Date ....

Strength of Material - (I)

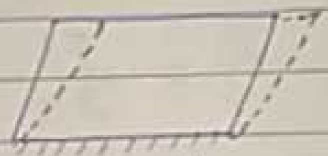
⇒ **Strength** :- It is the property of a material by virtue of which it can resist a external load up to rupture point or fracture point.

⇒ Types of Loading

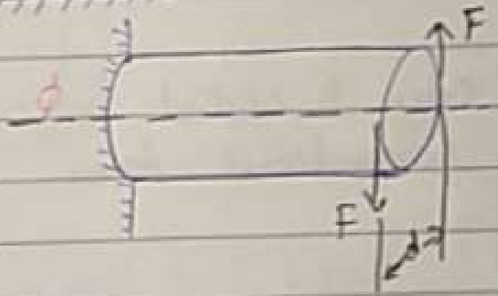
1. Tensile Loading :- Longitudinal axis

2. Compressive Loading :-

3. Shear Loading :-



4. Torsion Loading :-

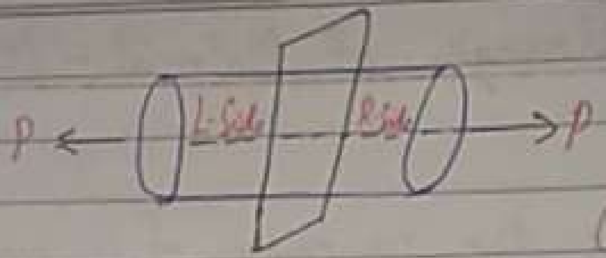


$T = F.d$

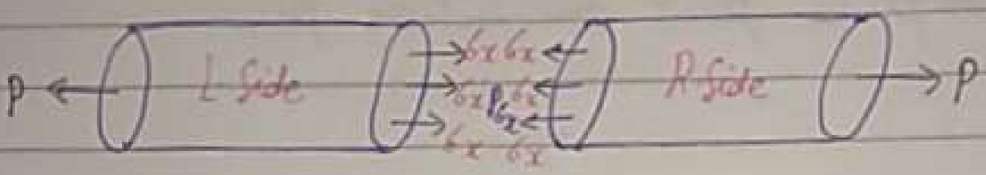
* Diffrent Types of Stress Induced in a Composite Bar

Stress :- It is defined as the magnitude of Internal resisting force developed at a point in a member under giving Conditions.

Tensile Stress :-



Crossection area $= \pi/4\, d^2$



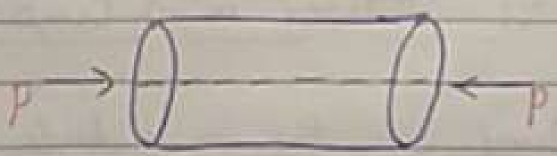
$$\sigma = \frac{F}{A} \quad , \quad p = \frac{F}{A}$$

⇒ Stress depends upon nature of diffrent types of loading

⇒ Stress is Calculated by $\sigma_T = \frac{F}{A}$ (+)ve nature

Where
$F =$ external applied Force
$A =$ Crossectional Area

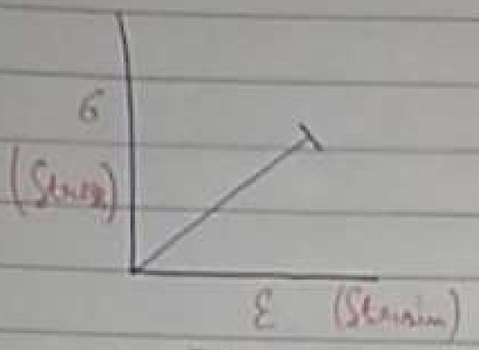
2. **Compressive Stress**



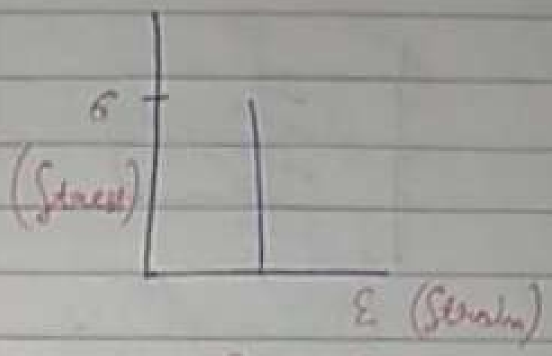
$$\sigma_c = \frac{F}{A} \quad \text{(-)ve nature}$$

Date..............

* Difference b/w Elastic Body & Rigid Body



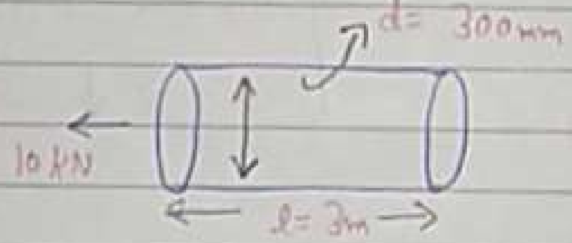
Elastic Body



Rigid Body

Que Calculate the tensile Stresses of a Bar is Subjected to 10 kN force & the length & diameter of the bar are 3m, 300 mm respectively.

Sol



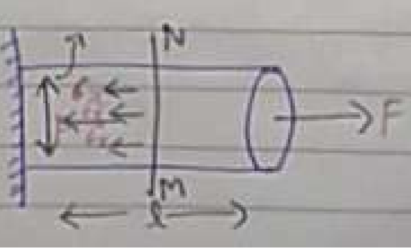
$$\sigma_T = \frac{F}{A} = \frac{4\times 10\times 10^3}{\pi(0.3)^2} = \frac{4\times 10^4}{0.2826} = 1.14\ (N/m^2)$$

$A = \pi/_4\, d^2$

Ans

* Tensile Stress Distribution

A Bar as Shown in fig is Subjected to a Tensile F having length l & its diameter d.



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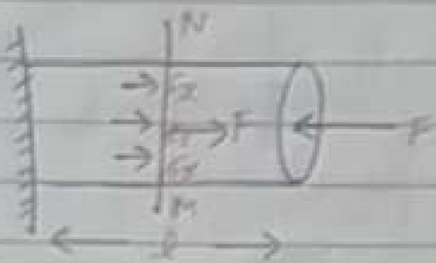
* Compressive Stress Distribution



* Axial Load Diagram

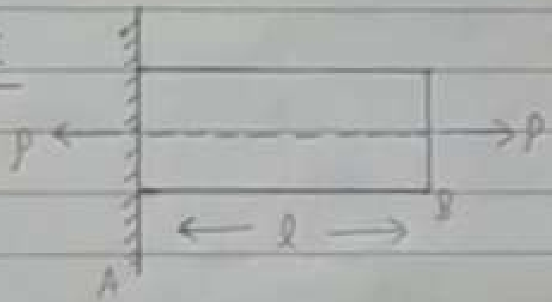
A Composite Bar is Subjected to different loading Conditions as Shown in fig.

Sign Convention

(+)ve ← ↑ (+)ve

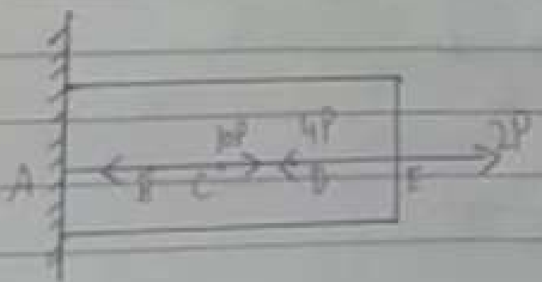
(−)ve → ↓ (−)ve

Case-I



ALD →

Case-II



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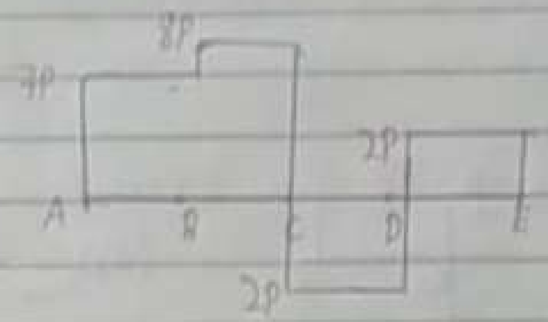
ALD :-

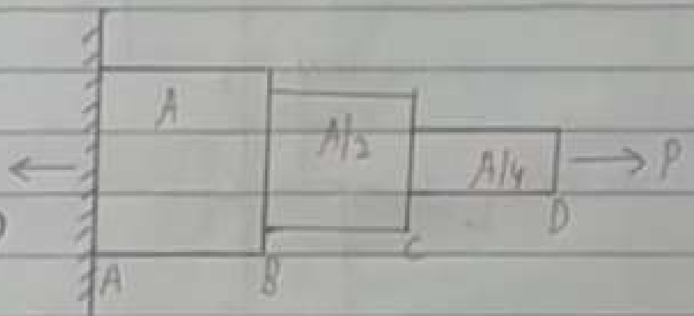


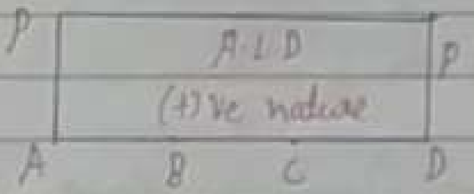
$$(\sigma_T)_{max}\ BC = \left(\frac{8P}{A}\right)_{BC} \qquad (\sigma_T)_{min}\ DE = \left(\frac{2P}{A}\right)_{DE}$$

$$(\sigma_c)_{CD} = \left(\frac{2P}{A}\right)_{CD}$$

Case – III

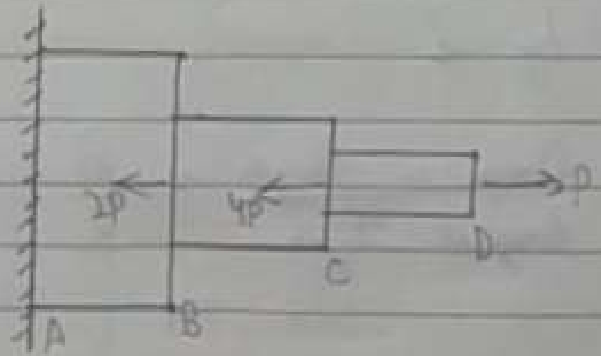
$R_A = P$





Case – IV

$R_A = -5p$



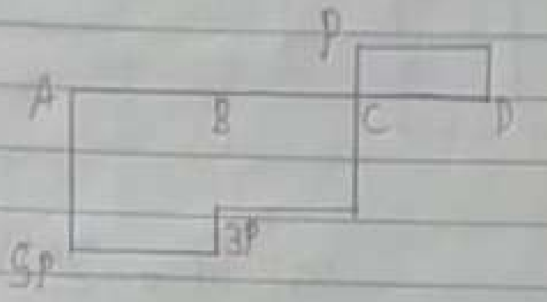
$\leftarrow 6p$

$\rightarrow P$

Bal = $-5p$

Date ....

ALD :-

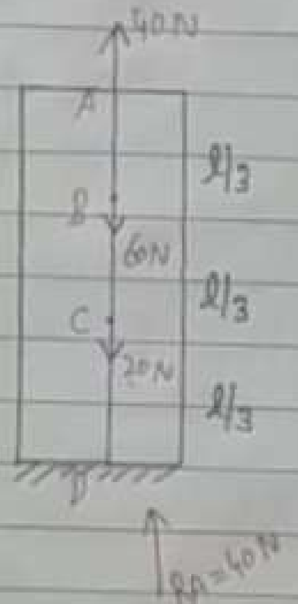


$(\sigma_t)_{max} = CD$

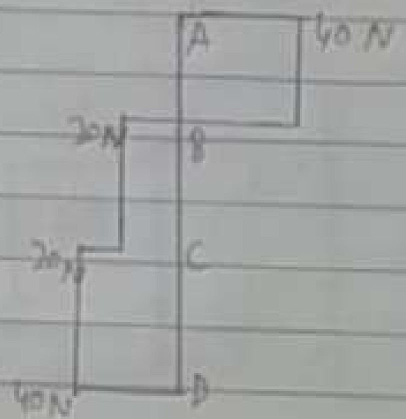
$(\sigma_c)_{min} = BC$

$(\sigma_c)_{max} = AB$

⇒ Draw ALD for the following Case.



⇒ ALD



* **Strain :-** It is the ratio of Change in length to Original length.

It is denoted by ε.

$$\varepsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{\delta l}{l}$$

Date ....

$$\varepsilon_c = \frac{\delta l}{l} \quad (-)ve \qquad , \qquad \varepsilon_T = \frac{\delta l}{l} \quad (+)ve$$

In Case of Compressive load ε_c is $(-)$ ve.

In Case of Tensile Load ε_T is $(+)$ ve.

* <u>Hook's</u> <u>Law</u> :-

According to Hook's Law

Stress $\propto$ Strain

Mathematically

$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$

$$\frac{P}{A} = \frac{E\,\delta l}{l}$$

E = Young Modulus

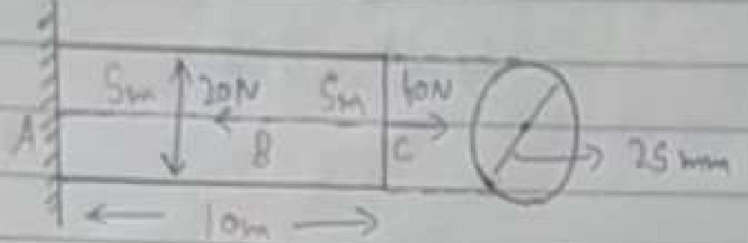
$$\boxed{\delta l = \frac{PL}{AE}}$$

Where,
A = Cross - Section Area
L = Length of the Bar
P = Applied Load

Que A Composite Bar as Shown in Fig.

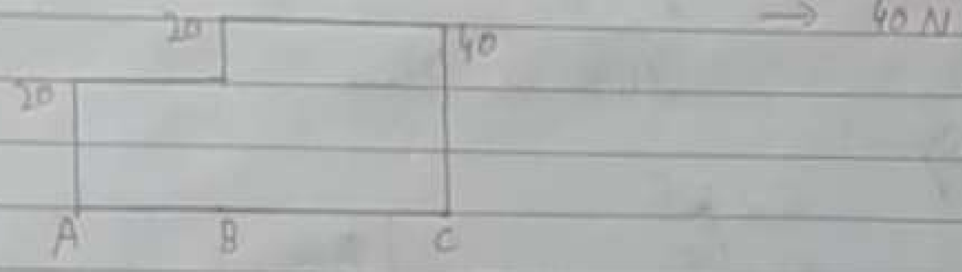
(i) Calculate the total elongation

(ii) Calculate maximum Stress. if $E = 2 \times 10^5$ P



$P_1 = 20N$

Sol



$$\delta L = \frac{PL}{AE}$$

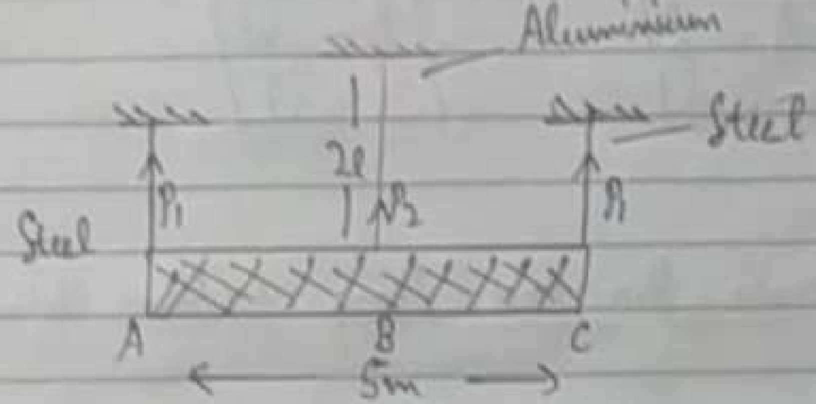
$$\delta l_{Total} = \left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC}$$

$$= \frac{40 \times 5 \times 4}{3.14 \times (12.5)^2 \times 2 \times 10^5} + \frac{20 \times 5 \times 4}{3.14 \times (12.5)^2 \times 2 \times 10^5}$$

$$\delta l_{Total} = 1.19 \times 10^{10} \text{ Ans}$$

Date ....

Ques A Composite Bar ABC is Suspended by three rod made of Steel and aluminium as Shown in Fig. If a Load P is applied at the middle of ABC Bar. Calculate the Force in Aluminium and Steel



Sol

$$(\Delta l)_s = (\Delta l)_{al}$$

$$2P_1 + P_2 = P \quad —— (1)$$

$$\left(\frac{Pl}{AE}\right)_s = \left(\frac{Pl}{AE}\right)_{Al}$$

$$\left(\frac{P_1}{E}\right)_s = \left(\frac{2P_2}{E}\right)_{Al}$$

We know that

$$E_s = 2\times10^5 \ N/mm^2$$
$$E_{al} = 10^5 \ N/mm^2$$

$$\frac{P_1}{2\times10^5} = \frac{2P_2}{10^5}$$

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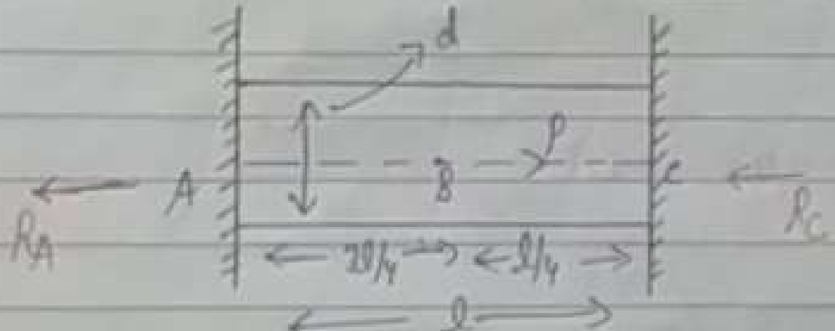
$$P_1 = 4P_2$$
$$8P_2 + P_2 = P$$
$$9P_2 = P$$

$$\boxed{P_2 = \frac{P}{9}} \qquad \boxed{P_1 = \frac{4P}{9}}$$

* Statically Interminate Bar and Beam (No of Unknown more)

A Composite Bar as Shown in Fig. Which is fixed at both the ends having length l and diameter d.



Note :-

This method is Used For Uniform area.

Step-I :- Net Load Direction

Step-2 :- Opposite Direction $\leftarrow R_A$ $\leftarrow R_C$

In Opposite direction (R_A Sign is (+ve))

In Same direction (R_A Sign is (−)ve)

Date

(i) $R_A = \frac{P \times l/4}{l} = \frac{\text{Force} \times \text{Distance}}{\text{Total length}}$

$\boxed{R_A = P/4}$

(ii) $R_C = \frac{+P \times 3l/4}{l}$

$\boxed{R_C = \frac{3P}{4}}$

⇒ Find Out the min Compressive and Tensile Stress in the Bar.

Axial load in Bar AB

$(\sigma_t) \Rightarrow P_{AB} = R_A = P/4$

$(\sigma_c) \Rightarrow P_{BC} = R_A - P$

$= P/4 - P$

$= -3P/4$

$(\sigma_t)_{AB} = \frac{P}{4A}$

$(\sigma_c)_{BC_{max}} = \frac{3P}{4A}$

Date

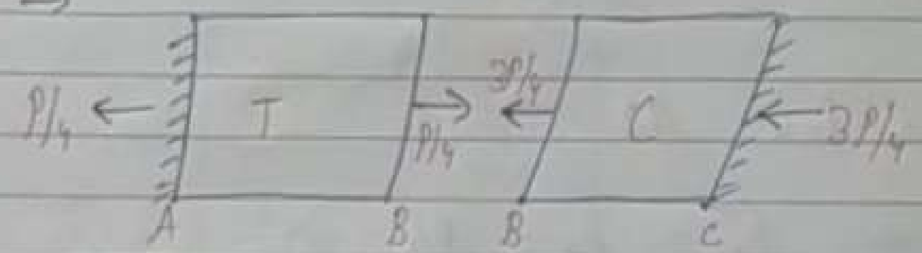
(ii) For AB Section

$$(\delta l)_{AB} = \left(\frac{Pl}{AE}\right)_{AB} = \frac{P}{4}\,\frac{3l}{4AE}$$

$$= \frac{3Pl}{16AE} \quad (+)\text{ve Tensile}$$

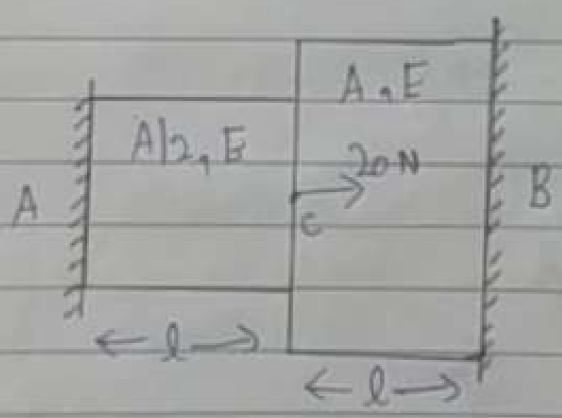
$$(\delta l)_{BC} = \left(\frac{PL}{AE}\right)_{BC} = \frac{-3P}{4}\,\frac{l}{4AE} = \frac{-3Pl}{16AE} \quad (-)\text{ve } C$$

FBD →



Que Calculate max tensile Stress and max Compressive Stress in Composite Bar.

Sol



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In this Situation, We Will use Compatibility Equation
(Total elongation Zero)

$$\boxed{\delta_{total} = 0}$$

$$\delta_{total} = \delta_{AC} + \delta_{BC} = \text{Zero}$$

$$= \left(\frac{PL}{AE}\right)_{AC} + \left(\frac{PL}{AE}\right)_{BC} = 0$$

$\rightarrow P_{AC} = R_A$

$\rightarrow P_{BC} = R_A - 20$

$$\delta_{total} = \frac{R_A \times \cancel{L}}{A/2 \times \cancel{E}} + \frac{R_A - 20 \times \cancel{L}}{A \times \cancel{E}}$$

$\Rightarrow 2R_A + R_A - 20 = 0$

$3R_A = 20$

$R_A = \frac{20}{3}$

$$R_A = 6.67N$$

$$P_{AC} = R_A = 6.67$$

$$\Rightarrow \quad P_{BC} = R_A - 20$$
$$= 6.67 - 20$$
$$P_{BC} = -13.33$$

$$(\sigma_1) = \frac{P}{A} = \frac{6.67}{A/2} = \frac{13.34}{A}$$

$$(\sigma_2) = \frac{P}{A} = \frac{-13.33}{A} \text{ Ans}$$

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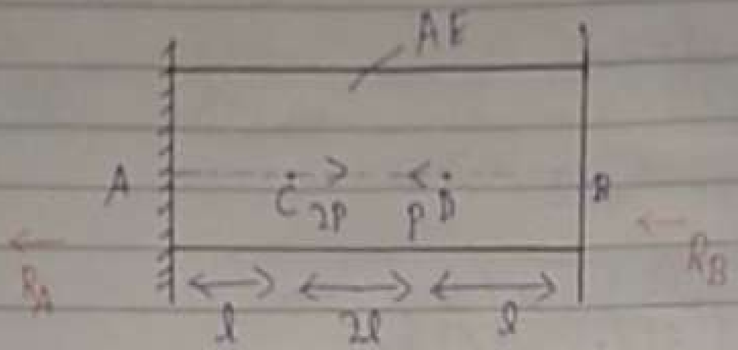
Que Calculate the Reaction force R_A and R_B



Net force P →

Step-1 :- Net Load Direction →

Step-2 :- $\leftarrow R_A \quad \leftarrow R_B$

$R_A = (+)ve$

(i) $R_A = \dfrac{+2P \times 3l - Pl}{4l}$

$$\boxed{R_A = \frac{5P}{4}}$$

(ii) $R_B = \dfrac{2P \times l - P3l}{4l}$

$$\boxed{R_B = \frac{-3}{4}}$$

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* Types of Strain

Two Types

Normal Strain (σ_t, σ_c)

Shear Strain (σ_s)

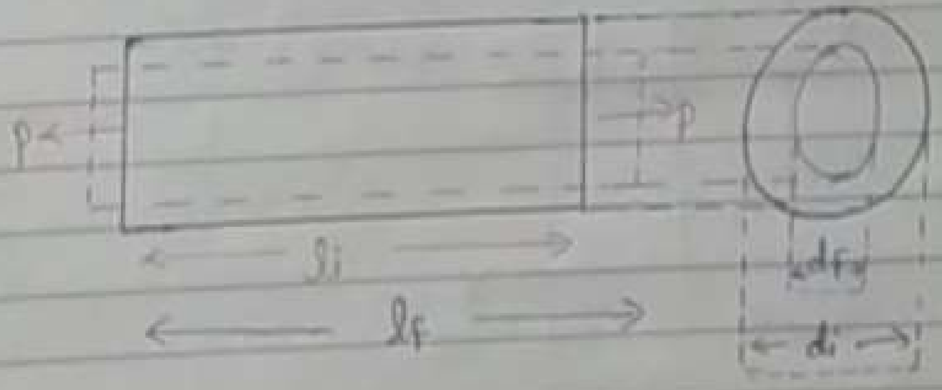
linear Strain

Volumetric Strain

Lateral Strain

1. linear Strain :- It is a normal Strain along the Load Direction.

2. Lateral Strain :- It is a Normal Strain perpendicular to Load Direction.



$$\varepsilon_{linear} = \frac{\delta l}{l} = \frac{l_f - l_i}{l} \quad (+)ve$$

$$\varepsilon_{lateral} = \frac{\delta d}{d} = \frac{d_f - d_i}{d} \quad (-)ve$$

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* Poisson Ratio (μ)

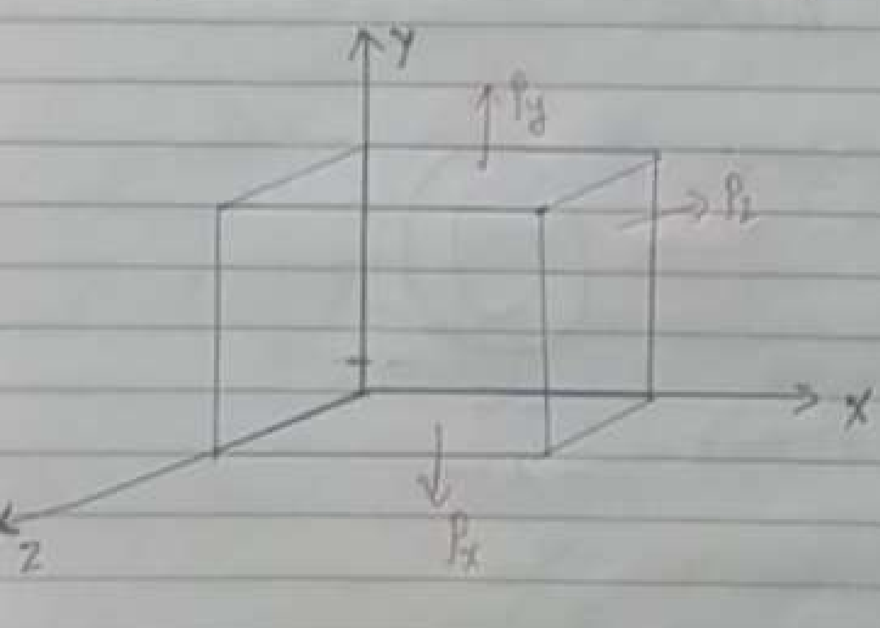
It is the Ratio of lateral Strain to linear Strain and it is denoted by μ

Mathematically

$$\boxed{\mu = \frac{\text{lateral Strain}}{\text{linear Strain}}}$$

$$\boxed{\mu = -\frac{\varepsilon_{\text{lateral}}}{\varepsilon_{\text{linear}}}}$$

* Expression For Volumetric Strain



Consider a Block having Volume lbt as Shown in Fig.

If P_x, P_y, P_z are the Load in x, y and z direction.

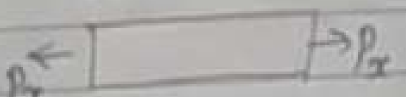
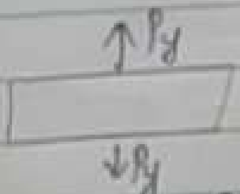
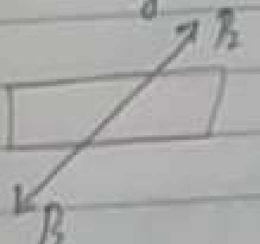
$$\varepsilon_V = \frac{\delta V}{V} \quad \frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$\varepsilon_x = \frac{\delta l}{l}$ Change in X-direction

$\varepsilon_y = \frac{\delta t}{t}$ Change in Y-direction

$\varepsilon_z = \frac{\delta b}{b}$ Change in z-direction.

$\delta l = \frac{Pl}{AE}$

Types of Load	X-direction	Y-Direction	Z-Direction
	(linear) $\frac{\sigma_x}{E}$	$-\mu\frac{\sigma_x}{E}$ (lateral)	$-\mu\frac{\sigma_x}{E}$ (lateral)
	$-\mu\frac{\sigma_y}{E}$	$\frac{\sigma_y}{E}$ (linear)	$-\mu\frac{\sigma_y}{E}$
	$-\mu\frac{\sigma_z}{E}$	$-\mu\frac{\sigma_z}{E}$	$\frac{\sigma_z}{E}$ (linear)

Total effect in X-direction

$$\varepsilon_x = \frac{\delta l}{l}$$

Date ....

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} = \frac{\sigma_x}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z)$$

Total effects in y- direction

$$\varepsilon_y = \frac{\delta t}{t} = -\mu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$= \frac{\sigma_y}{E} - \frac{\mu}{E}(\sigma_x + \sigma_z)$$

Total effects in Z- direction

$$\varepsilon_z = \frac{\delta b}{b} = -\mu\frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$= \frac{\sigma_z}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y)$$

Volumetric Strain

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \frac{\sigma_x}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z) + \frac{\sigma_y}{E} - \frac{\mu}{E}(\sigma_x + \sigma_z)$$

$$+ \frac{\sigma_z}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y)$$

Date ....

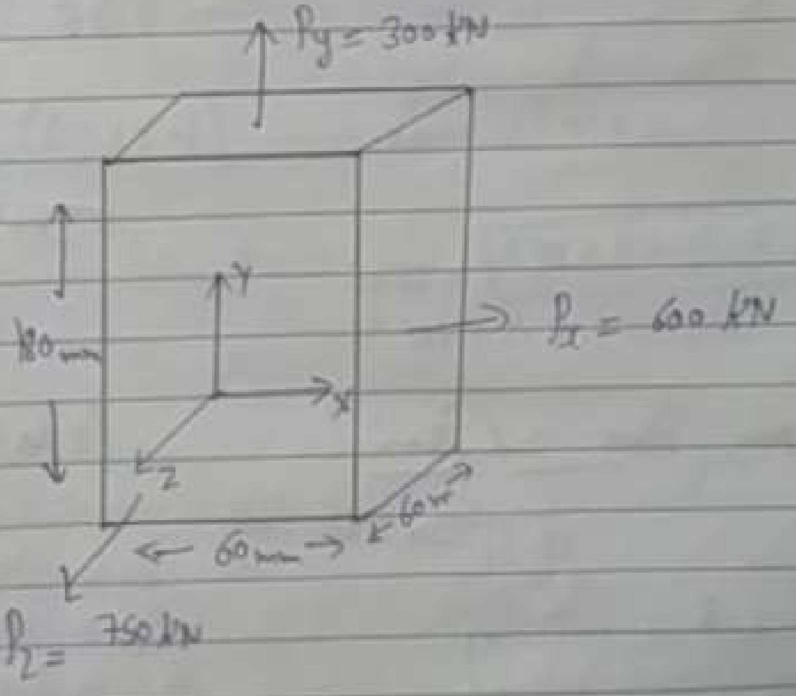
$$\varepsilon_v = \frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z)$$

$$- \frac{\mu}{E}(\sigma_x + \sigma_z) - \frac{\mu}{E}(\sigma_x + \sigma_y)$$

$$\boxed{\varepsilon_v = \frac{(1-2\mu)}{E}(\sigma_x + \sigma_y + \sigma_z)}$$

$$\sigma_x = \frac{P_x}{tb}, \quad \sigma_y = \frac{P_y}{lb}, \quad \sigma_z = \frac{P_z}{lt}$$

Que Calculate the Volumetric Strain, if A rectangular Block is Subjected to the following loading Condition:
Take poision ratio $\mu = 0.3$ $E = 200\,GPa$



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Sol

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \frac{(1-2\mu)\quad(\sigma_x+\sigma_y+\sigma_z)}{E}$$

$$\sigma_x = \frac{P_x}{A_x} = \frac{600 \times 10^3}{60 \times 180 \times 10^{-6}} = 1.8 \times 10^{12} = 1.8$$

$$\sigma_y = \frac{P_y}{A_y} = \frac{300 \times 10^3}{60 \times 60 \times 10^{-6}} = 3 \times 10^{11} = 0.3$$

$$\sigma_z = \frac{P_z}{A_z} = \frac{750 \times 10^3}{60 \times 180 \times 10^{-6}} = 2.25 \times 10^{12}$$

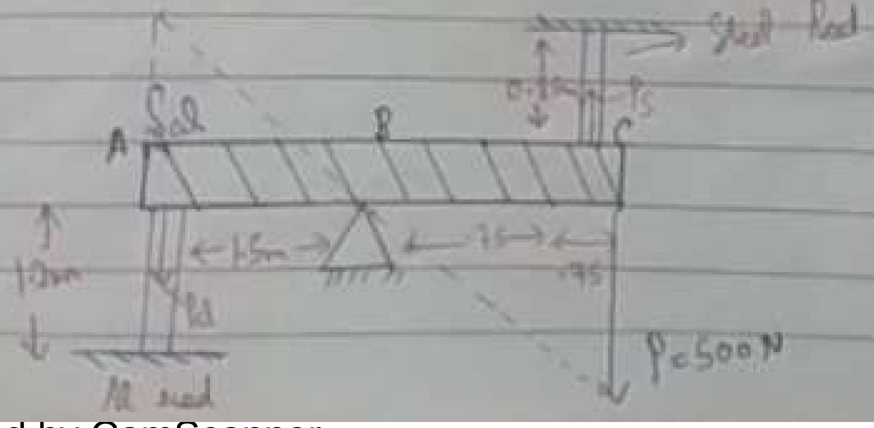
$$= 2.25$$

$$\varepsilon_v = 3.23 \times 10^{-3} V = \delta v \qquad \frac{(1-2\times0.3)\,(1.8+0.3+2.25)}{200 \times 10^9}$$

$$\boxed{\varepsilon_v = 2.09 \times 10^{-6}} = \cancel{1895}$$

$$1.875 \times 10^{-11}$$

* Problem Based upon Stress and Strain (I.A.S 10 Marks)



$E_s = 200\,GPa$

$E_{al} = 100\,GPa$

$A_{al} = 200\ mm^2$

$A_{steel} = 300\ mm^2$

Date

$$\sigma_{steel} = \frac{P_s}{A_s} \qquad \sigma_{al} = \frac{P_{al}}{A_{al}}$$

Using Similar triangle property By thales theorem

$$\frac{\delta_{al}}{\delta_s} = \frac{1.5}{0.75}$$

$$\delta_{al} = 2\delta_s \quad \text{—(1)}$$

Taking moments about B

Clockwise → +ve
anticlockwise → −ve

$$-P_{al} \times 1.5 - P_s \times 0.75 + \overset{500}{\cancel{300}} \times 1.5 = 0$$

$$-1.5 P_{al} - 0.75 P_s + 750 = 0$$

$$1.5 P_{al} + 0.75 P_s = 750 \quad \text{—(2)}$$

$$P_{al} + 500 = P_s \quad \text{—(3)}$$

$$P_{al} - P_s = -500$$

$$\left[\begin{array}{l} P_{al} = x \Rightarrow 166.6 \\ P_{st} = y \Rightarrow 666.6 \end{array}\right.$$

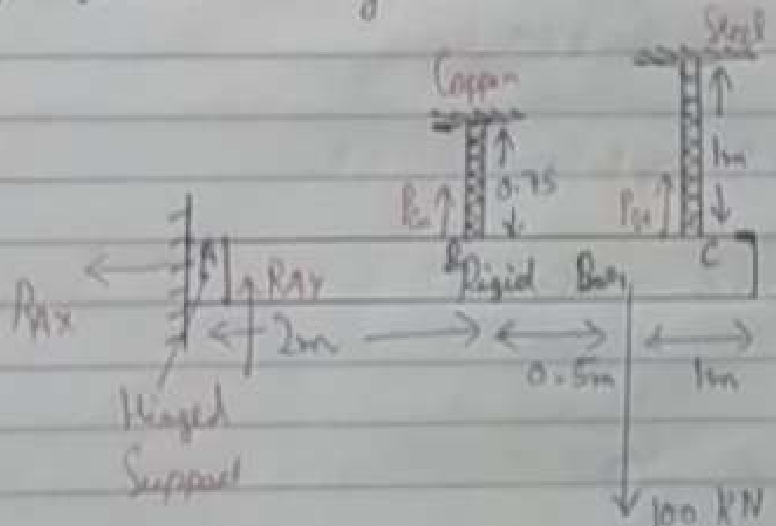
$$\delta_{al} = \left(\frac{Pl}{AE}\right)_{al} \quad \text{—(4)}$$

$$\delta_{al} = \frac{166.6P \times 1.2}{200 \times 100 \times 10^{9}}$$

$$\delta_{steel} = \left(\frac{Pl}{AE}\right)_{steel} = \frac{666.6P \times 0.85}{200 \times 200 \times 10^{9}}$$

$$\left[\begin{array}{l} \sigma_s = 1.6 \times 10^6 \ N/m^2 \\ \sigma_{al} = 1.1 \times 10^6 \ N/m^2 \end{array}\right. \quad \text{Ans}$$

Que Calculate elongation in Steel rod (10 Marks) CSPT:0



$E_s = 2 \times 10^8 \ kN/m^2$ $\qquad$ $A_{steel} = 2 \ cm^2$

$E_{Cu} = 1.2 \times 10^8 \ kN/m^2$ $\qquad$ $A_{Cu} = 4 \ cm^2$

$l_{steel} = 1m$

$l_{Cu} = 0.75 \ m$

Date ....

Sol $\delta l_{Copper} = \left(\frac{PL}{AE}\right)_{Cu}$ $\qquad$ $\delta l_{Steel} = \left(\frac{PL}{AE}\right)_{Steel}$

$\Sigma F_x = 0 \quad = R_{Ax} = 0$

$\Sigma F_y = 0$

$$R_{Ay} + P_{Cu} + P_{Steel} = 100 \times 10^3 N \quad \text{—} (1)$$

Taking moment about A point

$\Sigma M = 0$

$$-P_{Cu} \times 2 - P_s \times 3.5 + 100 \times 10^3 \times 2.5 = 0$$

$$2P_{Cu} + 3.5 P_s = 2.5 \times 10^5 \quad \text{—} (2)$$

⇒ By Using Similar triangular property By Thales theorem.

$$\frac{\delta_{Cu}}{\delta_{Steel}} = \frac{2}{3.5}$$

$$\frac{\left(\frac{Pl}{AE}\right)_{Cu}}{\left(\frac{Pl}{AE}\right)_{Steel}} = \frac{2}{3.5}$$

$$\frac{\frac{P_{Cu} \times 0.75}{4 \times 1.2 \times 10^5 \times 10^3}}{\frac{P_{Steel} \times 1}{2 \times 2 \times 10^5 \times 10^3}} = \frac{2}{3.5}$$

Date ....

$$\frac{0.75 \times P_{Cu} \times 2 \times 2 \times 10^{8} \times 10^{3}}{P_{Steel} \times 4 \times 1.2 \times 10^{8} \times 10^{3}} = \frac{2}{3.5}$$

$$P_{Cu}\ 10.5 = P_{Steel}\ 9.6$$

$$\frac{P_{Cu}}{P_{Steel}} = \frac{9.6}{10.5}$$

$$\boxed{P_{Cu} = 0.914\ P_s}$$

$$2P_{Cu} + 3.5\,P_s = 2.5 \times 10^5$$

$$2 \times 0.914\ P_s + 3.5 P_s = 2.5 \times 10^5$$

$$5.328\ P_s = 2.5 \times 10^5$$

$$P_s = \frac{2.5 \times 10^5}{5.328}$$

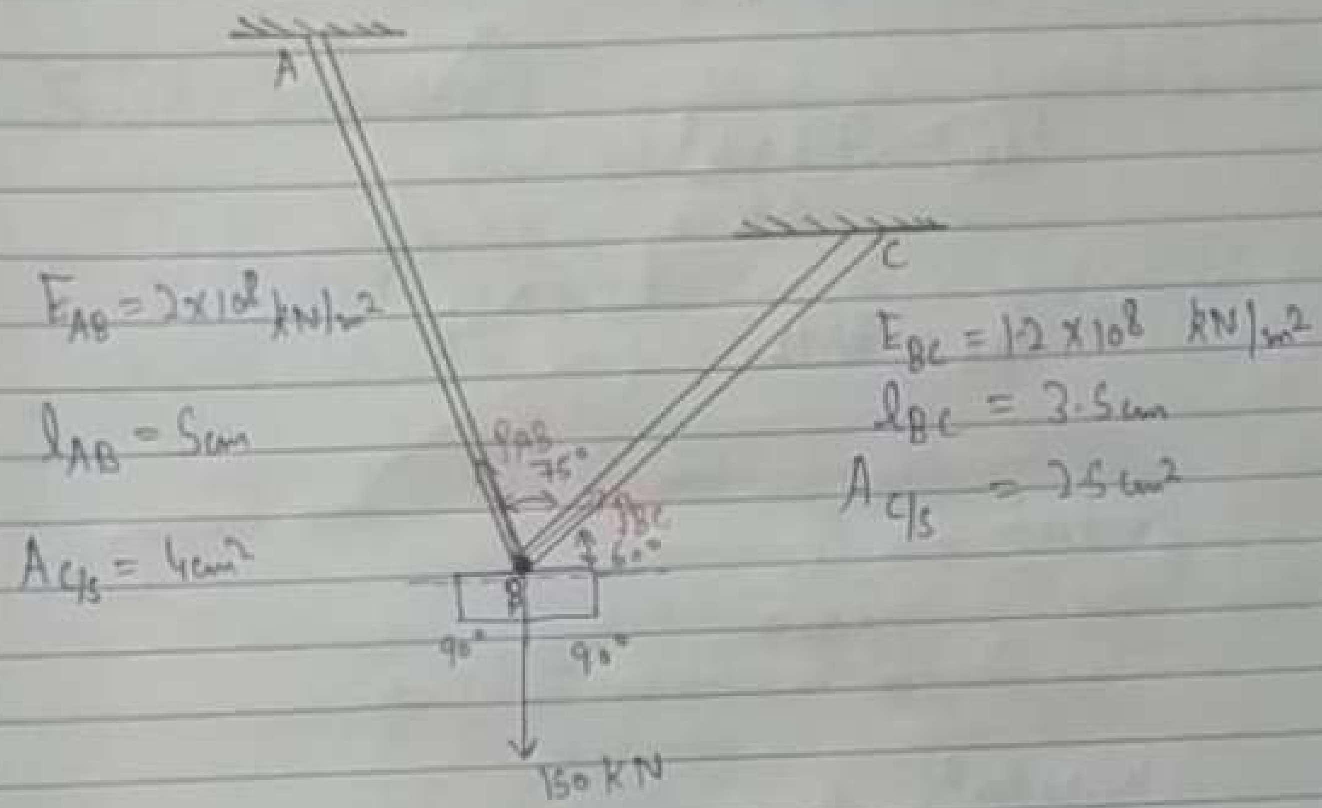
$$P_{Steel} = 0.46 \times 10^5$$

$$\delta_{Steel} = \frac{Pl}{AE} = \frac{0.46 \times 10^5 \times 1}{0.2 \times 2 \times 10^8} = \frac{0.46 \times 10^5}{0.4 \times 10^8}$$

$$\boxed{\delta_{Steel} = 1.15 \times 10^{-3}}$$ Ans

Date ....

Q. Calculate Stress in rod AB and CB Shown in Fig (5 marks)



Sol Apply Lami's Theorem

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

$$\frac{P_{AB}}{\sin 150} = \frac{150\times10^3}{\sin 75} = \frac{P_{BC}}{\sin 45+90}$$

$$\frac{P_{AB}}{0.5} = \frac{150\times10^3}{0.96} = \frac{P_{BC}}{0.707}$$

$$\Rightarrow \quad \frac{P_{AB}}{0.5} = \frac{150 \times 10^3}{0.96}$$

$$P_{AB} = \frac{75 \times 10^3}{0.96}$$

$$P_{AB} = 78.12$$

$$\Rightarrow \quad \frac{150 \times 10^3}{0.96} = \frac{P_{BC}}{0.707}$$

$$\frac{106.05 \times 10^3}{0.96} = P_{BC}$$

$$P_{BC} = 110.4$$

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{78.12}{0.4} = 19.53 \times 10^7 \ N/m^2 \quad Ans$$

1953

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{110.4}{2.5} = 44.16 \times 10^7 \ N/m^2 \quad Ans$$

4416

Date ....

Note :- E → Young Modulus Diffrent, Reaction diffrent
E → Young Modulus Same, Reaction Same

* Thermal Stress :- A Bar Which is Subjected both Loading and Temp Condition (ΔT).

The total Stress produce in the System, it is produce to high temp Condition ($T°C$)

α = Thermal expression Cofficient

$$\sigma_{total} = \underbrace{\sigma_{M \cdot E}}_{\text{Due to Loading Condition}} + \underbrace{\sigma_{thermal}}_{\text{Due to } \Delta T° \text{(temp) Condition}}$$

* Types of Expansion :-

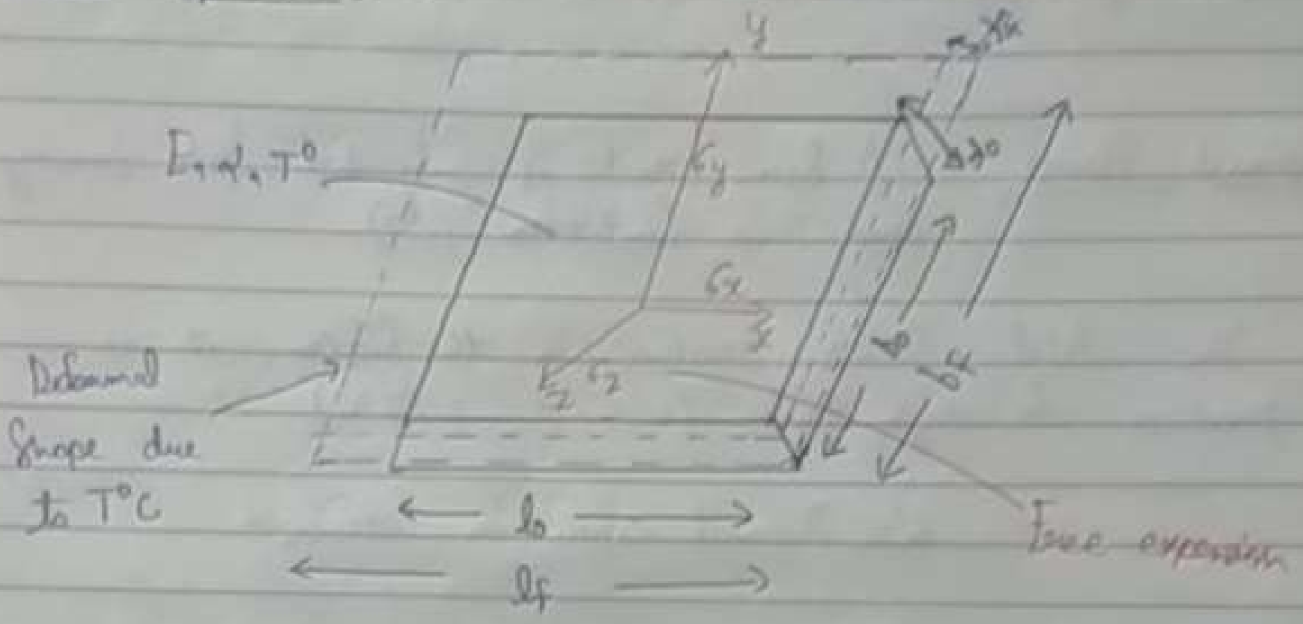
1. Free Expension (No Stress)

2. Partially Expension

3. Completely Closed Expansion / Restriction Expression.

Date.........

* Free Expansion :-



α = Thermal Expansion Coefficient

$$\varepsilon_{x\,th} = \frac{\delta l}{l_0} = \frac{l_f - l_0}{l_0} = \frac{\alpha T l_0}{l_0} = \alpha T$$

$$\varepsilon_{y\,th} = \frac{\delta b}{b_0} = \frac{b_f - b_0}{b_0} = \frac{\alpha T b_0}{b_0} = \alpha T$$

$$\varepsilon_{z\,th} = \frac{\delta t}{t_0} = \frac{t_f - t_0}{t_0} = \frac{\alpha T t_0}{t_0} = \alpha T$$

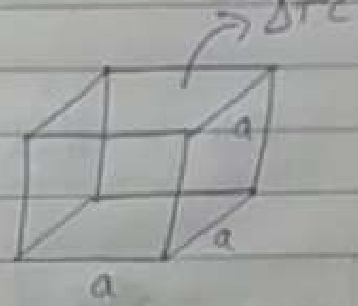
$$\varepsilon_{v\,th} = \frac{\delta V}{V_0} = \varepsilon_{x\,th} + \varepsilon_{y\,th} + \varepsilon_{z\,th}$$

$$= \alpha T + \alpha T + \alpha T$$

Date ....

$$\boxed{\varepsilon_{v_{th}} = 3\alpha T}$$

<u>Que</u> Calculate Change in Vol For a Cube. Which is Subjected to T°C

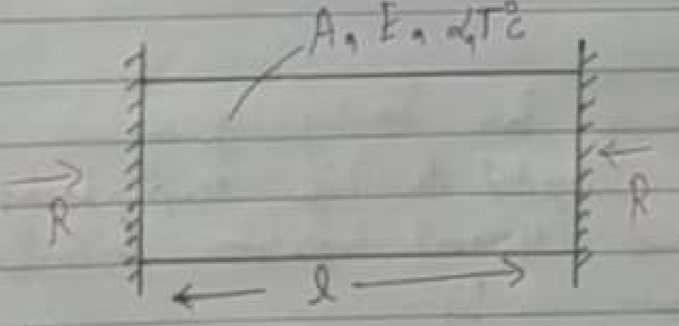


<u>Sol</u>

$$\Delta Vol = 3\alpha T a^3$$

* <u>Completely</u> <u>Restricted</u> <u>Expansion</u> :-

A Composite Bar as Shown in Fig. is Subjected to high temp, having length l, Area A, thermal expansion Coefficient α and Young's Modulus E.



$$\delta_{thermal} = \alpha T L$$

$$\delta_{axial} = \frac{PL}{AE}$$

$$\delta_{total} = \text{Zero} \quad \text{—(1)}$$

$$\delta_{axial\ load} + \delta_{thermal} = 0$$

$$\sigma_{thermal} \rightarrow \frac{R}{A}\frac{l}{E} + \alpha T L = 0$$

Date..........

$\frac{-R}{A} \Rightarrow (\sigma_{thermal}) \Rightarrow$ It is thermal Stress

$$-\frac{\sigma_{th} l}{E} + \alpha T l = 0$$

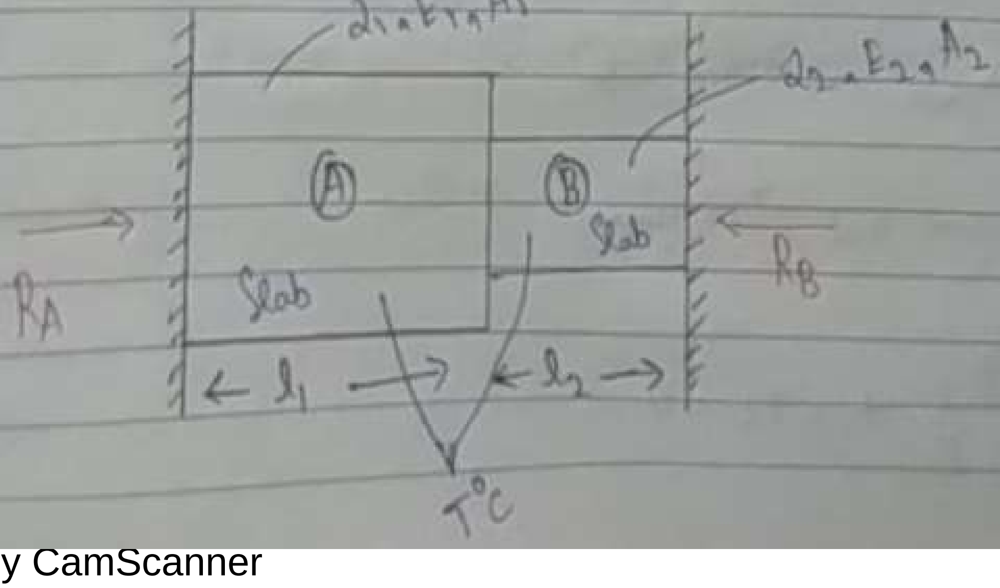
$$\sigma_{th} = \alpha T E$$

Stress $\boxed{\sigma_{th} = \alpha T E}$

Where σ_{th} is thermal Stresses
E is Young Modulus

Deformation $\boxed{\delta_{thermal} = \alpha T L}$

Case - II :- When A Composite Bar having Young Modulus E_1, E_2 Subjected to high temp having length l_1 and l_2 and thermal expansion Coefficient α_1 and α_2 as Shown in Fig.



Date ....

$\delta_{total} = \text{Zero} \Rightarrow \delta_A + \delta_B = 0$ — ①

For Slab Ⓐ

$$\delta_A = \delta_{thermal_{(A)}} + \delta_{axial_{(A)}}$$

$$= \alpha_1 T L_1 - \frac{R_A}{A_1}\frac{L_1}{E_1}$$

$$= \alpha_1 T L_1 - \sigma_{th_A}\frac{L_1}{E_1} \quad \text{— ②}$$

As Similarly For Slab Ⓑ

$$\delta_B = \alpha_2 T L_2 - \sigma_{th_B}\frac{L_2}{E_2} \quad \text{— ③}$$

$$\alpha_1 T L_1 - \sigma_{th_A}\frac{L_1}{E_1} + \alpha_2 T L_2 - \sigma_{th_B}\frac{L_2}{E_2} = 0$$

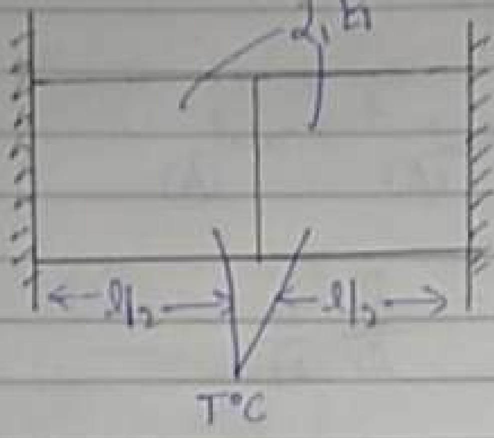
$$\boxed{\underbrace{\alpha_1 T L_1 + \alpha_2 T L_2}_{\text{Total Thermal}} = \underbrace{\sigma_{th_A}\frac{L_1}{E_1} + \sigma_{th_B}\frac{L_2}{E_2}}_{\text{Total axial}}}$$

$\sigma_{th_A} \rightarrow \frac{R_A}{A_A}$, $\sigma_{th_B} \rightarrow \frac{R_B}{A_B}$

Find $\sigma_A = ?$ $\sigma_B = ?$ $\sigma_A + \sigma_B$ — ①

$$\sigma_A = \frac{R_A}{A_A}, \quad \sigma_B = \frac{R_B}{A_B}$$

Que Calculate the thermal Expansion for a Composite Bar as shown in Fig.



$\sigma_1 = \sigma_2$

$$\alpha_1 T \frac{L}{2} + \alpha_1 T \frac{L}{2} = \sigma_1 \frac{l/2}{E} + \sigma_2 \frac{l/2}{E}$$

$$(\alpha_1 T + \alpha_1 T) = \frac{2\sigma_1}{E}$$

$$2\alpha_1 T = \frac{2\sigma_1}{E}$$

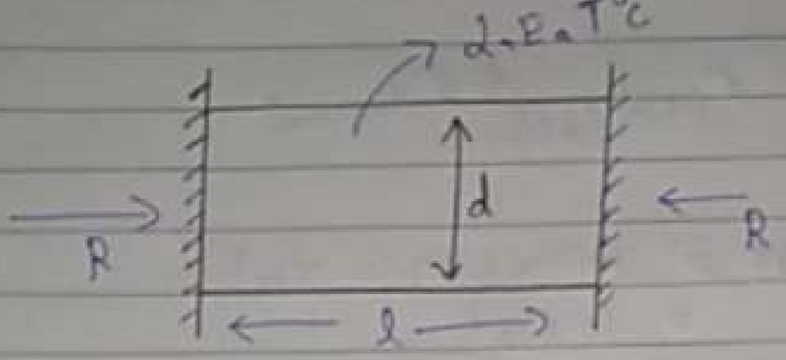
$$\boxed{\sigma_1 = T\alpha_1 E_1}$$

Case – III :- Thermal Stress [Deformation allowed] (λ)

A Composite Bar is Subjected to a high temp having thermal expansion Coefficient α and Young Modulus E.

In this Case thermal Deformation (λ) is allowed

Date ....



$$\delta_{total} = \delta_{axial} + \delta_{thermal} = \lambda$$

$$= \frac{-Rl}{AE} + \alpha T L = \lambda$$

$$-\frac{\sigma l}{E} + \alpha T L = \lambda$$

$$\boxed{\sigma_{th} = \frac{(\alpha T l - \lambda) E}{l}}$$

Note :- (i) $\sigma_{th} = 0$ free $\lambda = 0$
(ii) $\sigma_{th} = \alpha T E$ Complete

Que A Bar Which is Subjected to a Tensile Stress of 10 mPa at temp of 10°C. If a temp is increase by 15°C having $\alpha = 10^{-5}$ °C and $E = 200$ gPa. Calculate a total Stress induced in the bar if both the ends With the fix.
length = 500 mm

Date ....

Sol

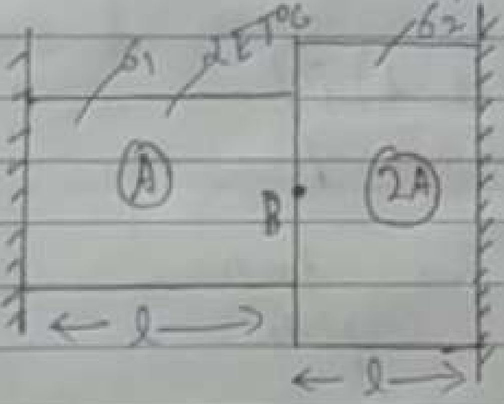
$$\sigma_{total} = \sigma_{axial\,(+ve)} + \sigma_{thermal}$$

$$\sigma_{th} = \alpha T E = 10^{-5} \times 5 \times 200 \times 10^{9}$$
$$= 10 \times 10^{6}$$

$$\sigma_{total} = 10 \times 10^{6} + \alpha T E = 0$$

$$\sigma_{total} = 10 \times 10^{6} - 10 \times 10^{6} = 0$$

Que A Composite Bar as shown in fig is Subjected to high temp. Calculate a thermal Stress Induced in the bar. Calculate Deformation at point B.



$\alpha = 10^{-5}$ °C
$E = 200$ GPa
$T = 50$°C
l $A = 100$ mm^2
A $l = 500$ mm^2
$= 500 \times 10^{-3}$

Sol

$$\sigma_1 = \frac{R}{A_1} \qquad \sigma_2 = \frac{R}{A_2}$$

$$\sigma_1 = \frac{R}{500} \qquad \sigma_2 = \frac{R}{500A}$$

Date ....

$$\frac{\sigma_1}{\sigma_2} = \frac{\frac{R}{500}}{\frac{R}{500 \times 2}}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{2}{1}$$

$$\sigma_1 = 2\sigma_2$$

$$\alpha_1 T L + \alpha_2 T L = \sigma_1 \frac{L_1}{E_1} + \sigma_2 \frac{L_2}{E_2}$$

$$10^{-5} \times 50 \times 500 \times 10^{-3} + 10^{-5} \times 50 \times 500 \times 10^{-3}$$

$$= \sigma_1 \frac{500 \times 10^{-3}}{200 \times 10^{9}} + \sigma_2 \frac{500 \times 10^{-3}}{200 \times 10^{9}}$$

$$25000 \times 10^{-8} = \sigma_1 2.5 \times 10^{-12} + \sigma_2 2.5 \times 10^{-12}$$

$$25 \times 10^{-6} \times 2 = \sigma_1 2.5 \times 10^{-12} + \sigma_2 2.5 \times 10^{-12}$$

$$= 2\sigma_2 \times 2.5 \times 10^{-12} + \sigma_2 \times 2.5 \times 10^{-12}$$

$$\sigma_2 = 0.67 \times 10^{8}$$

$$\sigma_1 = 133.3 \text{ mPa}$$

$$\delta_A = \alpha_1 T_1 L_1 - \sigma_1 \frac{l_1}{E_1} \qquad \delta_B = \alpha_2 T_2 L_2 - \sigma_2 \frac{l_2}{E_2}$$

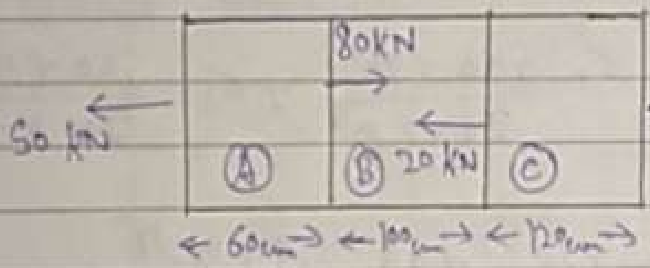
$$\delta_B = 10^{-5} \times 50 \times 500\times10^{-3} - 13.3 \times \frac{500\times10^{-3}}{200\times10^{9}}$$

$$= 25000\times10^{-8} - 13.3\times 2.5\times10^{-12}$$

$$= 25\times10^{-5} - 33.25\times10^{-12}$$

$$\boxed{\delta_B = -8.25\times10^{20}}$$

Que Calculate a total elongation if a bar is subjected to following Load Condition. Calculate δl.

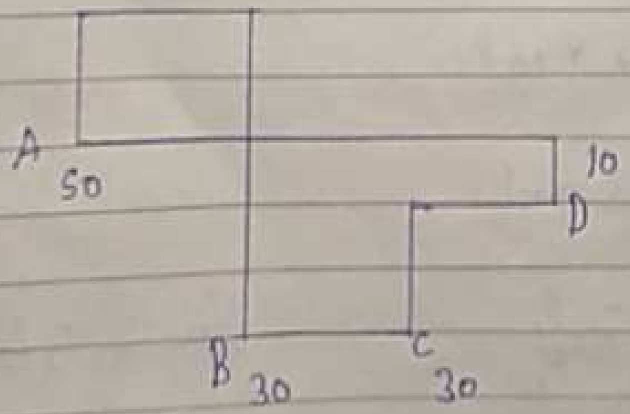


$E = 105\ GPa$
$A = 10\ cm^2$

← 20+10+50
← = 80
→ = 80

Sol

ALD:-



Scanned by CamScanner

$$\delta l - \delta l_{1_A} - \delta l_{2_B} - \delta l_{3_C}$$

$$= \left(\frac{PL}{AE}\right)_A - \left(\frac{Pl}{AE}\right)_B - \left(\frac{Pl}{AE}\right)_C$$

$$= \left(\frac{50 \times 60}{10 \times 105 \times 10^9}\right)_A - \left(\frac{60 \times 100}{10 \times 105 \times 10^9}\right)_B - \left(\frac{10 \times 120}{10 \times 105 \times 10^9}\right)_C$$

$$= \left(\frac{300}{105 \times 10^9}\right)_A - \left(\frac{600}{105 \times 10^9}\right)_B - \left(\frac{120}{105 \times 10^9}\right)_C$$

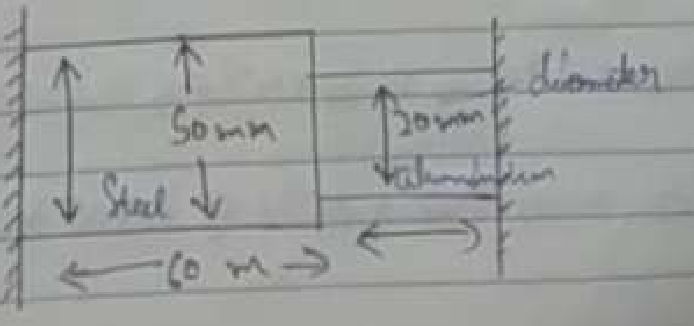
$$= \left(0.28 \times 10^{-9}\right)_A - \left(5.71 \times 10^{-9}\right)_B - \left(1.142 \times 10^{-9}\right)_C$$

$$= 0.28 - 6.852 \times 10^{-9}$$

$$= 6.572 \times 10^{-9} \quad \text{Ans}$$

Ques Calculate a thermal stress for a composite bar as shown in fig. If the young modulus for steel $E_s = 210$ GPa, for aluminium $E_{al} = 74$ GPa.

$\alpha_s = 11.7 \times 10^{-6}/°C$, & $\alpha_{al} = 23.4 \times 10^{-6}/°C$



$T_i = 21°C$
$T_f = 38°C$
$T = 17°C$

Date ....

Note :- If the external loading is present in a Composite Bar, Reaction will be same.

Sol
$$\sigma_1 = \frac{R}{A_1} \qquad \sigma_2 = \frac{R}{A_2}$$

$$\sigma_1 = \frac{R}{\pi (25)^2} \qquad \sigma_2 = \frac{R}{30 \times 20}$$

$$\sigma_2 = \frac{R}{\pi 100} \qquad \frac{\sigma_1}{\sigma_2} = \frac{\frac{R}{200}}{\frac{R}{600}}$$

$$\sigma_1 = \frac{2 \sigma_2}{3}$$

$$\sigma_1 = 0.66 \sigma_2$$

$$\alpha_s T_s L_s + \alpha_{al} T L_{al} = \sigma_1 \frac{L_s}{E_s} + \sigma_2 \frac{L_{al}}{E_{al}}$$

$$R.H.S = \sigma_1\, 0.28 \times 10^{-9} + \sigma_2\, 0.405 \times 10^{-9}$$

L.H.S

$$11.7 \times 10^{-6} \times 17 \times 60 + 23.4 \times 10^{-6} \times 17 \times 30$$

$$11934 \times 10^{-6} + 11934 \times 10^{-6} = \sigma_1\, 0.28 \times 10^{-9} + \sigma_2\, 0.405 \times 10^{-9}$$

$$11934 \times 10^{-6} \times 2 = \sigma_1\, 0.28 \times 10^{-9} + \sigma_2\, 0.405 \times 10^{-9}$$

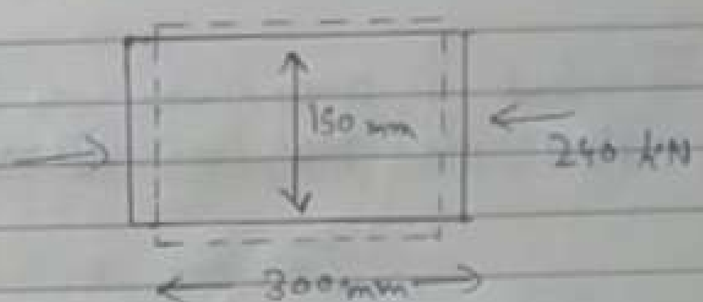
$\tau \propto \phi$

$\tau = G\phi$

Date ....

Ques Calculate Poisson ratio and young modulus if a Cylinder is Subjected to a Compressive Load (240 kN) having diameter & length & 150 mm & 300 mm. If decrease in length and increase in diameter are 0.127 mm (Δl) & 0.28 mm (Δd)

Sol

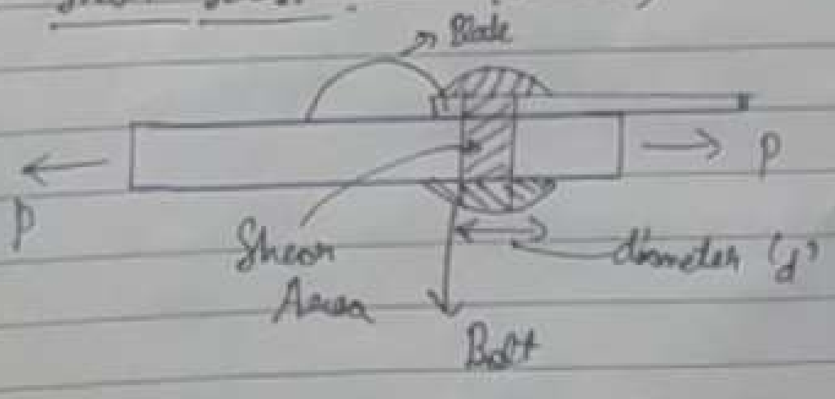


$$\mu = \frac{\frac{\Delta d}{d}}{\frac{\Delta l}{l}}$$

$$\mu = \frac{\frac{0.28}{150}}{\frac{0.127}{300}}$$

$$\mu = \frac{0.28 \times 300}{150 \times 0.127} = 0.071 \text{ Ans}$$

* Shear Stress :- (Parallel)



Before Shearing

After Shearing

If a load is perpendicular to longitudinal axis Shear Stress is produced. It is denoted by τ.

$$\tau = \frac{F}{A} = \frac{P}{A}$$

Where

A = Shear Area

$$A = \frac{\pi}{4} d^2$$

d = diameter of Bolts and rivets

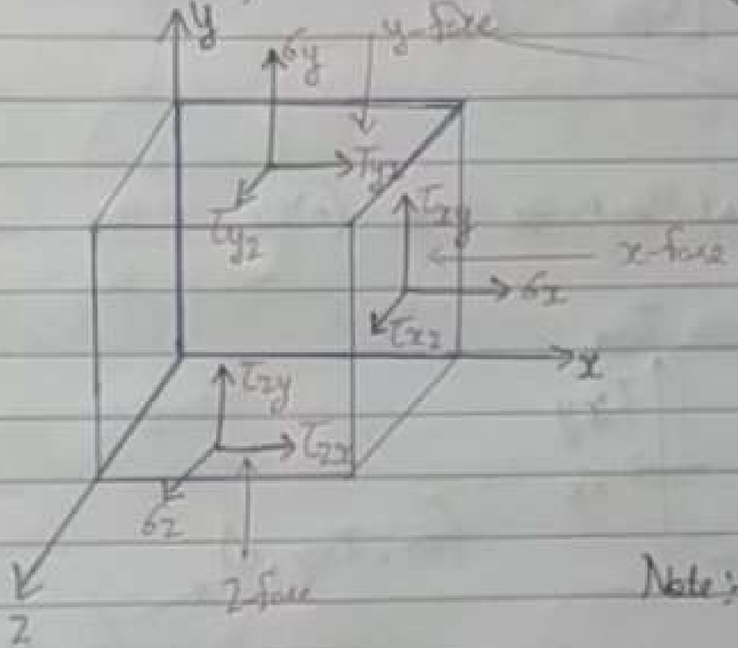
$$\boxed{\tau = \frac{P}{A} = \frac{4P}{\pi d^2}}$$

$\left(\frac{\pi}{4} d^2 \rightarrow \text{Circular area}\right)$

SOM Basic

* Stress Tensor

It is the Matrix representation of σ_x, σ_y and Shear Stress.



τ_{ab} — a: face, b: Direction

Note:- Max 9 Stress in a Body

For 3D Stresses

$$\begin{array}{cl} \left[\begin{matrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{matrix}\right] & \begin{matrix} \text{x-face} \\ \text{y-face} \\ \text{z-face} \end{matrix} \end{array}$$

Normal Stress

Where σ_x, σ_y, σ_z are the Normal Stress

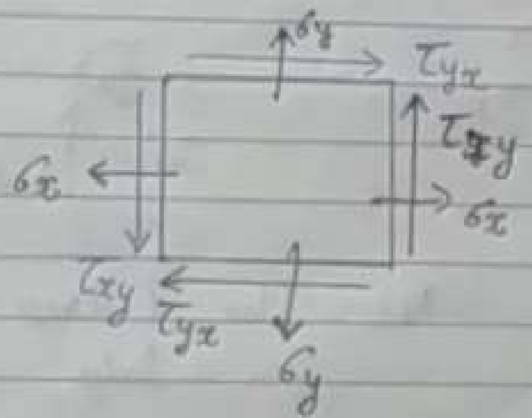
τ_{xy}, τ_{xz}, τ_{yx}, τ_{yz}, τ_{zx}, τ_{zy} are the Shear Stress. Which is produced parallel to the x, y, z faces.

Date ....

Stress Tension for 2D

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}$$

⇒ Shear Stress anticlockwise → (−)ve



$(\sigma_x, \tau_{xy}, 0)$ ← Coordinates

$(\sigma_y, \tau_{yx}, 0)$

⇒ Shear Stress Clockwise → (+)ve

If anticlockwise Shear Stress (−)ve
Clockwise Shear Stress (+)ve

(τ, σ) → General plane
(σ), $\tau \to 0$ Principle plane

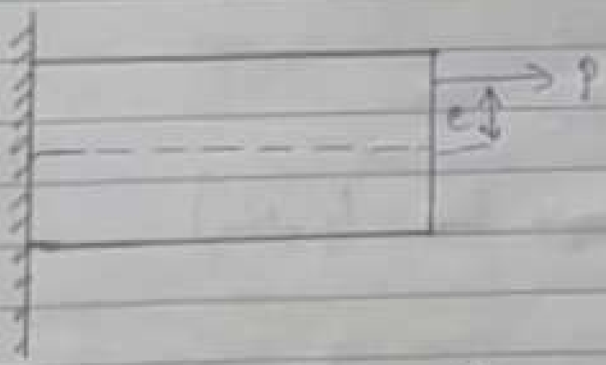
Major plane } Principle → max Stress
Minor plane } Principle → Min Stress

Date

* Different Types of Loading Condition

1. Pure Axial Load → (C, T) $\sigma = \frac{P}{A}$ ⊥ C/s
2. Eccentric Axial load
3. Pure Shear Load $\tau = \frac{P}{A}$ ∥ C/s
4. Eccentric Shear Load.
5. Pure Bending.
6. Pure ~~Tension~~ Torsion

* Eccentric Axial Load Condition :-

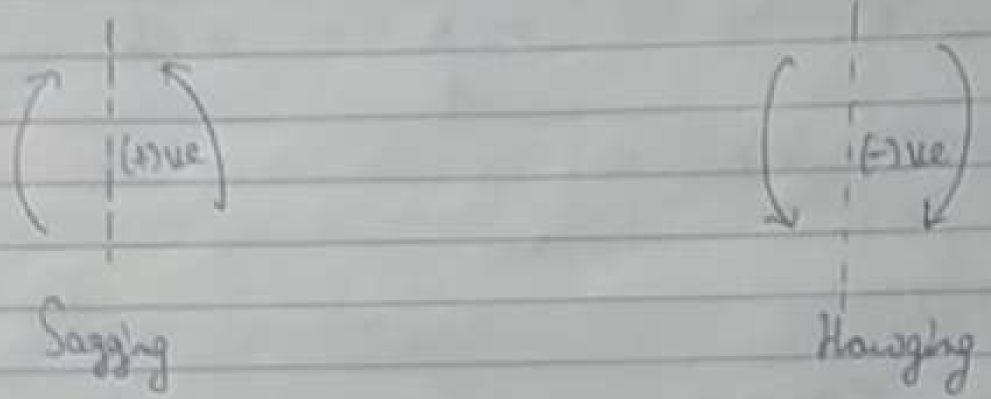


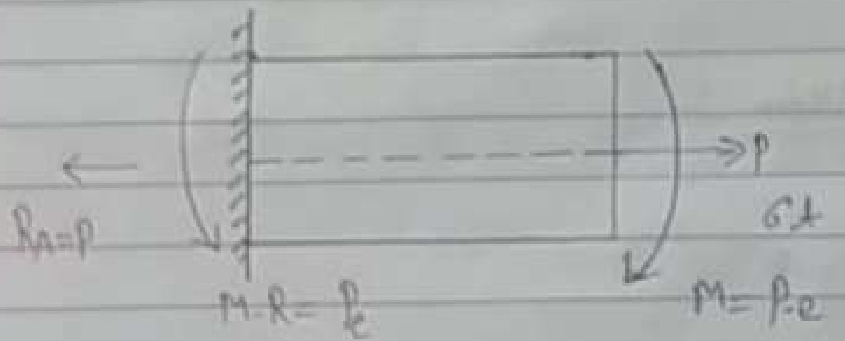
A Beam Which is Subjected to Eccentric axial load Condition as Shown in fig.

Where e = Eccentricity Distance from the Neutral axis

In this Case, 2 Stress is produced known as Bending Stress (σ_b) and axial Stress (σ_t, σ_c) depending upon the load Condition.

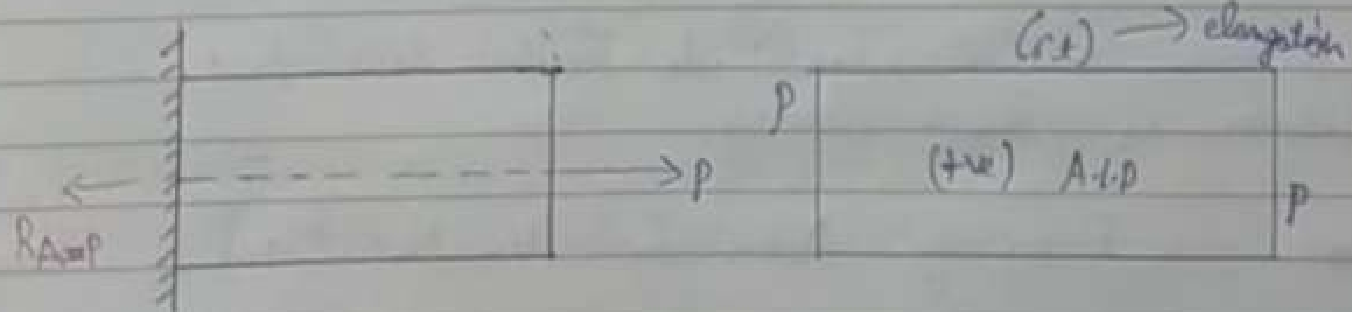
⇒ Sign Convention for Bending Moment Diagram.





M.R = Resisting Moment

$y = mx$ (Straight line Equation)



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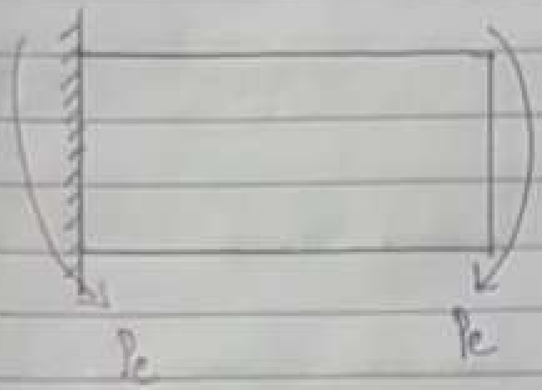
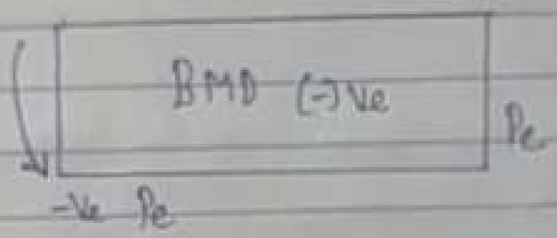




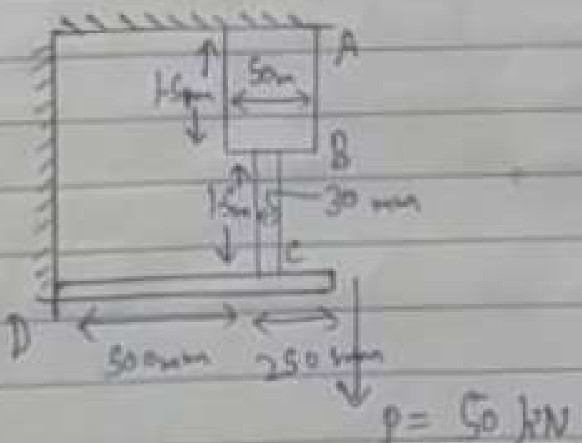
Total Stressed Produced In Eccentric Loading Condition

Total Stress Induced in this Case.

$$\sigma_{total} = \sigma_{(t,c)} + \sigma_b \leftarrow \text{bending Stress}$$

Ques A Beam Which is Subjected to a following Loading Condition. Calculate max Stress Induced and total elongation in the Bar.

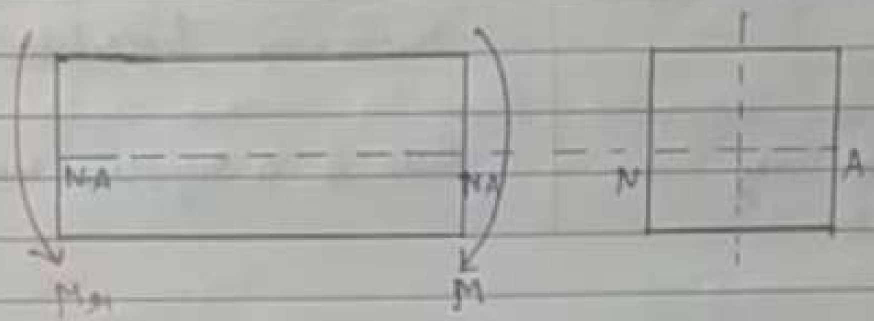
$E = 200\ GPa$ $\quad A = \frac{\pi}{4} d^2$



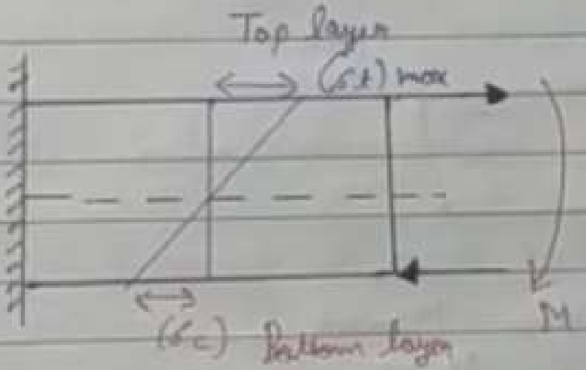
* Pure Bending :-

(i) ALD = 0
(ii) BMD = 0
(iii) SFD = 0

(a)



(b)



Pure Bending

$\boxed{-\text{Ve BMD}}$

Date ....

* Difference b/w Axial Stress Distribution and Bending Stress Distribution.

Axial Stress	Bending Stress
1. Stress is Same (-)ve or (+)ve at the Surface	In this Case Stress is Nonuniformed Distributed.
2. Stress is equal at every plane or point.	There is a resistive moment

* Pure Bending Equation

$$\boxed{\frac{M}{I} = \frac{\sigma_B}{y} = \frac{E}{R}}$$

Where

$M \rightarrow$ Resistive Moment

$\sigma_B \rightarrow$ Bending ~~Modulus~~ Stress

$E \rightarrow$ Young's Modulus

$I \rightarrow$ Moment of Inertia

$y \rightarrow$ Distance from the Neutral axis

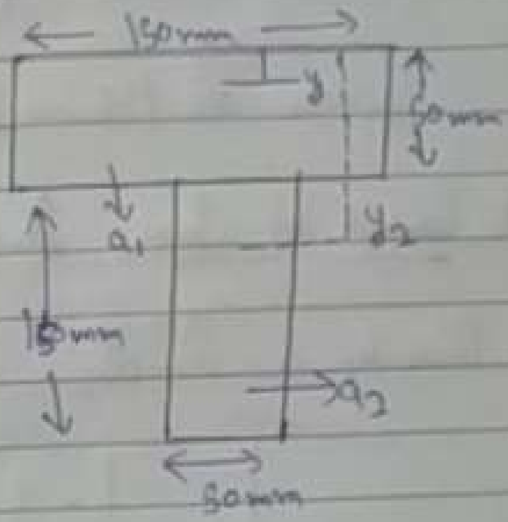
$R \rightarrow$ Radius of Curvature.

* Method for Finding the Position of N.A for I-Section and T-Section.

Consider T-Section as Shown in fig.

Date ....

Position of Neutral axis from = $\frac{\Sigma ay}{\Sigma a}$



When $a_1, a_2 \rightarrow$ are the area of diffrent Section

$y_1, y_2 \rightarrow$ are the Centroidal Distance from the Top.

$$N.A = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

⇒ Find the Neutral axis in fig.

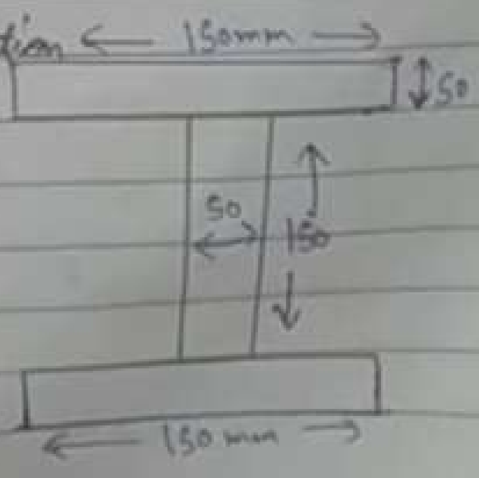
$$N.A = \frac{7500 \times 25 + 7500 \times 125}{7500 + 7500}$$

$$= 75$$

From bottom

$$N.A = 125$$

Que Calculate the position of N.A for I-Section



$$N.A = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

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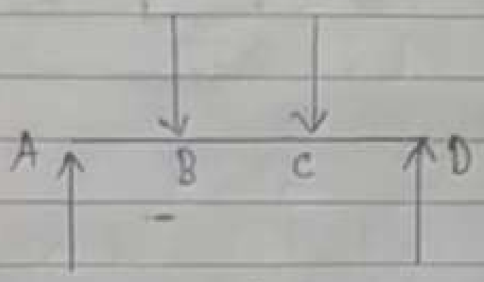
$$= \frac{150\times50\times25 + 150\times50\times100 + 100\times50\times220}{(150\times50)\times3}$$

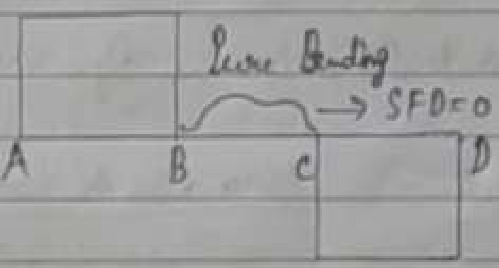
$= 125$ Ans

* Concept of Pure Bending in Simply Support Beam:-

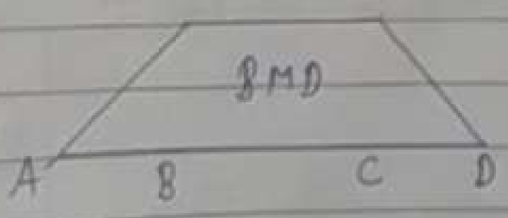
Consider a Simply Supported Beam as Shown in fig.



⇒



In this Condition a Beam position BC in under pure Bending Condition. We Can apply bending quation Only for BC portion Not AB not CB.

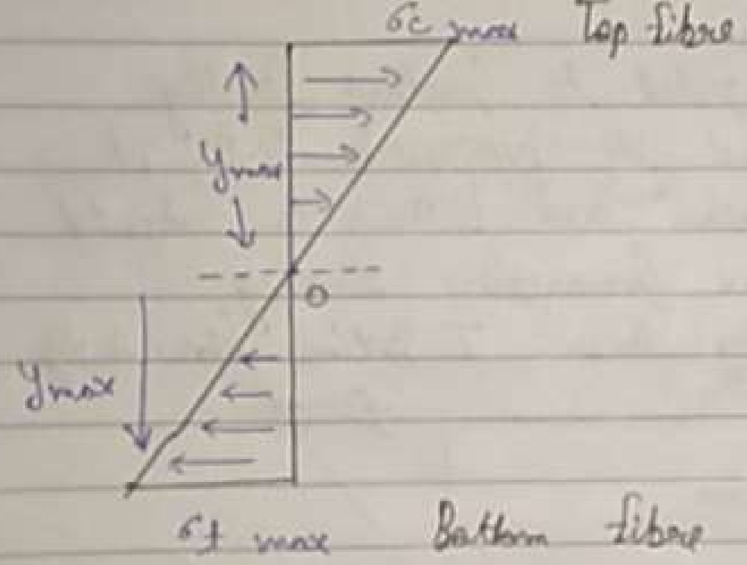


⇒ Pure Bending

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\boxed{\sigma = \frac{M}{I}} \qquad \sigma \propto \frac{y}{I}$$

In Case of Pure Bending we know that at N.A $\sigma_b = 0$



$$\sigma_{t\,max} \propto y_{max}$$

$$\sigma_{c\,max} \propto y_{max}$$

$$\frac{\sigma_{c1}}{\sigma_{c\,max}} = \frac{y_1}{y_{max}}$$

$$\sigma_{c1} = \sigma_{c\,max} \frac{y_1}{y_{max}}$$

$$\sigma_{t1} = \sigma_{t\,max} \frac{y}{y_{max}}$$

Date ....

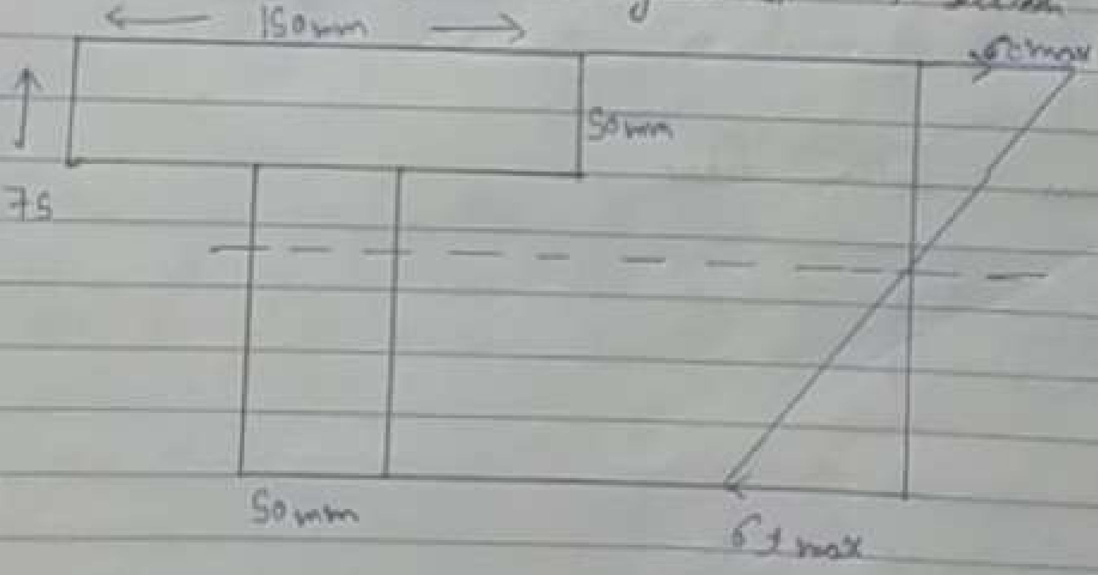
Que Calculate the ratio of σ_c to σ_t. When a Constant bending moment N is applied to up I-Section. Assume Top layer is Subjected to Compression and Bottom is Subjected to Tension.

Sol

$$\frac{\sigma_{c\,max}}{\sigma_{t\,max}} = \frac{y_{c\,max}}{y_{t\,max}}$$

1 : 1

Que A T-Section is Subjected to Constant Bending moment (3.4 kN m) around its horizontal N.A. If Top layer is Subjected to ~~tension~~ tensile and bottom is Subjected to Compression. Calculate max Compression Bending Stress and Tension Bending Stress. T-Section Show in fig.



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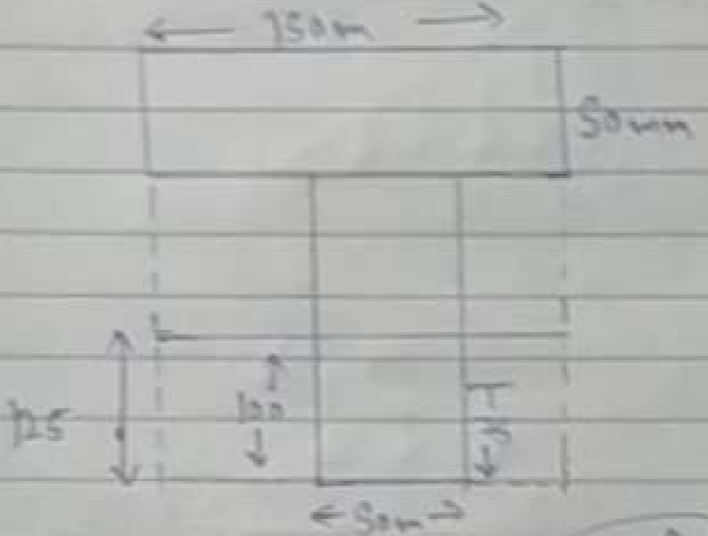
Note 1

For Rectangular Section $I = \frac{bd^3}{12}$

For Circular Section $I = \frac{\pi d^4}{64}$



$I \quad Ah^2$

$$I_{NA} = \frac{150 \times 200^3}{12} + 150 \times 200 \times 25^2 - 2\left[\frac{50 \times 150^3}{12} + 50 \times 150 \times 50^2\right]$$

$= 118750000 -$

$= 53125 \times 10^3$

$= 53125 \; mm^4$

$$\sigma_c = \frac{My}{I} = \frac{3.4 \times 75}{53125} = 4.8 \times 10^{-3} = 4.8 \; MN/m^2$$

$$\sigma_t = \frac{My}{I} = \frac{3.4 \times 125}{53125} = 8 \times 10^{-3} = 8 \; MN/m^2$$

Calculate Tensile force in T-Section.

$$\sigma = \frac{F}{A}$$

$$F = \sigma A = \frac{0+8}{2} \times 125 \times 50$$

$$= 4 \times 125 \times 50$$

$$\boxed{F = 25 \times 10^3 N}$$

* Section Modulus (Z)

It is the ratio of M.O.I to the distance from the N.A of bottom or Top fibre.

$$Z = \frac{M.O.I}{N.A \text{ distance}}$$