega^2 r^2}{8}\left[\frac{3 - 2/m}{1 - 1/m}\right]$$

$$\sigma_c = \frac{c_2}{2} - \frac{c_3}{r^2} - \rho \cdot \frac{\omega^2 r^2}{8}\left[\frac{1 + 2/m}{1 - 1/m}\right]$$

at $r_o = 200$ mm

$$0 = \frac{c_2}{2} + \frac{c_3}{(200)^2} - 7800 \cdot \frac{(314.16)^2 (200)^2}{8}\left[\frac{3 - 2(0.3)}{1 - 0.3}\right]$$

$$0 = \frac{c_2}{2} + \frac{c_3}{0.04} - 7800\left(\right.$$

$$0 = \frac{c_2}{2} + \frac{c_3}{0.04} - 1319732.75$$

$$1319.7 \times 10^3 = \frac{c_2}{2} + \frac{c_3}{0.04}$$

$$= 0.04\, c_2 + 2c_3$$

$$105.5 \times 10^4 = 0.04\, c_2 + 2c_3 \quad \text{--- (1)}$$

$$2c_3 = 105.5 \times 10^4 - 0.04\, c_2$$

$$0 = \frac{c_2}{2} + \frac{c_3}{(0.1)^2} - 7800 \times \frac{314.16^2 \times 0.1^2}{8}\left[\frac{3 - 2(0.3)}{1 - 0.3}\right]$$

$$\boxed{77.815 \times 10^3 = 0.01 c_2 + 2c_3}$$

$$77.815 \times 10^3 = 0.01 c_2 + 105.5 \times 10^4 - 0.04 c_2$$

$$/\ 972185 = 0.03\, c_2$$

$$\boxed{c_2 = 32.46}$$

$$\boxed{c_3 = 52.7}$$

$$\sigma_r = \frac{c_2}{2} + \frac{c_3}{r^2} - \rho\left(\frac{\omega^2 r^2}{8}\right)\left[\frac{3 - 2/m}{1 - 1/m}\right]$$

$$\sigma_c = \frac{c_2}{2} - \frac{c_3}{r^2} - \rho\left(\frac{\omega^2 r^2}{8}\right)\left[\frac{1 + 2/m}{1 - 1/m}\right]$$

$r_2 = 200$mm $r_1 = 100$mm
$= 0.2$m $= 0.1$m
$N = 3000$ rpm
$\rho = 7800$ kg/m^{-3} $\omega = \frac{2\pi N}{60} \Rightarrow \boxed{\omega = 314.16}$
$\frac{1}{m} = 0.3$

at $r = r_2$, $\sigma_r = 0$

$$0 = \frac{c_2}{2} + \frac{c_3}{r_2^2} - \rho\left(\frac{\omega^2 r_2^2}{8}\right)\left(\frac{3 - 2/m}{1 - 1/m}\right)$$

at $r_2 = 0.2$

$$0 = \frac{c_2}{2} + \frac{c_3}{(0.2)^2} - 7800\left(\frac{(314.16)^2 (0.2)^2}{8}\right)\left[\frac{3 - 2(0.3)}{1 - 0.3}\right]$$

$$0 = 0.5 c_2 + 25 c_3 - 1319732.75$$

=

at $r_2 = 0.1$

$$0 = \frac{C_2}{2} + \frac{C_3}{(0.1)^2} - (7800)\left(\frac{314.16^2 \times 0.1^2}{8}\right)\left(\frac{3 - 2(0.3)}{1 - 0.3}\right)$$

$$0 = 0.5C_2 + \frac{100C_3}{m^2} - \frac{3299283.187}{m}$$

$$329928.187 = 0.5C_2 + 100C_3$$

$$\frac{329928.187 - 100C_3}{0.5} = C_2$$

$$0 = \cancel{0.5}\left[\frac{329928.187 - 100C_3}{\cancel{0.5}}\right] + 25C_3 - 1319712.15$$

$$0 = 329928.187 - 100C_3 + 25C_3 - 1319732.15$$

$$75C_3 = 171.562$$

$$C_3 = 171562.72\ ,\quad C_2 = 3368268.63$$

$$(\sigma_2)_{r_1} = \frac{C_2}{2} + \frac{C_3}{(0.1)^2} - (7000)\left(\frac{314.16^2 \times 0.1^2}{8}\right)\left(\frac{3 - 2(0.3)}{1 - (0.3)}\right)$$

$$= 15539623.13$$

Q. From Notes (Photo copy)

Solution:

$r = 300\,mm = 300 \times 10^{-3} = 0.3\,m$

$N = 4500\ rpm$

$\rho = 7800\ kg/m^3$

$\frac{1}{m} = 0.3$

$\omega = \frac{2\pi N}{60}$

$\boxed{\omega = 471.24}$

We know

$$\sigma_r = \frac{C_2}{2} + \frac{C_3}{r^2} - \rho \cdot \frac{\omega^2 r^2}{8}\left[\frac{3 - 2/m}{1 - 1/m}\right]$$

$$\sigma_c = \frac{C_2}{2} - \frac{C_3}{r^2} - \rho \cdot \frac{\omega^2 r^2}{8}\left[\frac{1 + 2/m}{1 - 1/m}\right]$$

Due to solid cylinder

$$C_3 = 0$$

$$\therefore \sigma_r = \frac{C_2}{2} - \rho \cdot \frac{\omega^2 r^2}{8}\left[\frac{3 - 2/m}{1 - 1/m}\right]$$

$$\sigma_c = \frac{C_2}{2} - \rho \cdot \frac{\omega^2 r^2}{8}\left[\frac{1 + 2/m}{1 - 1/m}\right]$$

Boundary condition

$$r = r_2\ ,\quad \sigma_r = 0$$

$$0 = \frac{C_2}{2} - \frac{(7800) \times (471.24)^2 (0.3\,m)^2}{8}\left(\frac{3 - 2(0.3)}{1 - 0.3}\right)$$

$$\frac{C_2}{2} = 29.69 \times 10^6$$

$$C_2 = 59.38 \times 10^6\ N/m$$

$\boxed{\sigma_r = 66.51}$ $\boxed{\sigma_c = 44.54}$

(a) $r_1 = 0.5m$, $N = 2400$ rpm

$r_2 = 0.1m$ $\frac{1}{m} = \frac{1}{3}$, $\rho = 7850\ kg/m^3$

(i) $\sigma_r = \frac{c_1}{2} + \frac{c_2}{r^2} - \left(\frac{3+1/m}{8}\right)\rho\omega^2 r^2$

$\sigma_c = \frac{c_1}{2} - \frac{c_2}{r^2} - \left(\frac{1+3/m}{8}\right)\rho\omega^2 r^2$

at $r=0$

$\boxed{\sigma_r = \sigma_c = \frac{c_1}{2}}$

at $r = r_2$, $\sigma_r = 0$

$0 = \frac{c_1}{2} + \frac{c_2}{r_2^2} - \left(\frac{3+0.33}{8}\right)\left(7850\frac{kg}{m^3}\right)(251.32)^2 (r_2)^2$

$0 = 0.5c_1 + \frac{c_2}{(0.5)^2} - \left(\frac{3+0.33}{8}\right)\left(7850\frac{kg}{m^3}\right)(251.32)^2 (0.5)^2$

$0 = 0.5c_1 + 4c_2 - 51596235.27\ kg/m$

$51596235.27 = c_1 + 8c_2$

$\boxed{103.19 = c_1 + 8c_2}$ $\frac{kg}{m}\times10^6$

at $r = 0.1$, $\sigma_r = 0$

$0 = 0.5c_1 + \frac{c_2}{(0.1)^2} - \left(\frac{3+0.33}{8}\right)(7850)(251.32)^2(0.1)^2$

$0 = \frac{0.5c_1}{0.5} + \frac{100c_2}{0.5} - \cancel{206}\ 4127698.818$

$\Rightarrow c_1 + 200c_2 = 4.12\times10^6\ kg/m$

$\cancel{\sigma_1 = (\sigma_r)_{r=0} = (\sigma_c)_{r=0} =}$

$c_1 = 4.12\times10^6\ kg/m - 200c_2$

$\cancel{c_1 +8(4.12\times10^6\ kg/m - 200c_2) + 8c_2 = 103.19\times10^6\ \frac{kg}{m}}$

$\cancel{c_1 + 32.96\times10^6\ \frac{kg}{m} - 1600c_2 + 8c_2 = 103.19\times10^6\ \frac{kg}{m}}$

$103.19\times10^6\ \frac{kg}{m} = \left(4.12\times10^6\ \frac{kg}{m} - 200c_2\right) + 8c_2$

$103.19\times10^6\ \frac{kg}{m} = 4.12\times10^6\ kg - 200c_2 + 8c_2$

$99.07\times10^6\ \frac{kg}{m} = -192\ c_2$

$\boxed{c_2 = 0.516\times10^6\ kg/m}$

$\boxed{c_1 = -99.07\times10^6\ \frac{kg}{m}}$

$(\sigma_c)_{max} = \frac{-99.07\times10^6\ kg}{2\ m} - \frac{0.516\times10^6}{(0.1)^2} - \left(\frac{1+3(0.33)}{8}\right)(7850)(251.32)^2(0.1)^2$

$(\sigma_c)_{max} = 102.3\times10^6\ MN/m^2$

Unsymmetrical Bending

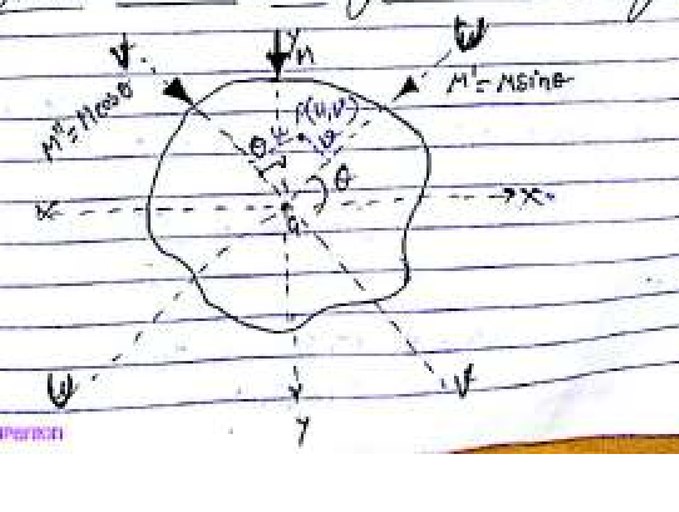
The plain of loading or that of bending does not lies in a plane that contains the principle centroidal axis of the cross section the loading is called unsymmetrical bending.

In Un-symmetrical bending the direction of neutral axis is not ⊥ to the plane of bending. following are the two reason of unsymmetrical bending.

(1) The section is symmetrical But the load line is inclined to the both principle axes.

(2) The section it self is unsymmetrical i.e. angle section or channel section vertical web. and the load line is along any centroidal axis.

Stresses due to Unsymmetrical bending



Let us suppose an arbitrary cross-section with a moment "M" on y-y axis.

⇒ Where G is the centroid of the cross-section
⇒ xx, yy are the co-ordinate axis passing through G
⇒ UU, VV are the principal axis inclined at any angle θ from xx, & yy respectively.

$$\boxed{\begin{aligned} u &= x\cos\theta + y\sin\theta \\ &\& \\ v &= y\cos\theta - x\sin\theta \end{aligned}}$$

The resultant bending stress at a point 'P' (u, v) is given by -

$$\boxed{\sigma_b = \frac{M'u}{I_{VV}} + \frac{M''v}{I_{UU}}}$$

$$\sigma_b = \frac{M\sin\theta \times u}{I_{VV}} + \frac{M\cos\theta . v}{I_{UU}}$$

To find the eqn of Neutral axis the locus of point on which resultant stress is zero

$$\sigma_b = M\left[\frac{u\sin\theta}{I_{VV}} + \frac{\cos\theta . v}{I_{UU}}\right] = 0$$

$$u = -v\frac{\cos\theta}{I_{UU}\sin\theta} \times I_{VV}$$

$$u = -\left[\frac{I_{VV}}{I_{UU}} \cdot \frac{\cos\theta}{\sin\theta} \times v\right]$$

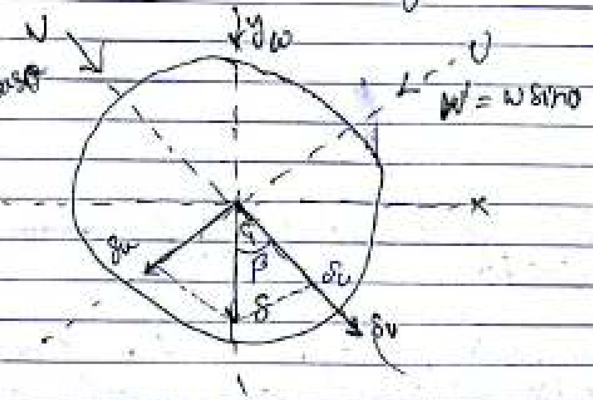
Similarly -

$$v = -\left[\frac{\sin\theta}{\cos\theta} \cdot \frac{I_{UU}}{I_{VV}}\right] u$$

$$\tan\alpha = \frac{v}{u} = \frac{(-)\sin\theta . I_{UU}}{\cos\theta . I_{VV}} \times u \times \left(-\frac{I_{UU}\sin\theta}{I_{VV}\cos\theta}\right) \times v$$

$$\boxed{\tan\alpha = -\left[\frac{I_{UU}}{I_{VV}}\tan\theta\right]}$$

* Deflection of Beam due to Unsymmetrical Bending



δu is the def deflection caused by component W sin θ on U axis

δv be the component of W cos θ on V axis

depending upon the end condn of the beam the deflection will be

$$\delta u = \frac{K W' L^3}{E I_{VV}}$$

$$\delta v = \frac{K\,W L^3}{E I_{uu}}$$

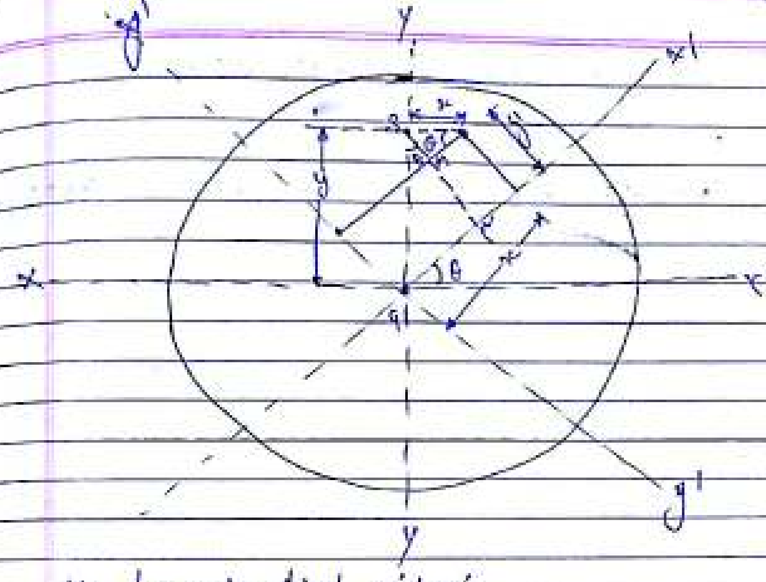
$$\delta = \sqrt{\delta u^2 + \delta v^2} =$$

$$\tan\beta = \frac{\delta v}{\delta u}$$

* Principle axis and principle ~~moment~~ moment of inertia.

If two axis about which the product of inertia is found are such that product of inertia becomes zero the two axis then called principle axis. The MOI about the principle axis is called principle moment of inertia.

→ Determination principle moment of inertia & direction of principle axis.



We have to find x' & y'.

$$\boxed{x' = y\sin\theta + x\cos\theta}$$

$$y' = MN$$

$$y' = SN - SM$$

$$\boxed{y' = y\cos\theta - x\sin\theta}$$

$$I_{xx} = \int y^2\,dA$$

$$I_{yy} = \int x^2\,dA$$

$$I_{x'y'} = \int x'y'\,dA$$

$$= \int (y\sin\theta + x\cos\theta)(y\cos\theta - x\sin\theta)\,dA$$

$$\boxed{I_{x'y'} = I_{xy}\cos 2\theta - \frac{I_{yy}-I_{xx}}{2}\sin 2\theta}$$

202

$$I_{x'y'} = \int (y^2 \sin\theta\cdot\cos\theta - xy\sin^2\theta + xy\cos^2\theta - x^2\cos\theta\cdot\sin\theta)\,dA$$

$$= \int (y^2 \cos 2\theta - xy\sin^2\theta + xy\cos^2\theta - x^2 \cos 2\theta)\,dA$$

$=$

This the general eq^n of product of moment of inertia about two ⊥ axis, if the M.O.I & product inertia about two ⊥ axis are known. So, when $I_{x'y'} = 0$, the axis ~~xx~~ xx' & yy' will become the principle axis.

$$I_{x'y'} = 0$$

$$I_{xy}\cos 2\theta - \frac{I_{xx}-I_{yy}}{2}\sin 2\theta = 0$$

$$I_{xy}\cos 2\theta = \frac{I_{xx}-I_{yy}}{2}\sin 2\theta$$

$$I_{xy} \times \frac{2}{I_{yy}-I_{xx}} = \frac{\sin 2\theta}{\cos 2\theta}$$

202 (9)

$$\boxed{\tan 2\theta = \frac{2I_{xy}}{I_{xx}-I_{yy}}}$$

$$\boxed{\begin{aligned} I_{x'x'} &= \int y'^2\, dA \\ I_{y'y'} &= \int x'^2\, dA \end{aligned}}$$

$$I_{x'x'} = \int (y\cos\theta - x\sin\theta)^2\, dA$$

$$= \int (y^2\cos^2\theta - x^2\sin^2\theta + 2y\cos\theta\cdot x\sin\theta)\,dA$$

$$= I_{xx}\cos^2\theta - I_{yy}\sin^2\theta + \int 2y\cos\theta\cdot x\sin\theta\, dA$$

$=$

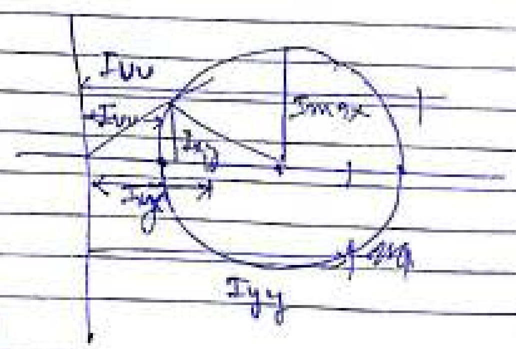
$\frac{dI_{x'x'}}{d\theta}$ ⇒ find, $\tan 2\theta$.

$$\boxed{\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{xx}}}$$ → condn for maxm. MOI.

$$I_{uu} = \frac{I_{yy}+I_{xx}}{2} + \sqrt{\left(\frac{I_{yy}-I_{xx}}{2}\right)^2 + (I_{xy})^2}$$

$$I_{vv} = \left[\frac{I_{yy}+I_{xx}}{2}\right] - \sqrt{\left(\frac{I_{yy}-I_{xx}}{2}\right)^2 + (I_{xy})^2}$$

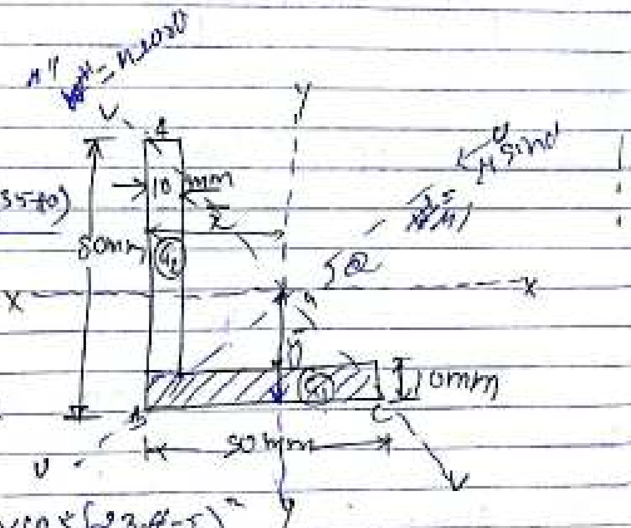
$\therefore$ $I_{uu} + I_{vv} = I_{xx} + I_{yy}$



Date
17/11/2017

A 80mm x 10mm angle section as shown in the fig. is used at a simply supported beam over a span of 2.4m. It carries a load of 400N along the line YG, where G is the centroid of the section. Calculate

(1) Stresses at the point A, B, C,
(2) Deflection of the beam at the mid-section & its direction with the load line
(3) Position of the neutral axis.



$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

$$= \frac{(800)(40) + (700)(35+10)}{800 + 700}$$

$$= 23.66\,mm$$

$$\boxed{\bar{x} = \bar{y}} = 23.66\,mm$$

$$I_{xx} = \frac{80\times10^3}{12} + 80\times10\times(23.66-5)^2$$

$$+ \left[\frac{10\times70^3}{12}\right] + 70\times10\times(45-23.66)^2$$

$$= 8.958\times10^5\,mm^4$$

$$I_{yy} = \frac{10\times80^3}{12} + 80\times10(40-23.66)^2 + \left[\frac{70\times10^3}{12} + 70\times10\times(23.66-5)^2\right]$$

$g_1\ (45,5)\ ,\ g_2\ (5,45)$

$\Rightarrow g_1[(40-23.66), -(28.66-5)] \Rightarrow g_1(16.34, 18.66)$

$g_2[-(23.66-5), (45-23.66)] \Rightarrow g_2(-18.66, 21.34)$

$\Rightarrow I_{xy} = A_1 x_1 y_1 + A_2 x_2 y_2$

$= 80 \times 10 \times$

$= (80\times10)(16.34)(-18.66) + (70\times10)(-18.66)(21.34)$

$\boxed{I_{xy} = -522666.6\ mm^4}$

$\Rightarrow \tan 2\theta = \dfrac{2 I_{xy}}{I_{yy} - I_{xx}} = \infty \quad [I_{yy} = I_{xx}]$

$2\theta = 90$

$\boxed{\theta_1 = 45^\circ}$

$\boxed{\theta_2 = 45 + 90 = 135}$

$\Rightarrow I_{uu} = \dfrac{8.983\times10^5 + 8.980\times10^5}{2} + \sqrt{(522666.6)^2}$

$= \cancel{922690.26} = 14.21 \times 10^5$

$I_{uu} + I_{vv} = I_{xx} + I_{yy}$

$14.81406\times10^5 + I_{vv} = 2(8.98\times10^5)$

$I_{vv} = 3.75140\times10^5$

$\Rightarrow M = \dfrac{wL}{4} = \dfrac{400\times2.4}{4} = 240\ Nm$

$\Rightarrow M' = M\sin\theta = 169.70 \quad | \quad M'' = M\cos\theta = M\cos(45) = 169.70$

$\boxed{\sigma = \dfrac{M'u}{I_{vv}} + \dfrac{M''v}{I_{uu}}}$

Point $A(-23.66, (80-23.66))$

$u = x\cos\theta + y\sin\theta$

$= -23.66\cos45 + 56.34\sin45$

$\boxed{u = 23.10}$

$v = y\cos\theta - x\sin\theta = 56.34\cos45 + 23.66\sin45$

$\boxed{v = 56.56}$

$\sigma_A = \dfrac{(169.70)(23.10)}{3.76140\times10^5} + \dfrac{(169.70)(56.56)}{14.21\times10^5}$

$\boxed{\cancel{\sigma_A = 3.655\times10^{-3}}}$ $\quad \sigma_A = 17.17\times10^6\ Pa$

$\boxed{\sigma_A = 17.17\ MPa}$

At point $c\ (56.34, -23.66)$

$u = 56.34\cos45 + (-23.66)\sin45$

$\boxed{u = \cancel{16.73}}\ 23.108$

$v = (23.66)\cos45 - 47.32(\sin45)$

$\boxed{v = \cancel{-50.19}}\ -56.56$

$\sigma_c = \dfrac{(169.70)(\cancel{16.73})\ 23.108}{3.75140\times10^5} + \dfrac{(169.70)(\cancel{-50.19})\ (-56.56)}{14.21\times10^5}$

$\sigma_c = 367\times10^{-3}\ N/mm^2$

$\boxed{\cancel{\sigma_c = 1.553\times10^{-3}}}$

Point X(-83.66, -23.66)

$u = -34.46$

$v = 0$

$$\sigma_B = \frac{(169.20)(-34.41)}{3.761\,40 \times 10^5} + \frac{(16\,120)(0)}{}$$

$\sigma_B = -0.0155\ N/m^2$

$\boxed{\sigma_B = 15.47\ N/mm^2}$

(ii) $\delta = \frac{KWL^3}{E}\sqrt{\frac{\sin^2\theta}{I_{uu}^2} + \cos^2\theta}$

From simply supported beam, $K = \frac{1}{48}$

$$\delta = \frac{1}{48} \times \frac{200 \times (2.4)^3 \, 10^3}{200 \times 10^3} \sqrt{\frac{\sin^2(45)}{(14.21 \times 10^5)^2} + \frac{\cos^2(45)}{\cancel{50}\,()^2}}$$

$\boxed{\delta = 14.56}$ mm

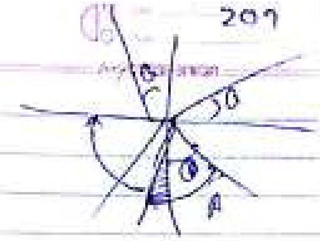
$$\tan\beta = \frac{I_{uy}}{I_w}\tan\theta$$

$$\beta = \tan^{-1}\left(\frac{I_{uy}}{I_w} \times \tan\theta\right) \Rightarrow \boxed{\beta = 75.17}$$

There for the deflation is at $\beta - \theta = 75.43 - 45$

$= 30.45$

(iii) Position of neutral axis can be determined



$\alpha_y \rightarrow 90 -$ after deflection

$= 90 - 30.43$

$= 57.58$

In CCW direction with the load line, hence the N.A is ⊥ to the line of deflection.